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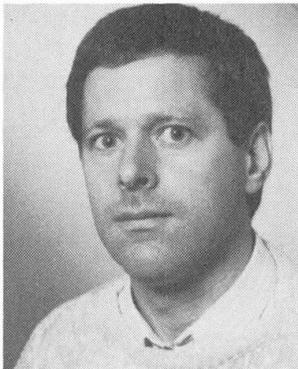
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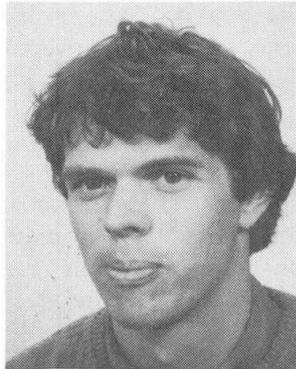
**Finite Element Supported Fracture Testing of Concrete**  
Essai de rupture du béton appuyé par un calcul par éléments finis  
Rechnergestützte Bruchuntersuchungen an Beton

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#### SUMMARY

This paper discusses the uniaxial tensile test on a concrete specimen. Two phenomena, i.e. non-symmetric crack opening and irregular descending branch as sometimes observed in this fracture test, were investigated by a numerical analysis. 'Structural behaviour' as already inferred from experimental results was confirmed by this analysis. It also showed how this behaviour influences the measured stress-deformation diagram. The paper demonstrates how experimental and numerical research can support each other when they encounter similar problems.

#### RÉSUMÉ

Cet article décrit l'essai de traction uni-axiale d'une éprouvette en béton. Deux phénomènes ont été examinés à l'aide d'une analyse numérique, notamment: l'ouverture de fissures non-symétriques et une courbe descendante irrégulière, telle qu'observée parfois dans de tels essais. Un 'comportement structural', expliqué par les résultats expérimentaux, a été confirmé par cette analyse. Il a également montré comment ce comportement influence le diagramme tension-déformation mesuré. L'article montre comment des recherches expérimentales et numériques peuvent se compléter lorsqu'elles rencontrent les mêmes problèmes.

#### ZUSAMMENFASSUNG

In diesem Beitrag wird der zentrische Zugversuch an Beton kritisch beleuchtet. Zwei Erscheinungen, nämlich die unsymmetrische Rissöffnung und ein unregelmäßig fallender Ast der Spannungs-Rissöffnungs-Linie, wie sie manchmal in diesem Versuch wahrgenommen werden, wurden in einer FE-Rechnung untersucht. Das 'Struktur-Verhalten', das schon früher aus Versuchsergebnissen abgeleitet wurde, konnte durch diese Berechnung bestätigt werden. Es wurde auch deutlich, wie dieses Verhalten die Spannungs-Rissöffnungs-Linie beeinflusst. Der Beitrag zeigt, wie sich experimentelle und numerische Forschung ergänzen, wenn ähnliche Probleme angetroffen werden.



## 1. INTRODUCTION

Due to developments in finite element techniques, research activities in laboratories for concrete structures are being more and more devoted to determining the material properties. Since the results of FE computations strongly depend on the correct input parameters it is very important to ascertain the actual material properties. One of these properties is the behaviour of concrete under tensile loading. The fact that concrete is a tension-softening material, which means that the stress beyond the peak load decreases with increasing deformation, makes investigation of concrete fracture rather difficult. Nevertheless, new achievements in the electro-hydraulic control of testing machines now enable complete stress-deformation curves to be determined. In addition, computational techniques have evolved so far that tracing the post-peak softening behaviour is no longer a problem.

The uniaxial tensile test is probably the most fundamental fracture test. It has been supposed that this test yields a stress-deformation diagram that includes all the fracture mechanics parameters, i.e. the tensile strength  $f_t$ , Young's modulus  $E_0$ , the fracture energy  $G_f$ , the shape of the descending branch and the maximum crack opening  $\delta_0$  at which stress can no longer be transferred [1]. Therefore, sometimes the name 'direct' tension test is used. Recently, however, some doubts have been raised about this assumption [2,3]. Due to a particular 'structural behaviour' as will be discussed in the next chapter, the crack opening in a uniaxial tensile test is non-symmetrically distributed over the specimen cross-section in some part of the loading path. As a result, the shape of the descending branch will be affected.

