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## Tests of Size Effect in Pull-Out of Reinforcing Bars from Concrete

Influence de la dimension lors de l'extraction des barres d'armatures du béton

Versuche zum Masstabseinfluss auf das Verbundverhalten von Bewehrungsstäben

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### SUMMARY

The paper gives a brief preliminary report on pull-out tests of geometrically similar specimens of very different sizes. The test results indicate a significant size effect which can be described by Bažant's approximate size effect law.

### RÉSUMÉ

L'article donne un bref rapport préliminaire d'essais d'extraction de spécimen géométriquement semblables mais de dimension très différentes. Les résultats d'essais montrent un effet très important de la dimension qui peut être décrit comme la loi de l'effet de la dimension approximative de Bažant.

### ZUSAMMENFASSUNG

Das Ziel dieses Beitrags ist, einen kurzen Bericht über Ausziehversuche an geometrisch ähnlichen Probenkörpern verschiedener Grösse zu geben. Die Ergebnisse zeigen einen deutlichen Massstabseinfluss, der mit dem Modell von Bažant beschrieben werden kann.

### Introduction

The pull-out failure of reinforcing bars embedded in concrete is known to be a brittle failure in which the maximum load decreases after its peak value rather than remaining constant. For this type of failure, theoretical analysis shows there should be a significant size effect, such that the nominal stress at failure decreases as the specimen size increases. Although the fracture mechanics aspects of pull-out failures have been pointed out by various authors, e.g., by S. P. Shah and A. Ingraffea, and the presence of fracture mechanics type size effect have been suspected, no test data which would clearly document this effect have apparently been obtained so far. Experimental verification requires testing specimens that are geometrically similar and a brief preliminary report on such tests which were carried out at Northwestern University is the purpose of this paper.

### Tests and Their Evaluation

The test specimen was a cube with a steel bar embedded in it, parallel to one edge of the cube and sticking out at the center of one face. The sides of the cubes tested were  $d = 1.5\text{ in.}$ ,  $3\text{ in.}$ , and  $6\text{ in.}$ , and the reinforcing bars were scaled in proportion to the size of the cubes, having nominal diameters  $0.125\text{ in.}$ ,  $0.25\text{ in.}$ , and  $0.5\text{ in.}$  respectively ( $1\text{ in.} = 25.4\text{ mm}$ ). The yield strength of steel was  $60,000\text{ psi}$  ( $1\text{ psi} = 6895\text{ N}$ ). The embedment lengths were  $0.5\text{ in.}$ ,  $1\text{ in.}$ , and  $2\text{ in.}$ , respectively, which ensured the specimen to fail before the yielding of the bars could occur. One specimen of each size was cast from the same batch of concrete. The concrete mix ratio of water:cement:sand:-gravel was  $0.6:1:2:2$ , by weight. The maximum gravel size was  $d_a = 0.25\text{ in.}$ , and the maximum sand grain size was  $0.132\text{ in.}$ . The aggregate was crushed limestone, and the sand was a siliceous river sand. The aggregate and sand were air-dried prior to mixing. Portland cement C150, of ASTM Type I, with no admixtures, was used. Companion cylinders of  $3\text{ in.}$  diameter and  $6\text{ in.}$  length were cast from each batch of concrete. The mean compression strength of these cylinders after standard 28-day moist curing was  $f' = 6,650\text{ psi}$ . The cubes were removed from their plywood forms  $1\text{ day}$  after casting and were subsequently cured in a moist room of relative humidity 95% and temperature  $78^\circ\text{F}$  for 28-days. Then the tests were made in a 60-ton Baldwin frame modified as a servo-controlled closed loop machine with an MTS controller. The ends of the steel bars were gripped in the machine, and the reaction was provided by a square sleeve made of split reinforcing bars as shown in Fig. 2, the sides of these squares being  $0.5\text{ in.}$ ,  $1\text{ in.}$ , and  $2\text{ in.}$  for the small, medium and large sizes. These reaction square sleeves were glued to concrete by epoxy shortly before testing.

Orangun et al. [1] developed for the bond strength the formula:

$$v_u = k_1 [1.22 + 3.23 C/d_b + 53d_b/\lambda_d] \sqrt{f'_c} \quad (1)$$

in which  $v_u$  = 28-day bond strength in psi,  $C$  = minimum clear cover of concrete in inches,  $\lambda_d$  = embedment length in inches,  $d_b$  = nominal bar diameter in inches,  $f'_c$  = standard 28-day cylinder strength of concrete in psi, and  $k_1$  = an empirical coefficient. According to Bažant's approximate size effect law for failures due to distributed cracking, this formula may be extended as follows:

$$v_u = C_1 k_1 \left(1 + \frac{d_b}{\lambda_0 d_a}\right)^{-1/2}, \quad C_1 = (1.22 + 3.23 \frac{C}{d_b} + 53 \frac{d_b}{\lambda_d}) \sqrt{f'_c} \quad (2)$$

in which  $d_a$  = the maximum size of aggregate and  $\lambda_0$  = a coefficient

characterizing the center of the transition from failures dominated by plastic limit analysis to failures dominated by linear elastic fracture mechanics. The formula of Orangun et al., yielded better agreement with the present test results than other formulas such as that of ACI [5] or of Aboona [6]. The nominal bond strength given by Eq. 2 is defined as  $v_u = T / s \ell_d$  where  $T$  = maximum tensile force in the bar and  $s$  = nominal surface area of the reinforcing bar embedded in concrete.

