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## Modelling of Bond

### Modélisation de l'adhérence

### Modellierung des Verbundes

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Mato Dragosavic, born 1928 in Yugoslavia, got his civil engineering degree at Delft University. Since 1961 he is a scientific-research member of IBBC-TNO, having been involved in various projects of bond research ever since. Since 1980 he has been carrying out the project presented here (physical accent), together with Hans GROENEVELD (computational accent).

#### SUMMARY

In micro-mechanic calculations of reinforced structural concrete it is necessary to treat bond explicitly. The corresponding bond-element and bond-model are described in this paper. The model is based on theoretical considerations and experimental observations of the bond zone. Two expressions are given for the model: one in material terms as usual, the other in periferic (stress/slip) relations. Numerical implementation into finite-element-program DIANA is discussed briefly.

#### RÉSUMÉ

Dans les calculs micro-mécaniques de béton armé, il est nécessaire de traiter l'adhérence de façon explicite. L'élément d'adhérence et le modèle y relatif sont expliqués dans l'article. Le modèle est fondé sur des considérations théoriques et des observations expérimentales dans la zone de l'adhérence. Deux expressions sont présentés pour ce modèle: l'une en des termes materiels, l'autre en relations d'adhérence (contrainte/glissement) périphériques. L'application numérique dans le programme DIANA est discuté brièvement.

#### ZUSAMMENFASSUNG

In mikro-mechanischen Berechnungen des konstruktiven Stahlbetons muss der Verbund explizit betrachtet werden. Das betreffende Verbundelement und Verbundmodell werden in diesem Artikel beschrieben. Das Modell beruht auf theoretischen Betrachtungen und experimentellen Beobachtungen. Zwei Ausdrücke für das Modell werden vorgestellt: einmal in Materialkennwerten, wie üblich, zum anderen als Beziehung zwischen Verbundspannung aus Stabumfang und Verschiebungen. Die numerische Anwendung im Finite-Element-Programm DIANA wird kurz diskutiert.



## 1. INTRODUCTION

Bond is an essential property in reinforced structural concrete, influencing the behaviour and the bearing capacity of a structure, in particular of many crucial structural details. In macro-mechanic calculations the bond is processed implicitly: in terms of anchoring length of the reinforcement, crack width, tension stiffening, etc. In micro-mechanic calculations an explicit model is required.

In micro-mechanic finite-element-program DIANA, a simplified model has already been defined to process bond. That model expressed an axial bond stress/slip constitution, but was unable to honour relevant radial components introduced by the bond zone [1, 2]. A follow-up project of study and experiments is carried out to provide a better one.

This project started with definition of a bond zone around a reinforcing bar, with the inner diameter equal to the nominal diameter  $\emptyset$  of the bar and the outer diameter twice as large. Over a limited axial length, the bond zone is considered as a bond-element. This length is facultatif; for didactic reason it can be here assumed equal to the bar diameter. In other words: a bond-element is a hole cylinder, with the inner diameter  $\emptyset$ , the outer diameter  $2\emptyset$  (thus thickness  $0,5\emptyset$ ), and the axial length  $\emptyset$  (Figure 1).

The behaviour of a bond-element was studied:

- theoretically, based on constitution of concrete and adhesion, in particular with respect to deformation-controlled post-failure behaviour, and
- experimentally, by series of tests where the behaviour of the bond zone was measured under various practical conditions (cracking of the surrounding concrete, cyclic and sustained loading).

Some results (with an accent on the experimental ones) have already been presented in [3]. A final report on the experiments is given in [4]. Further results (with the accent on the modelling) are briefly discussed in this paper; more information will be given in a final report [5].