In this paper it is investigated whether this 'structural behaviour' can also be determined by means of a numerical analysis. Details of this analysis have been given before by Rots, Hordijk and de Borst [4]. The purpose of this paper is to discuss these numerical results in close relation to the experimental results. Therefore, the underlying computational and constitutive aspects will not be discussed in detail here. For more information on these aspects the reader is referred to de Borst [5] and Rots et al. [6] respectively.

## 2. THE BEHAVIOUR OF CONCRETE IN A UNIAXIAL TENSILE TEST

### 2.1 General

Concrete is an elastic-softening material (Fig. 1a). Straining concrete in uniaxial tension displays a linear stress-strain relation almost up to the peak. Then, beyond the peak a steep decay occurs which gradually evolves into a long tail. This decay is due to the development of one single crack in the specimen. The intention of a uniaxial tensile test is to create a crack, while the crack surfaces remain parallel to each other from the instant at which the first micro cracks are initiated until a crack opening  $\delta_0$  is reached at which no more stress can be transferred. In that case we assume the  $\sigma$ - $\delta_{tot}$  relation to be a material property. It should be noted, however, that a visible crack starts as a cluster of micro cracks which coalesce during further deformation. The deformation measurement is taken over a specimen slice which contains the softening zone. Consequently the measured  $\sigma$ - $\delta_{tot}$  relation is linked up with the applied measuring length of the gauges. Subtracting the

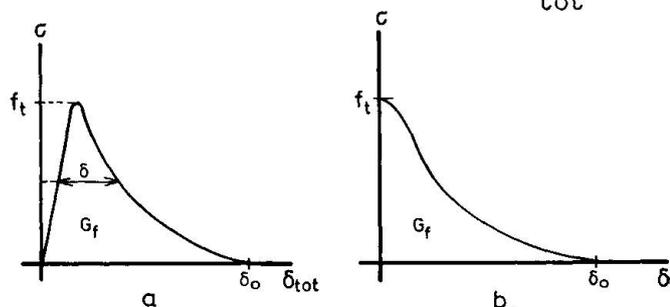


Fig. 1a a) Stress-deformation diagram.  
b) Stress-crack opening diagram.

elastic deformation over the gauge length yields the stress-crack opening displacement (Fig. 1b) which may serve as a basis for a crack model.

## 2.2 Non-symmetric crack openings and 'bump' in the descending branch

Two peculiar aspects have sometimes been observed in uniaxial tensile tests. First, a non-symmetric crack opening may occur in some part of the loading path. Fig. 2b shows a typical test result obtained on a lightweight concrete [2]. In this test a prismatic specimen 250 mm long, 60 mm wide and 50 mm thick was used. Two saw cuts 5 mm x 5 mm reduced the critical cross-sectional area to 50 mm x 50 mm.

