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Ultimate Load and Stability Analysis of Reinforced Concrete Shells

Charge de rupture et stabilité de coques en béton armé

Traglast- und Stabilitätsberechnung von Stahlbetonschalen

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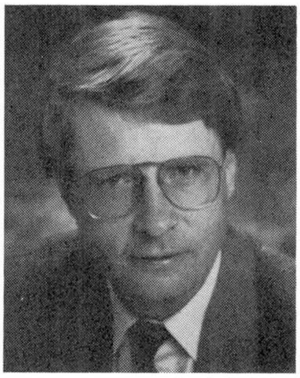


Ekkehard Ramm, born in 1940, earned his Dipl.-Ing. degree in civil engineering at the Technical Universities of Darmstadt and Stuttgart. He finished his Ph.D. in 1972 and his habilitation in 1976. Being a professor since 1976 he is now head of the Institute of Structural Analysis at the University of Stuttgart.

SUMMARY
The paper firstly discusses the fundamental behaviour of RC-shells in the ultimate load range which is characterized by a strong interaction of buckling and strength. It reviews current design procedures, few reported structural failures as well as RC model tests and finite element formulations for geometrically and materially nonlinear finite element analyses of RC-shells. Finally a brief description of one specific numerical model is given. It is applied to the ultimate load and stability analyses of conically shaped cooling towers.

RÉSUMÉ
Le rapport traite du comportement fondamental des coques en béton armé dans le domaine de la charge de rupture qui est caractérisé par l'interaction de la résistance moindre du matériau et du voilement. Il présente la pratique de projet actuelle, quelques cas de dommage et des expériences à l'aide de modèles en microbéton. Des calculs de coques en béton armé sur la base de la méthode des éléments finis sont présentés, en tenant compte de la non-linéarité géométrique et matérielle. Finalement un modèle numérique est décrit brièvement et appliqué aux calculs de charge de rupture et de stabilité des tours de réfrigération de forme conique.

ZUSAMMENFASSUNG
Der Beitrag diskutiert zunächst das prinzipielle Verhalten von Stahlbetonschalen im Grenzlastbereich, der durch kombiniertes Beul- und Materialversagen charakterisiert ist. Es wird ein Überblick gegeben über die gegenwärtige Entwurfspraxis, einige Schadenfälle, Modellversuche aus Mikrobeton und finite Elementformulierungen für geometrisch und materiell nichtlineare Berechnungen von Stahlbetonschalen. Schließlich wird ein numerisches Modell kurz beschrieben und auf Traglast- und Stabilitätsberechnungen von kegelförmigen Kühlturmschalen angewandt.

1. INTRODUCTION: BUCKLING OR STRENGTH?

RC - shells are extremely thin structures with radius to thickness ratios from 300 to 800, in particular if they are compared to classical domes or even natural egg shells with ratios up to 50 and 100, respectively. Therefore, it is obvious that each designer immediately is concerned that buckling may be a dominant phenomenon. However, most engineers have in mind the classical elastic stability problems when they think of buckling where the failure is usually caused by extreme symmetry in geometry, load, boundary conditions, stress state (uniform membrane) etc. Typical examples are the diamond shaped buckling of axially loaded cylinders or the snap - through behaviour of spherical shells under external pressure. It is natural that problems associated with buckling like imperfection sensitivity then have to be considered. This is the reason that for many RC - shell structures elastic model tests have been carried out in order to investigate the safety against buckling.

The question has to be raised whether this kind of buckling phenomenon can be met with RC - shells. It is well-known that the material behaviour may have a severe influence on stability, f.e. in the range of plastic buckling. The strong interaction can already be seen in the simple formula for classical linear buckling of shells with double curvature under external pressure:

$$p_{cr,ideal} = c \cdot E \cdot t^2/R_1 \cdot R_2$$

The buckling load depends on the material stiffness (Young's modulus E), the thickness t and the Gaussian curvature $1/R_1 \cdot R_2$. The factor c varies from one shell to the other, it is 1.15 for spheres. Quality and nonlinear behaviour of the concrete, creep and shrinkage, yielding of the reinforcement enter the formula via the material property E. The effective thickness is influenced by cracking, the percentage of the reinforcement and the number of layers (single or double). Moreover, creep may drastically change the original shape (flattening). All together these material effects may contribute more to the failure of the structure than the purely geometrical phenomenon of buckling.

Even if most people call a collapse of a shell structure in analysis, test or reality a buckling problem it is better to distinguish between the influence of material and geometrical nonlinearities. Therefore, let us call the collapse of a shell a buckling phenomenon when it is a finite deformation problem with little influence of the material failure and a strength problem when it is just the other way around (Fig. 1).