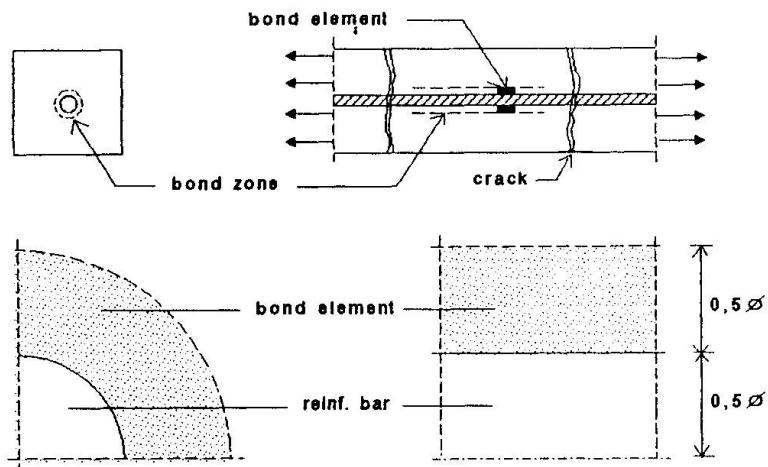


Fig. 1 Bond element

## 2. TWO VERSIONS OF MODELLING

When with respect to element mesh acceptable, the bond-element as defined in 1. will be considered as one mesh element; compatible with the bar element inside, the concrete element(s) outside and the bond elements aside.

With respect to the constitution of the bond-element, being the bond-model required, various appearances of the model are considered here, depending on the aims involved (Figure 2).

When the model is defined in material terms as for a concrete model usual, it is indicated as a "material" model. For more reasons it is also significant to define the model in periferic constitution only; than it is indicated as a "periferic" model.

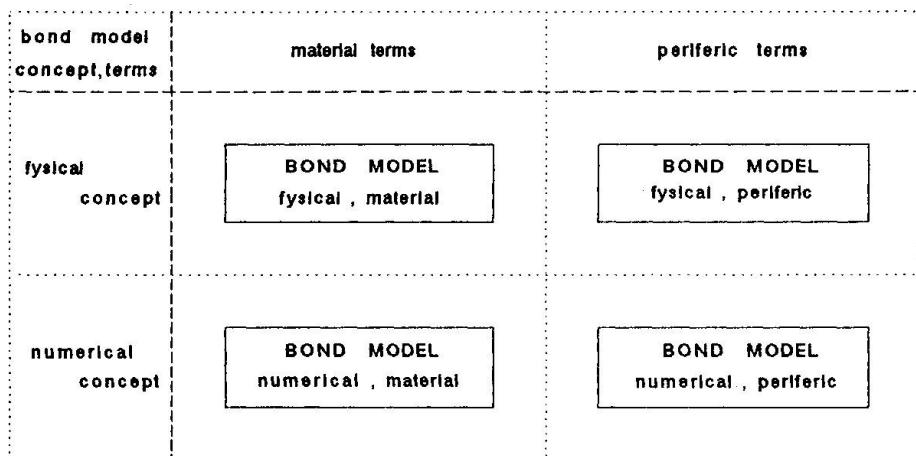


Fig. 2 Appearances of the bond model

The relevant periferic variables are (Figure 3):

- the axial and radial stress components ( $\tau$  and  $\sigma$  respectively) in the inner surface of the bond-element (in the outer surface  $0.5\tau$  and  $0.5\sigma$  are assumed),
- the axial and radial displacements ( $\Delta$  and  $\nabla$  respectively) of the outer surface of the bond-element with respect to the bar axis, and the bar contraction  $\nabla$ .

A distinction shall also be made between a "fysical" model and a "numerical" one. A fysical model aims to describe the real behaviour as close as possible, without simplifications which a numerical program might require. With these simplifications committed, the model is called the numerical model. It is evident that a numerical model is program-dependent; suggestions will be given here for DIANA.