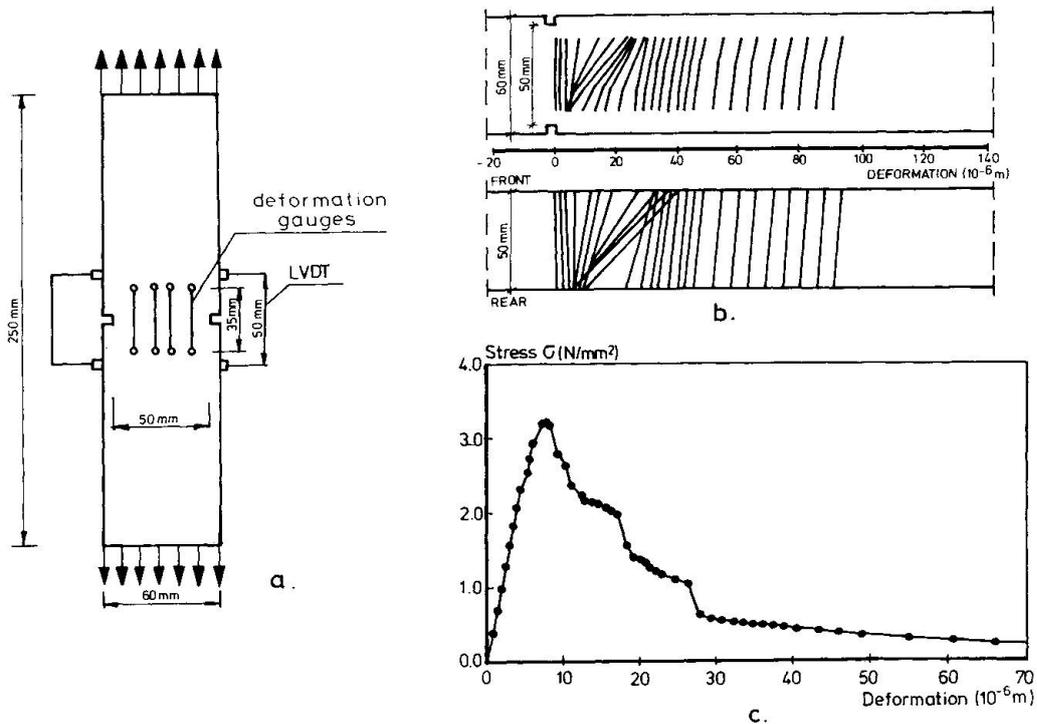


Fig. 2 Experimental result; a) specimen, b) deformation distribution and c) stress-deformation diagram.

For the deformation distribution eight deformation gauges with a gauge length of 35 mm were used, four on the front and four on the rear side of the specimen. It can be seen in Fig. 2b that in some part of the loading history the deformation distribution is non-symmetric even though rotation of the loading platens was prevented by a stiff guiding system. Such non-symmetric tensile fracture modes have also been reported by other investigators [3,7,8,9]. Gopalaratnam and Shah [10], however, reported symmetric modes.

The second peculiar aspect is that sometimes a 'bump' can be observed in the descending branch. As an example Fig. 2c displays two of these bumps. Similar results of bumps in the descending branch of stress-deformation diagrams have been reported by, for example, van Mier [3], Willam et al. [11] and Budnik [12].

## 2.3 Explanation of the observed phenomena

In [2] a qualitative model has been given that can possibly explain the phenomena described above. The basic idea of that model will be briefly summarized. Suppose that the stress-deformation relation for a small slice of concrete comprising the critical cross-section is as shown in Fig. 3a. In Fig. 3b it can



be observed that, for an average deformation  $\delta_{peak}$ , the applied force must be larger in case of a symmetric deformation distribution (solid line) than in case of a non-symmetric one (dashed line). Nevertheless a non-symmetric deformation as indicated by an angle  $\phi$  is limited by the rotational stiffness of the remaining part of the specimen in combination with the rotational stiffness of the loading platens.

In an experimental programme this model was verified by varying the specimen length [2]. It was concluded that there is a direct relation between the non-symmetric crack opening and the bump in the descending branch, on the one hand, and between the degree of non-symmetric crack opening and the rotational stiffness of the surrounding of the softening zone, on the other hand. Furthermore it became clear that the deformations must be studied three-dimensionally as appears also from Fig. 2b, in which deformation distributions projected on two perpendicular planes were plotted. By using very short specimens symmetric crack openings and a smooth descending branch were obtained.

The said phenomena were also observed by van Mier [3]. He explains them by the growth of a crack from one side of the specimen to the other. In his opinion the crack-arresting effect of large inclusions may cause a pronounced plateau (bump) in the descending branch. In addition, he suggests that the boundary condition is responsible for it because non-rotatable end-platens compel the crack, starting from one side, to jump to the other side.

### 3. FINITE ELEMENT ANALYSIS OF A UNIAXIAL TENSILE TEST

#### 3.1 General

A numerical analysis of a uniaxial tensile test may shed some new light on the response of concrete in such a test. Therefore a test performed in the Stevin Laboratory was simulated by Rots, Hordijk and de Borst [4]. Some typical results of their study will be discussed here in relation to the experimental observations. It was intended to investigate the observed phenomena rather than to fit an experimental result exactly. This would not even have been possible, since a two-dimensional analysis was performed, while the specimen in an experiment reacted three-dimensionally.