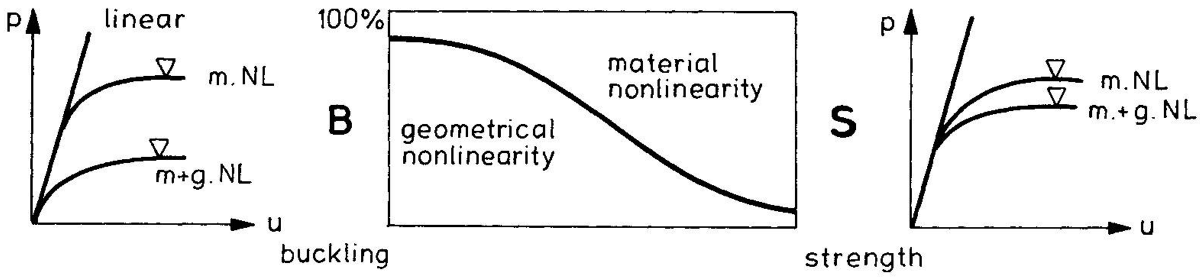


Figure 1: Contribution to Collapse

Unfortunately it is often not known in advance in what range the real structure has to be classified. However, certain parameters exist which qualitatively indicate the tendency to the one or the other kind of failure (Fig. 2). Many practical cases are located in the intermediate range where both effects influence each other.

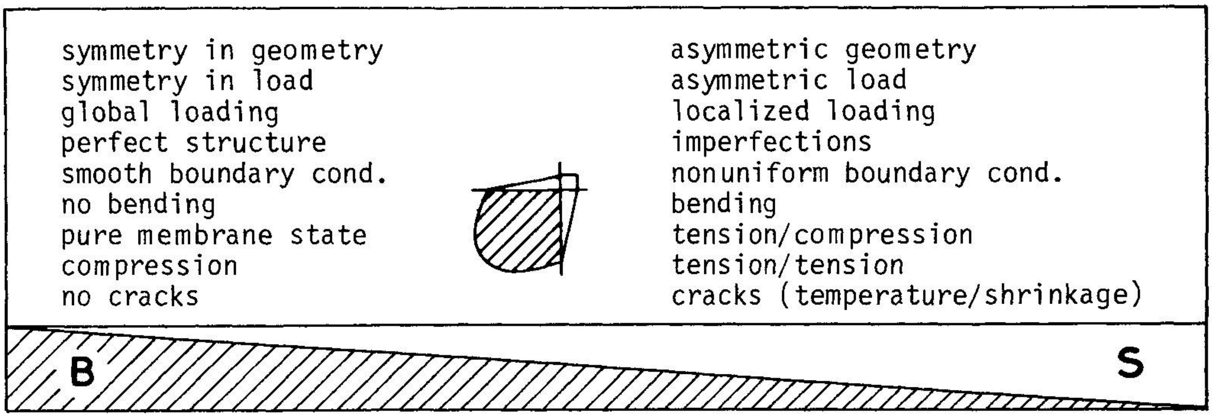


Figure 2: Buckling versus strength

The purpose of this paper is to review the literature with respect to this topic, to give some remarks to existing finite element models and first of all to call attention to this problem.

2. BASIC NONLINEAR STRUCTURAL BEHAVIOUR OF RC - SHELLS

An excellent compilation of the current state of understanding of concrete shell buckling is the ACI publication [1]. But the report also makes clear that beyond the classical type of buckling a considerable lack of information exists.

2.1 Current Design Procedures

Most codes on concrete structures only briefly stress the importance of shell buckling, enumerate several buckling load reducing effects and specify high safety factor, e.g. 5, in order to indicate the uncertainty of parameters and analysis (DIN 1045, ACI Standard 318). No details are given how the check against buckling has to be made. An exception are the IASS Recommendations [2] which are mostly based on the work of Dulácska [3]. The procedure contains five steps reducing the linear elastic buckling load of the homogeneous uncracked shell to the design load p (Fig. 3).