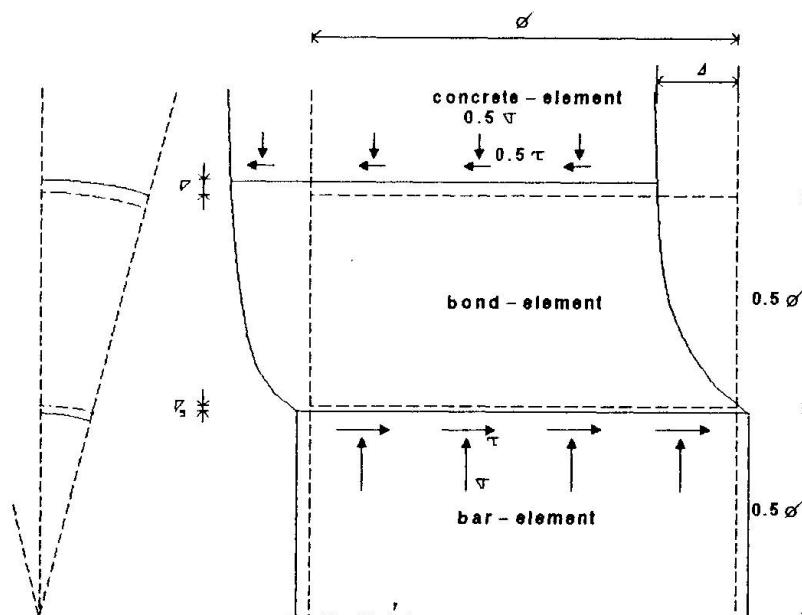


Fig. 3 Periferic stress/displacement components:  $\tau$ ,  $\sigma$ ,  $\Delta$ ,  $\nabla$



### 3. BASIC ASPECTS

From the definition of the bond-element it is obvious that it consists of concrete. The reason to require another model for bond is that attention must be paid to specific features:

- adhesion between concrete and the bar,
- stress concentrations at bar ribs,
- post-failure damages and deformations of the concrete, and
- peculiar loading in bond.

The concrete-to-steel adhesion strength is inferior to the concrete tensile strength. Thus, a bond-element behaves as an orthotropic body with a weak plane. After the adhesion failure the frictional capacity is very limited and another mechanism is necessary to resist bond of deformed bars.

After the adhesion failure, stress concentrations occur at the ribs. But the stresses are much higher and the deformation much larger there, then from the assumed uniformly distributed  $\tau$  over the rib distance would follow.

Because of the adhesion failure and the stress concentrations, post-failure stage of concrete occur at low values of  $\tau$ , too. Equilibria are still possible due to the deformation-controlled situation of the bond zone, but the damages and deformations are excessive in relation to those of pre-failure.

The loading in bond, i.e. history and level of  $\tau$ , is very different with the loading of a structural member in mind. Because of tensile axial stresses in tensile zone of a member, and the difference in radial contraction of the bar and concrete, the adhesion failure or concrete tensile failure occur at a very low (or even zero) value of  $\tau$ . Due to a (sudden) appearance of a crack,  $\tau$  suddenly becomes all values possible, depending on the crack distance.

A cyclic or sustained loading of a (cracked) member, should have a tremendous influence (cyclic or sustained creep) on the bond where post-failure stages already occurred. But this is tempered as tremendously by the deformation-controlled condition (relaxation and redistribution). Because of that, the usual fatigue or creep parameters, defined for constant (amplitudes of)  $\tau$  do not fit for bond.

More in detail the above aspects and their consequences for modelling of bond will be shown in [5].

### 4. FYSICAL BOND-MODEL

#### 4.1 Material version

It follows from paragraph 3 that the bond-model is similar to that for concrete. The conformity is evident because of the concrete involved, the difference is understood from the specific features mentioned.

Suppose the adhesion strength is  $\xi_1$  times the concrete tensile strength, and after the adhesion failure  $\xi_2$  times the rib distance is resisting  $\tau$  (so,  $1/\xi_2$  higher stresses occur at the ribs). Such an element, with the concrete having a nominal tensile strength  $f_{ct}$  and a nominal compressive strength  $f_{cc}$ , behaves (rough about) as a concrete with a tensile strength  $\xi_1 \cdot f_{ct}$  and a compressive strength  $\xi_2 f_{cc}$ . With respect to other uncertainties even may be assumed that a bond-element behaves as concrete one with by  $\xi \ll 1.0$  reduced concrete grade. Due to uncertainties, cyclic and sustained loading can be treated the same way: by further reduction of  $\xi$ .

Reduction factors  $\xi_i$  to various parameters of the concrete model separately are being studied, comparing them with the experimental results. Up to now the bond-model can be assumed as a concrete-model with reduced concrete grade only. Values of about  $\xi = 0.4$  for a first loading and  $\xi = 0.3$  for (long time)

cyclic or sustained loading promise reasonably good results. Corresponding envelopes in Mohr-diagram and uniaxially presented deformations are drawn in Figure 4.