#### 3.2 Constitutive modelling

A smeared crack model as proposed by Rots et al. [6] has been used with a linearly elastic model for the concrete and a softening model for the crack. For the tensile softening a bilinear diagram was adopted as shown in Fig. 4. The fracture energy  $G_f$  was assumed to be a fixed material constant. As a smeared crack model was employed this energy is related to a certain crack band width, which in turn is related to the element configuration. The

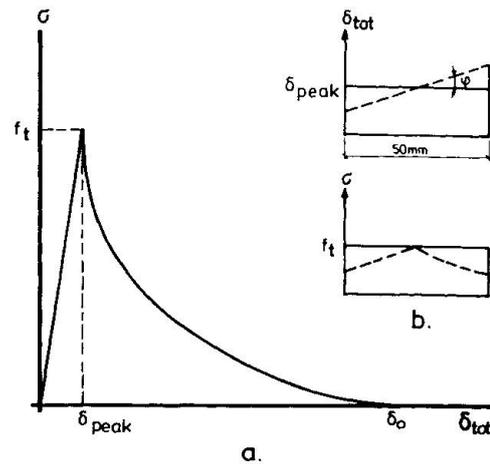


Fig. 3 a) Assumed  $\sigma$ - $\delta_{tot}$  relation.  
b) Deformation and stress distributions.

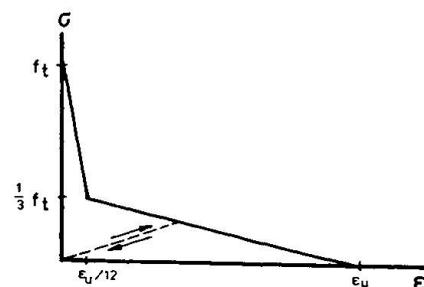


Fig. 4 Bilinear softening diagram.

necessary provisions were included to correctly release the fracture energy over the crack band width [13]. For the unloading and reloading a secant approach was used (Fig. 4).

### 3.3 Finite element idealization and material properties

The finite element mesh of the specimen as shown in Fig. 2a is given in Fig. 5. Four noded bilinear elements were used which were integrated using four-point Gauss quadrature. For the centre elements, where the fracture was expected to occur, a reduced centre-point integration scheme was used [6]. The lower boundary was assumed to be fixed, whereas the upper boundary was provided with a translational spring ( $k_t=148000 \text{ N/mm}$ ) and a rotational spring ( $k_r=10^9 \text{ Nmm/rad}$ ) in order to simulate the experimental conditions. Dependence relations were used to prevent distortion of the upper boundary which is in agreement with a rigid loading platen in an experiment.

In the experiment the load was applied at the upper boundary and was servo-controlled by a feedback signal from two LVDTs mounted on the sides of the specimen (measuring length 50 mm). In the analysis this control mechanism was simulated by using the averaged crack opening displacement as a control parameter. This procedure of 'indirect displacement control' has recently been proposed by de Borst [5]. Further, a full Newton Raphson iterative procedure was employed with the tangent matrix being updated before each iteration.

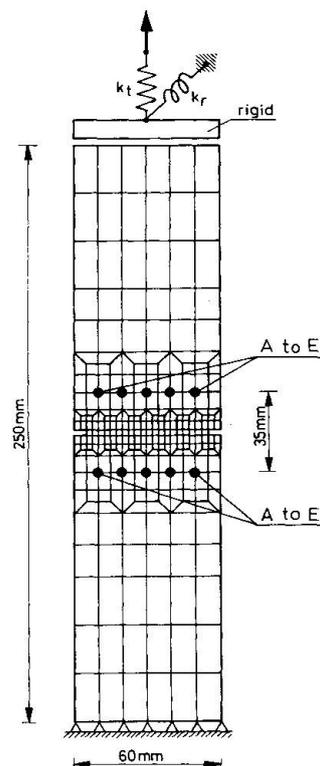
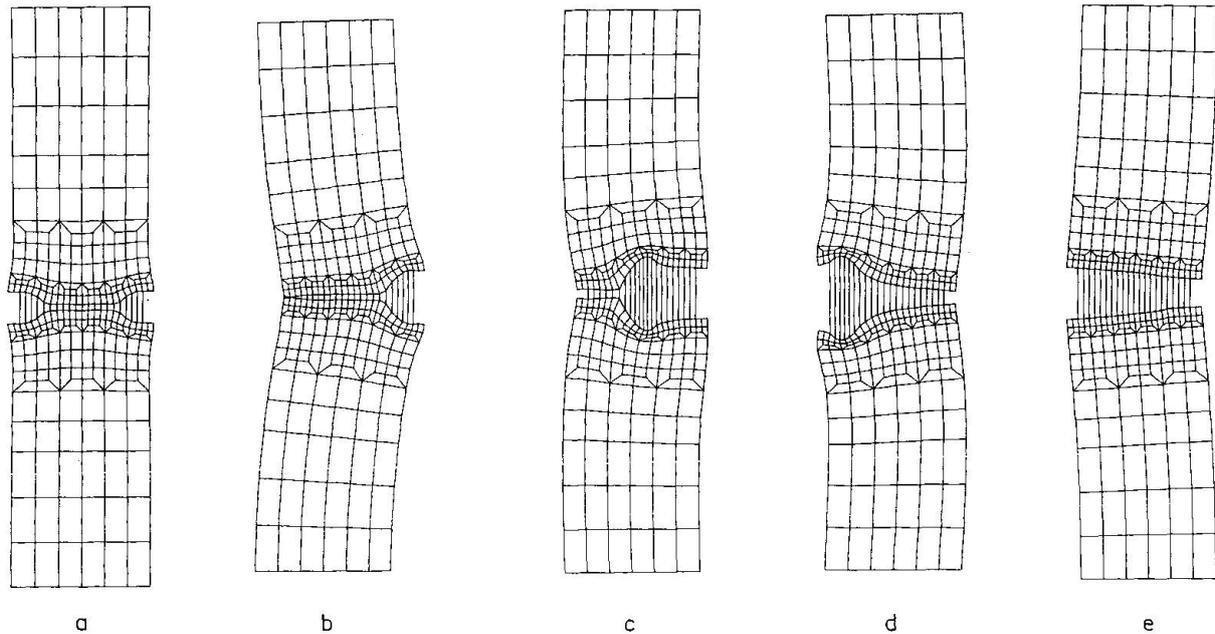