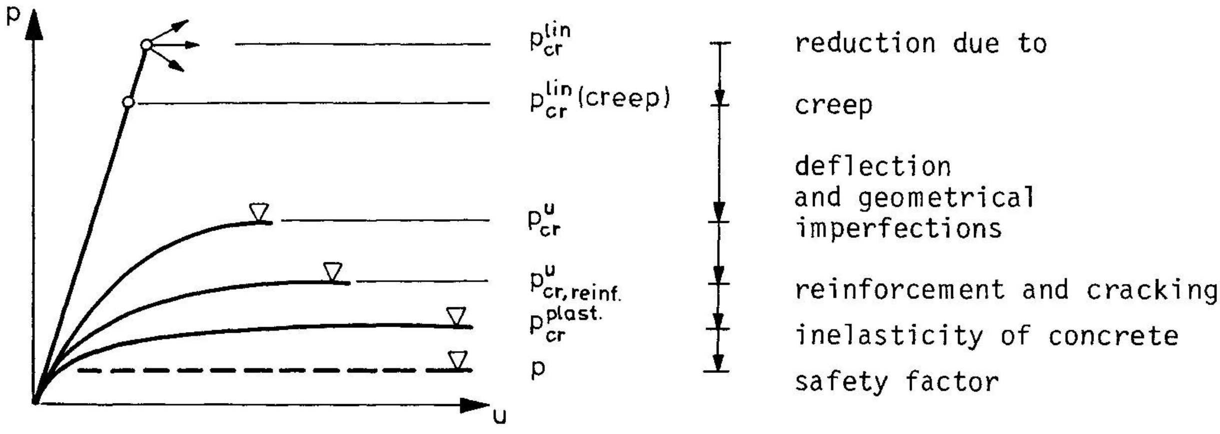


Figure 3: Buckling load according to IASS Recommendation [2]

The calculation which is essentially based on a local failure criterion does not cover the realistic situation because it accumulates all effects neglecting their different interactions. Despite the fact that it leads to a conservative design the scatter of results may be very large depending on the size of imperfections assumed. Reduction factors of less than 0.01 are possible. The Recommendations also address the possibility of one middle layer of reinforcement, a case which should not be used in practise due to unexpected local bending effects like concentrated loading (wind gusts), temperature change etc.

The situation with concrete cooling towers - even though more extensively investigated - is nearly the same [1], [4], [5]: independent design procedures against buckling on one side and yielding on the other side determining the wall thickness and the amount of reinforcement, respectively. The buckling analysis is mostly based on linear stability analyses or elastic model tests using reduction factors to account for imperfections, nonlinear behaviour, creep, cracking etc. In addition, high safety factors, e.g. 5, compared to the regular values of 1.75 for the yielding or the reinforcement, are introduced. In [6] it has been demonstrated through nonlinear finite element analyses that this discrepancy in safety factors is unrealistic since both effects strongly interact. A factor of safety 2.8 against buckling is proposed.

Although the more empirical approach is not satisfactory from the scientific point of view it has nearly always led to safe designs. A perfect example is the Swiss engineer Isler who has built more than 1400 concrete shells without any failure [7], [8].

2.2 Structural Failures

Very few failures of RC - shells have been reported (Table 1). Non of them can be attributed to buckling in the real sense. In most cases poor design and/or

Hungary [9], [10]	1954	EP 19 x 18 m	near collapse after 2 years, shell weakened by small glass skylights
Ferrybridge, GB	1965	cooling towers	poor design (membrane theory, working load design, one layer, no ring reinforcement)
Virginia [11]	1970	HP-gable shell 31 x 31 m	collapse after 7 years due to creep
Ardeer, GB	1973	cooling towers	low circumference reinforcement, vertical cracking due to thermal gradients
Latin America [12]	1975	EP 27 x 27 m	collapse after 4 days, poor concrete quality, significant geometrical imperfections, earthquake excitation
Port Gibson, USA	1978	cooling towers	damaged by toppling tower crane due to tornado
Berlin [13]	1980	HP	not a shell design, partly collapsed due to corrosion of tendons

Table 1: Failures of RC - shells

manufacturing can be made responsible, so that finally material failure caused the damage. On the other side there are several examples where well designed RC - shell structures withstood unexpected loadings, f.e. tornado (Port Gibson, 1978) or earthquake (Mexico, 1985).

2.3 Model Tests

The literature on small scale buckling tests of shells made of elastic material or metal is immense. In contrast to this very little information exists on model tests of RC - shells using microconcrete or mortar with and without reinforcement. In Table 2 some documented experiments are classified with respect to their kind of failure. This underlines the statements given in Fig. 2. If the structure is thick the crushing strength is decisive. If certain cracking is possible, for example due to boundary conditions, a combined buckling/material failure takes place. The more cracking is excluded and the thin structure is in a uniform compression state buckling becomes dominant. In this case the tangent modulus approach for buckling can be applied [18]. In [20] the important influence of creep on instability is stressed.

Schubiger [14]	ellipsoid (R), lateral load	628 *	combined buckling/strength
Bouma et al. [15]	cylindrical roof (R), lateral load	100	material failure (bending), small influence of geometrical nonlinearity
Distefano et al. [16]	HP (R), lateral load	a) shallow b) deep	a) buckling with material cracking b) pure buckling
Haas et al. [17]	cylinders (U), axial compression	50 - 120	material failure (crushing)
Griggs [18]	cylindrical roof (R), lateral load	238	combined buckling/strength
	spheres (R), ext. pressure	340	buckling with some material influence
	cylindrical panel (R), biaxial compression	200	
Müller et al. [19]	spheres (R), lateral load	a) 218 - 370 b) 303 perfect	a) material failure (bending) b) material failure (comp. strength)
Vandepitte et al. [20]	spheres (U), external pressure	~ 350	a) combined buckling/strength b) creep buckling

Table 2: RC - model tests (U = unreinforced, R = reinforced, * = r/t)

2.4 Nonlinear Analyses

Very few existing RC - shells have been investigated by a fully nonlinear analysis taking into account geometrical as well as material nonlinearties. The reason is that numerical models which are to some extend reliable came up only recently (see Chapter 3.1). Here few selected examples are mentioned. In the work of Scordelis and Chan [11], [21] an HP - gable shell is investigated which is patterned from a real structure. The collapse analysis indicates a strong interaction between both nonlinearities; it also points out the severe influence of creep on the ultimate load.