Eq. 2 can be algebraically rearranged to a linear plot in which  $Y = (C_1/v_u)^2$  is plotted versus  $X = d_b/d_a$ . The present test results, listed in Table 1, are plotted in this manner in Fig. 3 (left). The optimum fit can be obtained by linear regression, and the regression parameters are given in the figure along with the coefficient of variation of the vertical deviations from the regression line  $w_{Y|X}$ , the correlation coefficient  $r$ , and the coordinates  $X$  and  $Y$  of the data centroid. Based on linear regression, one may then plot the diagram of  $\log(v_u/C_1)$  versus  $\log(d_b/d_a)$ , as shown in Fig. 3 on the right. The test results in these plots represent the individual tests made. If the averages for each size are plotted, the scatter is of course considerably reduced.

#### Conclusions

Despite the significant scatter of the test results, probably inevitable in this type of test, the size effect is clearly apparent. Although the test results cannot be said to validate the applicability of the size effect law, they are nevertheless described by this law adequately.

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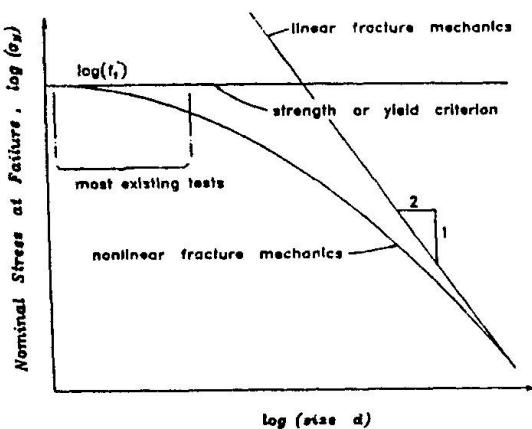


Fig. 1 - Size Effect Law

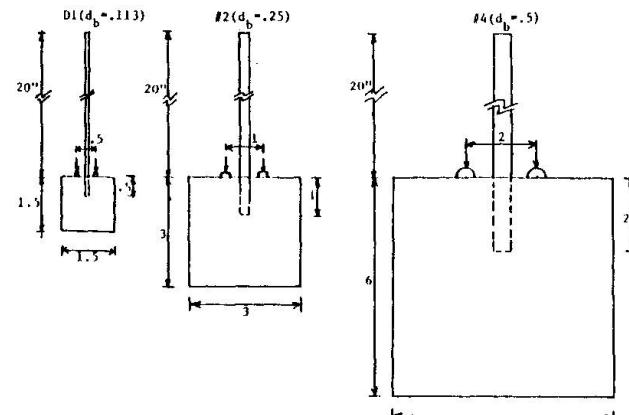


Fig. 2 - Specimen Geometry

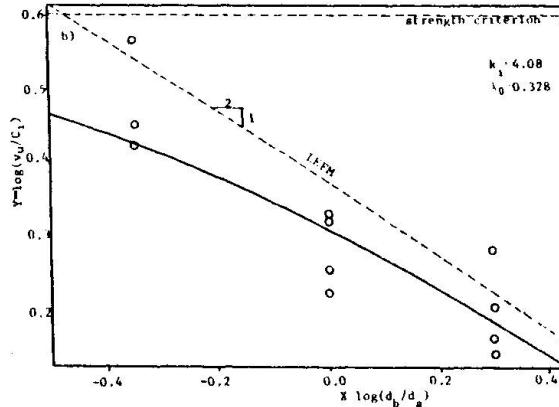
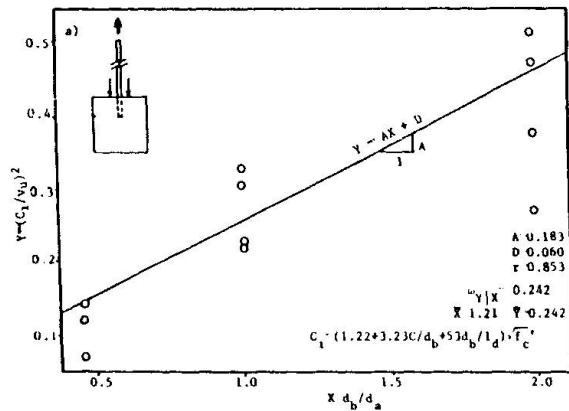


Fig. 3 - Size Effect Observed in pullout Tests

Table 1. Pullout Test Results

Beam No	$d_b$ in.	$l_d$ in.	$d_a$ in.	$f'_c$ psi	$F_u$ p
A1	.5	2.	.25	6650	5040
A2	.5	2.	.25	6650	4801
A3	.5	2.	.25	6650	5592
A4	.5	2.	.25	6650	6624
A5	.25	1.	.25	6650	1512
A6	.25	1.	.25	6650	1800
A7	.25	1.	.25	6650	1836
A8	.25	1.	.25	6650	1560
A9	.113	0.5	.25	6650	502
A10	.113	0.5	.25	6650	468
A11	.113	0.5	.25	6650	656