Fig. 5 Finite element mesh.

The elastic concrete properties were assumed to be: Young's modulus  $E=18000 \text{ N/mm}^2$  and Poisson's ratio  $\nu=0.2$ , corresponding to a lightweight concrete. The softening properties were taken as: tensile strength  $f_t=3.4 \text{ N/mm}^2$ , fracture energy  $G_f=59.3 \text{ N/m}$  and crack band width  $h=2.5 \text{ mm}$ . One element in front of the righthand notch was given a material imperfection by means of 1 percent reduction of  $G_f$ . The importance of this will become clear from the sequel of this paper.

### 3.4 Computational results

In conformity with the experiments, stress is given as the applied force divided by the central cross-sectional area, whereas deformation is the mean of five values measured between points A to E (Fig. 5). In previous experiments five instead of four deformation gauges were used on each side of the specimen. In Fig. 6 the incremental deformations are shown which refer to key events in the fracture localization process. Pre-peak deformations appeared to be symmetric (Fig. 6a). At an average stress  $\sigma=2.856 \text{ N/mm}^2$  a limit point was encountered. For this point a negative eigenvalue was calculated for the tangent stiffness matrix. In the corresponding eigenmode (Fig. 6b), which is identical to the incremental deformation field, a non-symmetric behaviour can be observed. Obviously, the side with the small imperfection opens, while the other side unloads. Upon further increase of the control parameter the load was decremented and a genuine equilibrium path was obtained. The fracture localization was propagated to the other side of the specimen (Fig. 6c) till the left side of the specimen tended to open suddenly, which resulted in a temporarily unloading of the right side (Fig. 6d). Subsequently, after a slight increase the load could decrease to zero while further the deformations were symmetric (Fig. 6e).

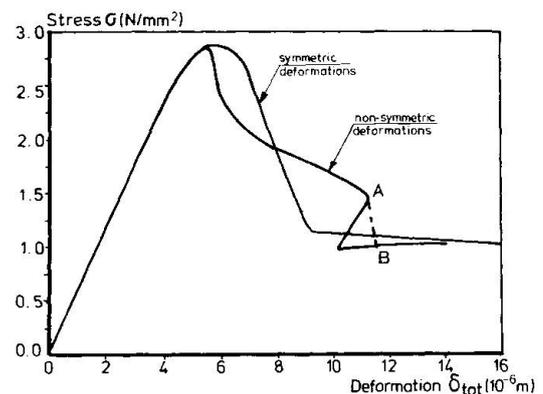


**Fig. 6** Eigenmodes for different loading points.

- |   |   |
|---|---|
| a) pre-peak , $\sigma=2.837 \text{ N/mm}^2$ | b) at peak , $\sigma=2.856 \text{ N/mm}^2$  |
| c) post-peak, $\sigma=1.865 \text{ N/mm}^2$ | d) post-peak, $\sigma=1.101 \text{ N/mm}^2$ |
| e) post-peak, $\sigma=1.026 \text{ N/mm}^2$ |   |