Significant work on the analysis of cooling towers under dead and wind load has been reported [4] but, as already mentioned in Chapter 2.1, most is based on elastic bifurcation and geometrical nonlinear analyses. For example a buckling criterion, the so-called buckling stress state (BSS), in conjunction with an equivalent axisymmetric stress approach is proposed in [22]. It has already been pointed out by Mang [23] that this assumption leads to the wrong conclusion that the structure would fail by buckling due to biaxial compression. Through elaborate materially and geometrically nonlinear analyses of two built cooling towers the authors in [24], [25] demonstrated that the loss of structural integrity is caused by cracking of the concrete on the windward side with some subsequent redistributions of stresses in the postcracking range. It is rather a material failure with little influence of large deformation effects than a buckling problem. This has been confirmed for the same cooling tower in [26] where in addition the noticeable influence of tension stiffening has been investigated.

For conical type of cooling towers see Chapter 3.2.

3. FINITE ELEMENT MODELS FOR RC - SHELLS

3.1 Review

The brief review is restricted to large deformation finite element models of general RC - shells. It is not considered to be complete. Either flat, curved shell or degenerated solid elements are applied. Due to the size and complexity of the problem neither a microscopic nor a macroscopic modelling is used. It is rather an intermediate type of idealization. That means that neither discrete cracks, strain localization or individual rebars on one side nor material laws defined in stress resultants like moment curvature relationships on the other side are introduced. To the author's knowledge all models use a smeared crack, layered approach. In each individual concrete layer a 2D stress state is assumed, in most cases referring to Kupfer's 2D failure envelope. The majority (Table 3) applies a nonlinear elastic, orthotropic material model introducing the equivalent uniaxial strain concept by Darwin and Pecknold. The fact that this semi-empirical formulation violates invariance requirements seems to be of little consequence since the principal stress direction does not rotate very much. Nearly all models assume a tension stiffening effect, either referred to the concrete or to the steel and use a fixed or variable shear retention factor after cracking. The steel layers always have uniaxial properties based on a bi- or multilinear stress - strain curve with hardening and elastic unloading. Large deformation effects are covered in the conventional way as in elastic analysis.

The assumption of a 2D stress state is certainly justified for most shell problems in which the load is mainly carried by membrane action. But it has to be noted that certain limitations exist: All stress states which deviate from the 2D situation like concentrated loading or localized support conditions cannot be properly analysed. For such local problems the design anyway requires special care, f.e. stirup reinforcement. In this case it is necessary to increase locally the strength reflecting the existence of confined concrete. Otherwise premature failure occurs. Another possiblity is to resort to a 3D concrete model.

	ref.	material model	tension stiffening	extras
Ghoneim/Ghali	[33]	NE	C	prestress, creep, shrinkage
Arnesen/Bergan	[28]	EC	-	combined tensile strain/stress, criterion f. cracking
Floegl/Mang	[24], [29]	NE	S (bond slip)	influence of stress gradient
Chan/Scordelis	[21], [11]	NE	S	creep, shrinkage
Kompfner/Ramm	[30], [31]	NE	C	-
Figueiras/Owen	[32]	P (Kupfer)	C	w. & w/o hardening
Milford/ Schnobrich	[26], [27]	NE	S	rotated crack model
Cervera/Abdel Rahman/Hinton	[34]	P (v.Mises)	C	rotated crack model

Table 3: Large deformation RC - shell models
(NE: nonlinear elastic model, EC: endochronic model,
P: plasticity model, C/S: concrete/steel referred)

3.2 Present Model

The present model is described in detail in [30], see also [31]. The main characteristics of the formulation and the concrete model which is essentially an extended Darwin/Pecknold model are summarized in Tables 4 and 5.

formulation	arbitrarily large deformation, material mode (T.L.), incremental/iterative
iteration scheme	standard, modified, quasi Newton, load-, displacement-, arc-length-control, line search
shell element	isoparametric displacement model, degenerated solid, linear, quadratic or cubic interpolation (serendipity, Lagrange), full or reduced integration (Gauss), layered model (Simpson's integration)
material model	concrete: short time, nonlinear elastic, orthotropic, equivalent uniaxial strain concept (Darwin/ Pecknold), tension stiffening steel: smeared layers with uniaxial properties, multilinear, elasto-plastic, isotropic hardening

Table 4: RC - shell element formulation [30], [31]

The failure envelope renders the limit stresses σ_{ic} for each stress ratio α. Together with the limit strain ε_{ic} taken from test results it defines the corresponding stress - equivalent uniaxial strain curve, from which the material stiffness E_i is taken. In the finite element formulation nonproportional loading in each individual point cannot be avoided. In this case the stress - strain curve for that point varies. E_i is found according to the actual stress instead of the actual uniaxial strain; in the descending portion E_i is set to zero.

