

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 54 (1987)

**Artikel:** Significance of crack models for bond-slip studies  
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**DOI:** <https://doi.org/10.5169/seals-41923>

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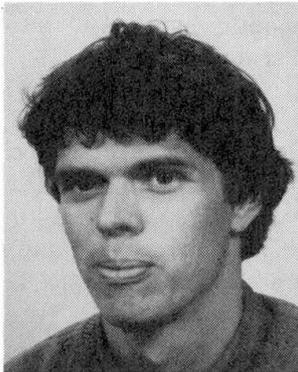
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## Significance of Crack Models for Bond-Slip Studies

Valeurs des modèles de fissuration pour les études adhérence-glisserment

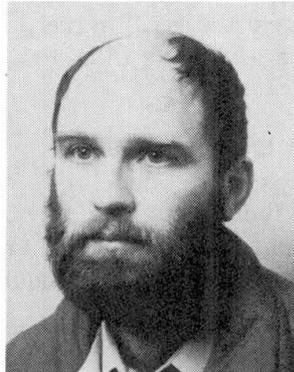
Die Bedeutung von Rissmodellen für Verbundstudien

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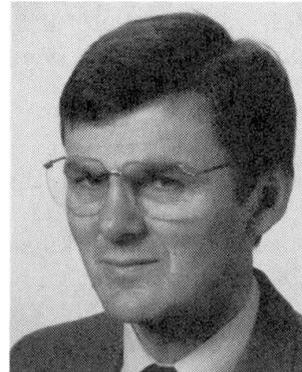
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### SUMMARY

The bond-slip phenomenon in reinforced concrete is investigated using finite elements and an elastic-softening crack model. Computational examples demonstrate the capability of this approach for predicting bond-slip related cracks such as primary cracks, secondary cracks and longitudinal splitting cracks. Attention is furthermore drawn to 'tau-delta' curves and their possible non-uniqueness. The paper is concluded by a more practical example involving the application of bond-slip interface elements.

### RÉSUMÉ

La phénomène adhérence-glisserment dans le béton armé est étudié à l'aide des éléments finis et d'un modèle de fissuration élastique souple. Des exemples de calcul démontrent la capacité de cette approche à prédire les fissures dues à cette situation adhérence-glisserment, telles que fissures primaires, secondaires et longitudinales. L'attention est attirée sur les courbes 'tau-delta' et sur leur éventuelle non-unicité. L'article conclut par un exemple pratique comportant l'application d'éléments d'interface adhérence-glisserment.

### ZUSAMMENFASSUNG

Der verschiebbare Verbund im Stahlbeton wird mit Hilfe von Finiten Elementen und einem elastischen Entfestigungsmodell untersucht. Beispiele zeigen die Fähigkeiten des Modells in Bezug auf die Voraussage von Primär- und Sekundärrissen und Längsspaltrissen. Schubspannungs-Verschiebungskurven werden kritisch beleuchtet, vor allem hinsichtlich ihrer Nicht-Eindeutigkeit. Der Beitrag wird mit einem Beispiel aus der Praxis abgeschlossen.