A stress-deformation diagram of two analyses has been plotted in Fig. 7. The first refers to the analysis with the imperfect material, the second belongs to an analysis with symmetric deformations. In the latter case the material imperfection was omitted. It appears that as a result of the non-symmetric deformations the peak load is reduced. More interesting, however, is the consequence for the shape of the descending branch which seems to be drastically affected. The resemblance with the experiment is evident. The 'bump' in the descending branch is now proven to be merely the result of the non-symmetric deformations.

In Fig. 7 an interesting phenomenon can be observed in the stress-deformation diagram for the non-symmetric solution. Beyond point A the deformation as well as the stress decreased. This phenomenon is called 'snap-back'. The importance of snap-back behaviour in elastic-softening materials was recognized before by, for example, Carpinteri [14] and de Borst [5]. It should be noted that the snap-back in the descending branch was found because the pure crack opening displacement was controlled. In the experiments the deformation was controlled by the average signal of the LVDTs ( $l_{\text{meas.}}=50 \text{ mm}$ ). Therefore a sudden drop as indicated by the dashed line A-B is mostly observed in the experiments (see Fig. 2). From the results of a post-peak cyclic test a snap-back as in Fig. 7 was already inferred [2].



**Fig. 7** Stress-deformation diagrams.



#### 4. DISCUSSION

In the numerical analysis for the non-symmetric solution a small imperfection was given to one of the elements. Of course concrete is a heterogeneous material containing a lot of imperfections and it is therefore concluded that the response of concrete in a tension test will always be associated with a non-symmetric state of deformations. The numerical analysis clearly demonstrates that the stress-deformation relation from a uniaxial tensile test can strongly be influenced by these non-symmetric deformations. The tensile strength and the shape of the descending branch no longer necessarily represent the actual material behaviour. In the event of sudden jumps, due to snap-back behaviour, also the area under the stress-deformation relation may be measured incorrectly. If this is so, the fracture energy  $G_F$  is incorrect.

In the numerical analysis with the imperfection non-symmetric deformations could be observed between  $\delta_{tot} = 5\mu\text{m}$  and  $\delta_{tot} = 14\mu\text{m}$ . For other values of  $\delta_{tot}$  the deformation distribution as well as the stress was equal to that in the symmetric solution (Fig. 7). Therefore we still assume that for crack openings in which these openings are symmetric the  $\sigma$ - $\delta_{tot}$  relation can be regarded as a material property. In this respect it can be mentioned that it is known from experiments [2] that the non-symmetric crack openings can be restricted to a small part of the loading history by means of a high rotational stiffness of the boundary of the softening zone. With very short specimens it was even possible to obtain symmetric crack openings in every loading stage. Further research activities should clarify whether the  $\sigma$ - $\delta_{tot}$  relation obtained on such short specimens can be regarded as a material property.

It has been shown that the observed 'bump' is caused by the boundary conditions, as suggested by van Mier [3]. In the case of hinges instead of non-rotatable end-platens the crack will probably continue to open from one side, resulting in a smooth descending branch, as has been discussed by van Mier [3]. It should not be difficult to check this numerically.

#### 5. CONCLUSIONS

- In a uniaxial tensile test on an elastic-softening material like concrete the crack opening will be non-symmetric in some part of the loading history.
- The phenomenon of non-symmetric crack openings can be regarded as 'structural behaviour'.
- Due to the structural behaviour in a uniaxial tensile test the measured stress-deformation relation cannot directly be regarded as a material property.
- A numerical analysis simulate the behaviour of concrete in a uniaxial tensile test, including the structural behaviour. Such an analysis can be used to investigate the influence of this structural behaviour on the material models derived from such a fracture test. Furthermore it can be used to improve the fracture test.
- In order to obtain post-peak stable softening behaviour in a displacement controlled uniaxial tensile test, very short gauge lengths have to be used.
- In numerical and experimental research the same types of problem can be encountered. Therefore the co-operation of these research fields should be stimulated.



## ACKNOWLEDGEMENTS

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