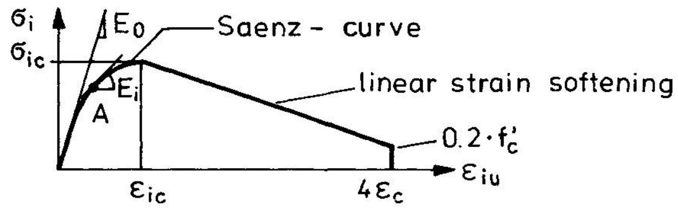
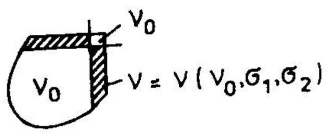
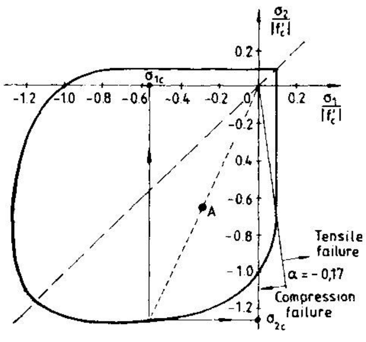
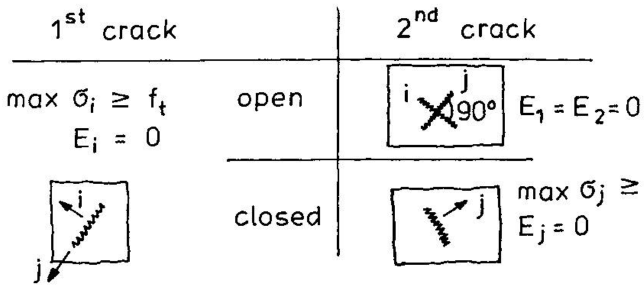
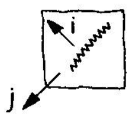
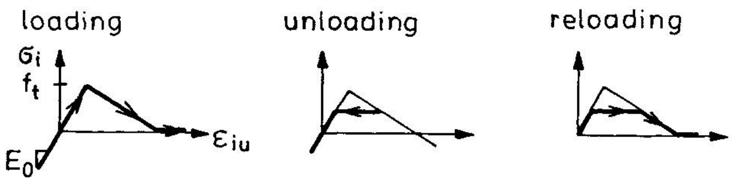
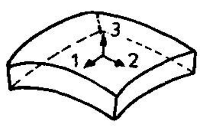
stress-equivalent uniaxial strain curve in compression	
biaxial failure envelope (Kupfer/Gerstle) 	
cracking fixed crack approach	
tension stiffening 	
constitutive matrix for shell element	 $\sigma_{33} = 0$; $\varepsilon_{33} = 0$ $G_{transverse} = \frac{5}{6} \cdot G_{inplane}$

Table 5: Concrete material model

Cracking follows the usual maximum principal stress criterion. Tension stiffening is included in a straightforward way. In the locally defined constitutive matrix the zero stress/strain condition is enforced. The inplane and transverse shear moduli are automatically adjusted according to the incremental orthotropic material tensor.

4. NUMERICAL EXAMPLES

In [30] ultimate load analyses of cylindrical roof shell and an HP-gable shell are described applying this model.

Recently in two power plants near Stuttgart two so-called hybrid (dry/wet) cooling towers have been built (Figures 5 and 7). Both towers with different size consist of a conventional (frame/ring wall) base structure with many openings and a shell shaped as a conical frustum, the latter being investigated in the following study, for details see [35]. The shell thickness is 16 cm (30 cm) for tower I (II); only in the lower part 3.06 m (4.50 m) it is increased to 30 cm (50 cm). In each production cycle one quarter section with a height of 1.45 m is poured. The extreme loading is dead load, wind - taken constant along the meridian - and concentrated loads at the free edge due to scaffolding on 90° with added life load and fresh concrete. Preliminary studies have shown:

* The structural response is completely different from that of conventional hyperbolic cooling towers.
* The critical period is the phase before the upper ring is built, leading to a free edge boundary condition at the critical height h_{cr}.
* The concentrated loading at the top could be localized at the free edge in order to simplify the input data.
* Linear elastic buckling analyses restricted to axisymmetric modes lead to unrealistic high buckling loads.
* The results are almost not influenced by the boundary conditions of the lower edge (clamped or hinged).