The variables of the bond-model remain the infinitesimal (principal) stresses and strains, as for a material model usual.

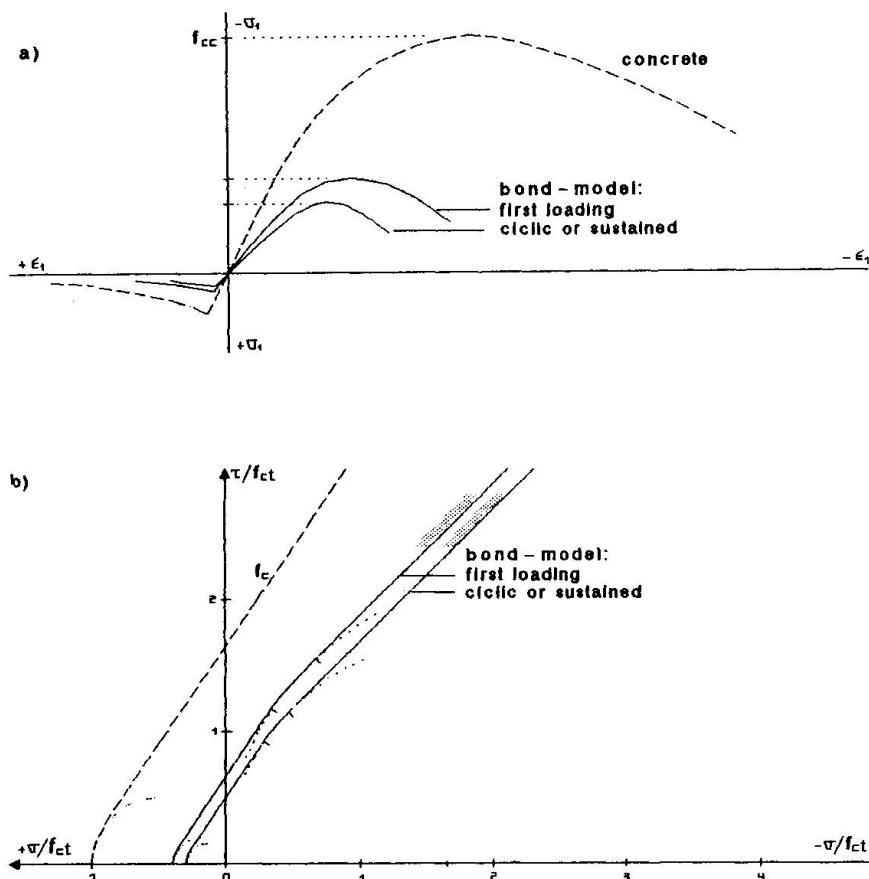


Fig. 4 Fysical bond-model, in material terms:  
a) uniaxial constitution  
b) Mohr envelop

#### 4.2 Periferic version

In principle, the periferic bond model is exactly the material one, but transformed into the periferic components as mentioned before. But, because of:  
- imperfections of the material model,  
- numerical complications of the transformation, and  
- direct comparability of the periferic model with the experimental results, the periferic model is here defined independently. By comparing the periferic model with the material one on one side, and with the experimental results on the other, both models can be improved, too (Figure 2).

It is easy to understand that the  $\tau/\Delta$  relation is very similar with an arctan-function. So it can be written:

$$\tau = \tau_{\infty} \frac{\arctan D\Delta}{0.5 \pi} \quad \dots \quad \text{for } \Delta = 0$$

For practical values of  $\Delta$ , the  $\tau/\Delta$  relation is approximated by

$$\tau = \tau_{00} \left( 1 - \frac{\arctan \frac{N(\nabla + \nabla_s)}{s}}{0.5 \pi} \right) \dots \dots \text{ for } \Delta = \infty$$

Together for  $\tau/\Delta, \nabla$  (see Figure 5a,b):

$$\tau = \tau_{00} \left( \frac{\arctan \frac{D\Delta}{0.5 \pi}}{0.5 \pi} \right) \left( 1 - \frac{\arctan \frac{N(\nabla + \nabla_s)}{s}}{0.5 \pi} \right)$$

$D, N$  en  $\tau_{00}$  are parameters to be defined. A first comparison with the experimental results gave  $\tau_{00} \approx 20 \text{ N/mm}^2$  (seems very little dependent on the concrete grade, due to dominant post-failure behaviour), with  $D \approx 30 \text{ }^1/\text{mm}$  and  $N = 300 \text{ }^1/\text{mm}$  (for deformed bars). Further comparative calculations may result in better values (or will approve these ones).