The material properties of both towers are given in Figure 4.

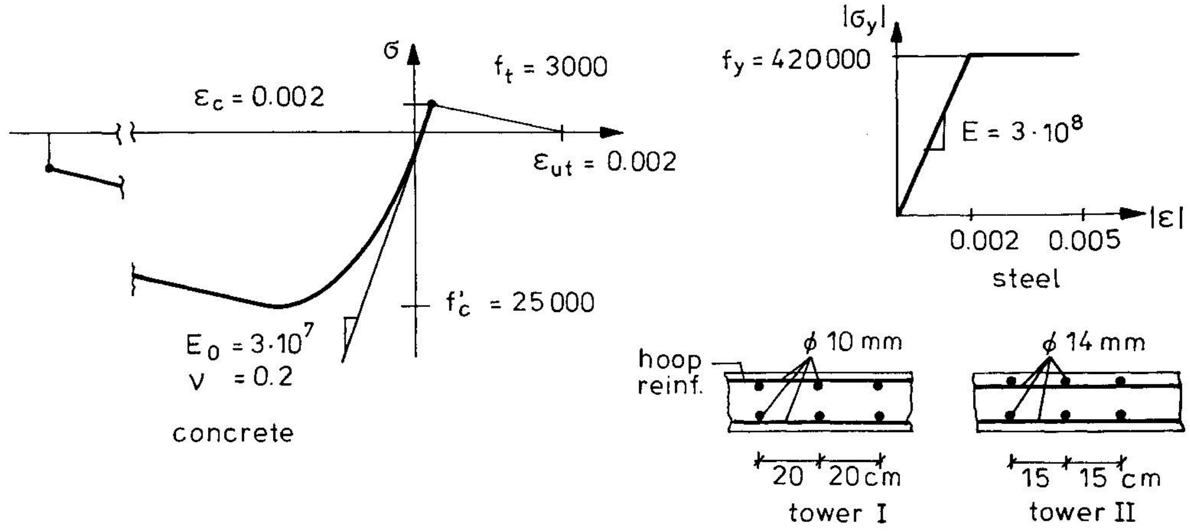


Figure 4: Material properties [kN/m²]

4.1 Cooling Tower I (Altbach)

The base structure of the small tower (Fig. 5) could be considered as very stiff. Therefore, clamped boundary conditions at the lower edge are introduced. As a conservative approach uniform thickness and axisymmetric loading is assumed. Linear elastic buckling analyses lead to a critical load factor λ = 15.7 with

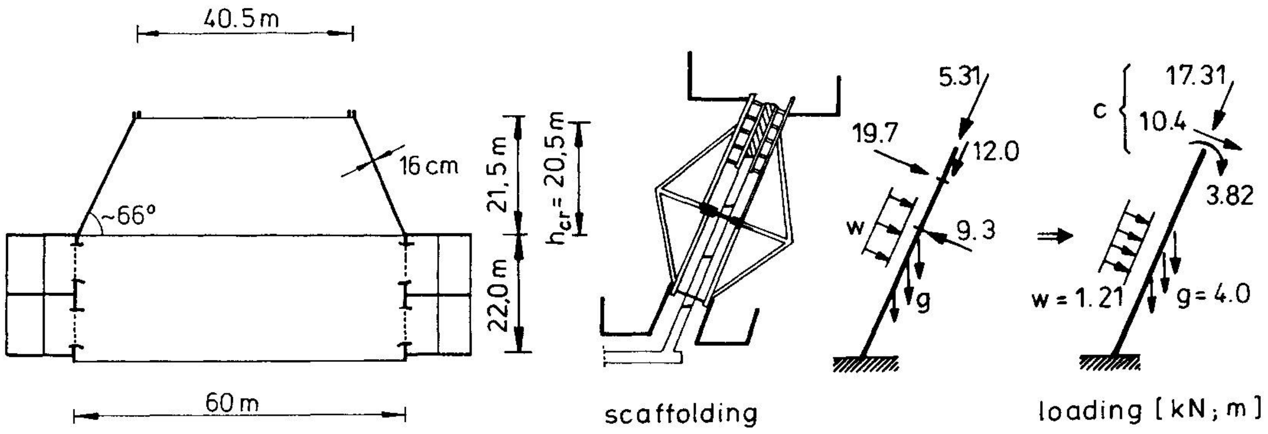


Figure 5: Cooling tower I (Altbach)

a buckling mode of 8 waves in hoop direction concentrated near the free edge. The corresponding stresses are far beyond the compressive strength of the concrete. A geometrically and materially nonlinear axisymmetric analysis of the perfect shell resulted in a failure load g + 5.4 (w + c). The load factor is reduced to 5.0 if tensile strength and ultimate strain are reduced (f_t = 10 kN/m²; $\varepsilon_{ut} = f_t / E_o$). Next nonsymmetric geometrical imperfections corresponding to the buckling wave pattern (n = 8) with a maximum amplitude of ± 5 cm are introduced. One half wave sector is modelled, assuming a reduced Young's modulus $E_o = 2.8 \cdot 10^7$ kN/m² for the upper 1.4 m and tension cut-off (f_c = 10 kN/m²; ε_{ut} = 0). These extreme conditions lower the load factor to 4.3 (Fig. 6). Cracking is concentrated to the upper ring portion.