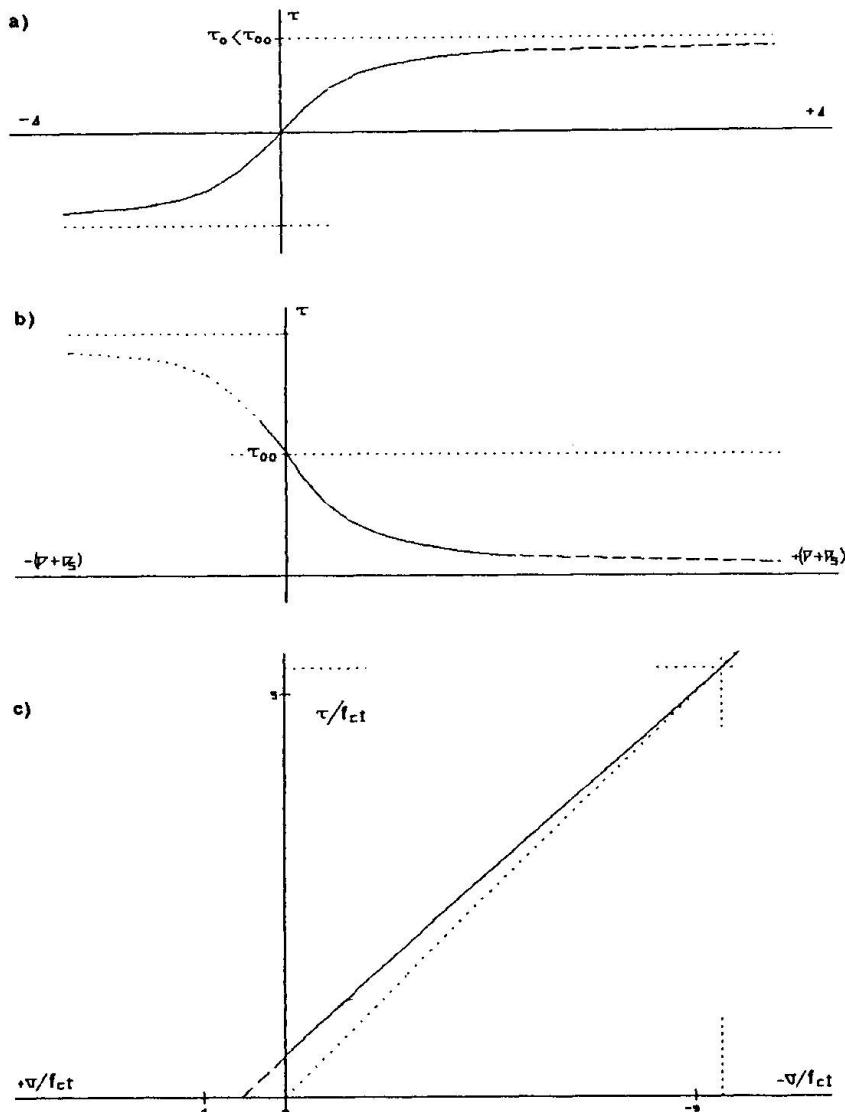


Fig. 5 Fysical bond-model, in periferic terms  
 a)  $\tau/\Delta$    b)  $\tau/(\nabla + \nabla_s)$    c)  $\tau/\sigma$

An equation similar to  $\tau/\Delta, \nabla$  can be written for  $\sigma/\Delta, \nabla$ ; but it appeared from the theoretical and experimental results that approximately (Figure 5c):

$$\sigma = 0.5 f_{ct} - \tau \frac{f_{cc} + f_{ct}}{f_{cc}}$$

For  $\tau > f_{ct}$  can even be accepted:

$$\sigma = - \tau$$

Axial concrete stresses can be neglected within the bond zone (mind the post-failure stages when concrete tension is inferior). Tangential stresses are inferior, too (as already assumed with  $0,5 \tau$  and  $0,5 \sigma$  in the outer surface, paragraph 2). Thus, the periféric variables as applied and the parameters  $D$ ,  $N$  and  $\tau_{oo}$  are sufficient to complete the periféric model.