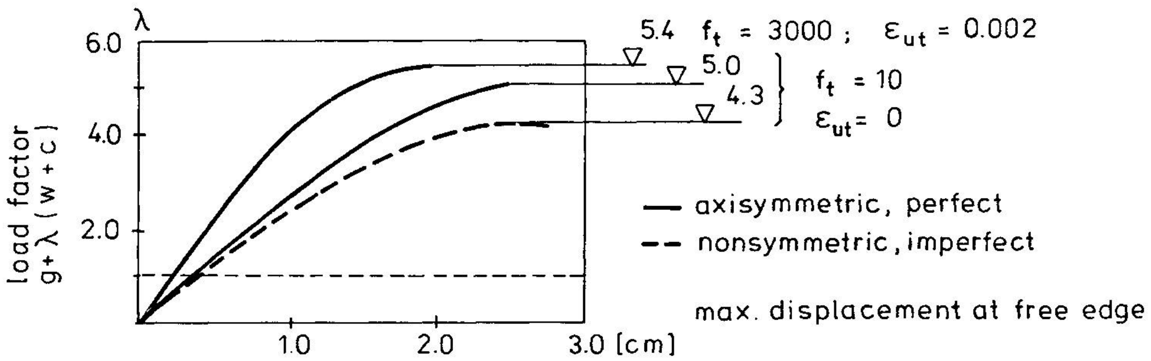


Figure 6: Load displacement diagram

4.2 Cooling Tower II (Neckarwestheim)

In contrast to tower I the base structure of cooling tower II (Fig. 7) is very flexible. Despite its size and dimensions it is a relatively slender contruction in which many precast elements are incorporated. Therefore, the shell itself has

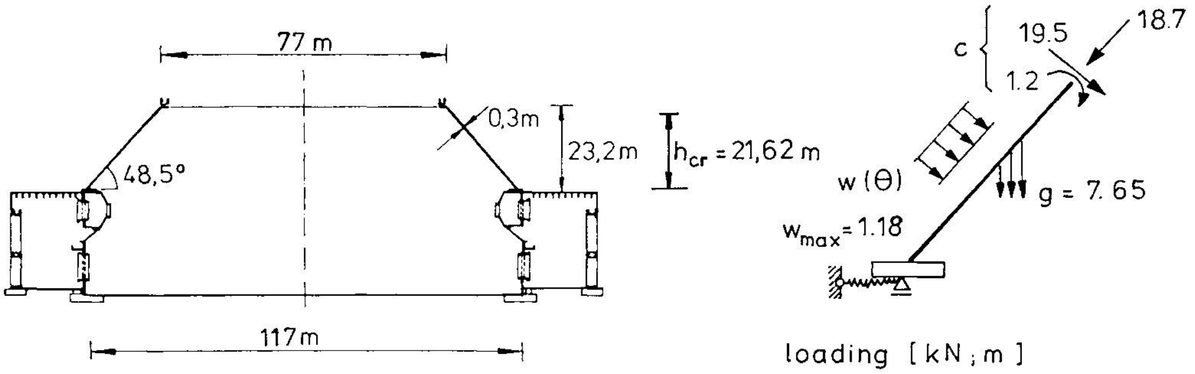


Figure 7: Cooling tower II (Neckarwestheim)

a lower ring beam (4.00 x 0.75 m; A_s = 482 cm²), but already under dead load the beam is partly cracked (state II) so that a reduced membrane and bending stiffness is introduced for the linear buckling analyses (EI^I/EI^{II} = 4.4; EA^I/EA^{II} = 8.5). Assuming that the ring/shell structure is hinged and radially as well as tangentially unrestrained - i.e. neglecting the stiffness of the base structure - a linear buckling analysis with axisymmetric loading leads to bifurcation loads 6.17·(g + w + c) or g + 13.50 (w + c) with five buckling waves in hoop direction. In this case a nonlinear study of a half wave sector of the imperfect shell - as for tower I - did not reflect the real situation since unsymmetric loading of wind and scaffolding causes considerable inextensional deformations (ovalization) due to the flexible base. Therefore, one half of the shell was modelled by a non-uniform mesh 15 x 32 eight-node shell elements and 32 quadratic beam elements for the edge beam which are compatible to the shell elements. Again the bending stiffness of the edge beam is reduced to that of state II; the membrane stiffness is restricted to the steel reinforcement alone. Regarding the different age of concrete the Young's modulus of the upper portion (2.9 m) of the shell is lowered by 20 percent. Now the base structure is simulated by radial and tangential springs at each node. The spring stiffnesses of k_r = 25675 kN/m and k_t = 8190 kN/m taken from a preliminary linear study of the entire structure under unfavourable conditions have been found essential for the safety of the structure. The concentrated load c of the scaffolding over a 90°-sector was located at the free edge on the windward side. Wind load and suction in hoop direction are defined in the following way:

$$w = (c_p \cdot 1.01 + 0.53)\ \text{kN/m}^2$$

$$\text{with} \quad c_p = \begin{cases} 1 - 2.1 \cdot [\sin(\Theta \cdot 90/71)]^n & 0 \leq |\Theta| \leq 71^\circ \\ -1.1 + 0.6 \cdot [\sin(90 - 71) \cdot 90/22]^n & 71^\circ \leq |\Theta| \leq 90{,}4^\circ \\ -0.5 & 90{,}4^\circ \leq |\Theta| \leq 180^\circ \end{cases}$$

$$n = 2.395$$

The wind load is assumed constant along the meridian. Geometrical imperfections correspoding to the first buckling mode with a maximum horizontal amplitude of ± 10 cm were superimposed. For this a linear buckling analysis of the structure under nonsymmetric loading assuming a fixed lower boundary has been performed. Few circumferential waves are concentrated at the windward compression zone. In Figure 8 two materially nonlinear analyses with and without large deformation effects are compared indicating the considerable influence of the geometrical

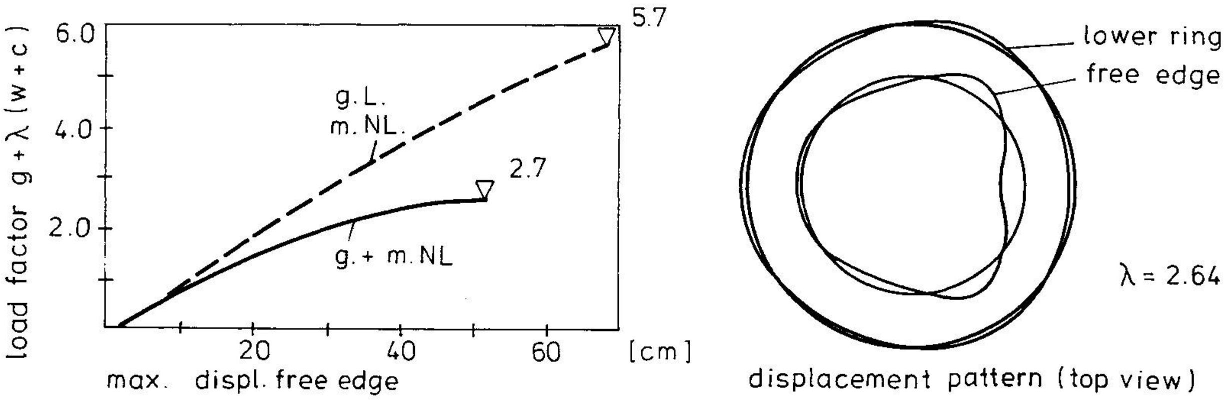


Figure 8: Load displacement diagram/ failure mode

nonlinearity. The ultimate load is g + 2.70 (w + c). The ovalization of the entire shell can be seen in the failure mode (Fig. 8). A supplementary study with imperfections of ± 15 cm and a different loading sequence rendered an ultimate load of 1.45 g + 1.45 w + 2.51 c.

According to Figures 1 and 2 tower I could be classified as a primary strength problem whereas due to the flexible base structure tower II is located in the combined buckling/strength range.

5. CONCLUSION

The present study has shown:

* The knowledge of the fundamental response of RC-shells in the ultimate load range is still limited. Therefore, the question of reliable safety factors against failure is not yet answered.

* The current design procedures with a more or less empirical coupling of buckling with material failure is unsatisfactory.

* Elastic buckling analyses or tests are of limited value for RC-shells. They are necessary but not sufficient.

* The current development of numerical oriented RC material models including large deformation effects are a promising alternative to the current procedure.

* High quality of analysis based on conservative assumptions in loading, imperfections, boundary conditions, material properties allows to reduce safety factors against failure, e.g. to 2.5. However, these analyses are still expensive and need a lot of experience.

Further research is needed

* to gain further information on the basic nonlinear structural behaviour of RC - shells and

* to further improve existing or to develop new nonlinear material formulations.

ACKNOWLEDGEMENTS

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