## 5. NUMERICAL BOND-MODEL

### 5.1 Material version

In DIANA, the parameters of the model for concrete are [2]:

- Mohr-Coulomb envelop, with a cohesion  $c$  and a friction angle  $\theta$ , both adapted for dominant compression,
- cut-off criterion with a tensile strength  $f_{ct}$ , suitable for dominant tension;
- stiffness ratios  $E$  and  $\mu$  for pre-failure
- yield rule beyond the Mohr-coulomb envelop, and softening terms beyond the cutt-off;
- additional parameters related to cracking.

An obstacle to the numerical bond-model in material terms is the incompatibility of the numerical concrete model in compression and tension. Another one is the dominancy of post-failure stages, when  $E$  and  $\mu$  are irrelevant and the flow rule is not very satisfactory.

As long as the concrete model remains as now, it has not much sense to add the bond factors  $\xi_i$  very exactly. A reduction of the concrete grade as mentioned in 4.1 with  $\xi \approx 0.4$  and  $\xi \approx 0.3$ , can be applied. A better definition of the post-failure constitution is necessary, however.

### 5.2 Periféric version

The fysical model can be applied, but DIANA requires an incremental approximation of the artan-functions. At a level  $\tau_k$ ,  $\sigma_k$ ,  $\Delta_k$ ,  $\nabla_k$  follows ("d" asigns an increment):

$$\begin{aligned} \tau &= \tau_k + K_{11} & K_{12} & \cdot d\Delta \\ \sigma &= \sigma_k & K_{21} & K_{22} & \cdot d\nabla \end{aligned}$$

The partial derivatees  $K_{11} \dots K_{22}$  can easy be calculated from the equations in 4.2 because for any (artan  $X \cdot x$ ), with  $X = D$  or  $N$ , and  $x = \Delta_k$  or  $\nabla_k$ :

$$\frac{d(\text{artan } X \cdot x)}{dx} = \frac{X}{1 + (X \cdot x)^2}$$

Herewith the bond part of the model is defined. For the bar inside the bond element, with the axial steel stress  $\sigma_s$ , Young-modulus  $E_s$  and axial strain  $\epsilon_s$ , follows simply:

$$\sigma_s = E_s \cdot \epsilon_s$$

Evident is the relation  $\tau_k / \sigma_s$  over an incremental bar length  $dl$ :

$$\tau_k \cdot \pi \phi \cdot dl = d\sigma_s \cdot \frac{1}{4} \pi \phi^2 \quad \text{or: } \tau_k = \frac{1}{4} \phi \cdot \frac{d\sigma_s}{dl}$$

For  $\nabla_s$  holds:  $\nabla_s = \mu_s \epsilon_s \cdot 0.5 \phi$

Evident is also that with the bond model as given here, the obstacles mentioned in 5.1 are passed by. Uncertainties of the values of  $D$ ,  $N$  and  $\tau_{oo}$  remain instead, and a adequat numerical bond-element is to be defined.



## 6. FINAL REMARKS

With the model(s) proposed, the parameters as the crack distance from a bond-element, eventual external compression perpendicular to the reinforcing bar etc., are avoided. Such parameters would be in contradiction with a model, too.

The results presented in this paper will be checked and improved by comparative calculation of the behaviour measured in the experiments mentioned before. Such calculations are only incidentally carried out until this paper.

It is evident that a numerical model is dependent on the capacity and sofistication of a finite-element-program. With better programs, the model(s) for bond can be improved, too. For three- or two-dimensional application a further adaption of the axissymmetrical model is required.

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