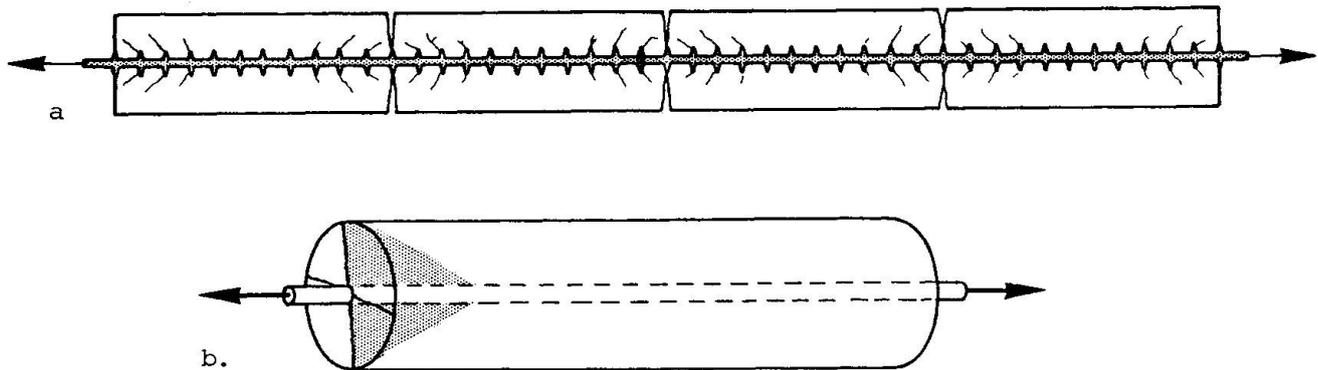


## 1. INTRODUCTION

Bond-slip in reinforced concrete is known to be closely related to, or even stronger, caused by cracking. This was probably first emphasized by Goto [13], who presented experimental results on tension-pull specimens, revealing the existence of primary cracks surrounded by cone-shaped secondary cracks (Fig. 1a). While the primary cracks are generally visible at the outer surface, the secondary cracks are formed internally behind the ribs of the (deformed) reinforcing bar at both sides of a primary crack. In some cases one may also observe longitudinal splitting cracks, the importance of which depends on the precise ratio of concrete cover versus bar dimensions (Fig. 1b). Secondary cracks and longitudinal splitting cracks are generally considered as being the dominant factors contributing to bond-slip.

Experimental measurements of bond-slip and bond-slip related cracks have been the subject of much controversy, primarily because of the significant scatter emerging from different specimen configurations and different testing techniques, e.g. [1,10,11]. Within this context computational models may be helpful for investigating bond-slip behaviour, especially when we consider the progress which has been recently made in the field of fracture mechanics and crack models for concrete.

The purpose of this paper is to apply this increased knowledge of concrete cracking to the bond-slip problem. To this end, the crack model developed within DIANA [6,18] has been applied to a tension-pull configuration and the focus is placed on obtaining a detailed resolution of primary cracks, secondary cracks and longitudinal splitting cracks. The paper is concluded by a more practical example involving the application of special bond-slip elements [9,19] in addition to cracking options.



**Fig. 1.** Crack formation in tension-pull configuration.

(a) primary and secondary cracks, (b) longitudinal cracks.

## 2. CRACK MODEL

The crack model adopted was based on a decomposition of the total strain into a concrete part and a crack part. This procedure has the notable advantage that it allows crack behaviour to be treated separately from the behaviour of the intact concrete between the cracks (de Borst & Nauta [6], Rots et al [18]). In this paper a linearly elastic model has been assumed for the concrete and a softening model for the crack, so that we arrive at an elastic-softening model for the cracked concrete.

The softening was assumed to occur in mode-I and was controlled by three parameters, viz. the uniaxial tensile strength  $f_{ct}$ , the mode-I fracture energy  $G_f$  and the shape of the softening diagram. In this paper a non-linear tensile softening function [17] has been adopted. The

tensile strength and the fracture energy  $G_f$  were assumed to be fixed material constants and provisions were included to correctly release the fracture energy over the given crack band width  $h$  which is related to the particular finite element configuration [2]. A constant crack shear modulus was inserted to control mode-II crack sliding behaviour. This technique corresponds to the use of a constant shear retention factor  $\beta$  [18].

Multi-directional cracking in an integration point was modelled according to the ideas of Litton [15] and de Borst & Nauta [6]. For axi-symmetric configurations like tension-pull specimens this option is essential as it allows us to simultaneously detect one or more tangential cracks as well as a radial splitting crack. It is further noted that the crack model includes a crack closing and re-opening option in the form of a secant unloading/reloading branch.

### 3. COMPUTATIONAL SET-UP

This paper considers a tension-pull specimen, which consists of a center-placed reinforcing bar surrounded by a cylinder of concrete. The particular dimensions correspond to a portion of the specimen tested by Broms & Raab [4], which also served as a basis for a numerical study by Ingraffea et al. [14]. The axi-symmetric finite element configuration is shown in Fig. 2 and consists of quadratic elements which have been integrated using nine-point Gauss quadrature. The concrete elements have been rigidly connected to the steel elements in order to simulate the mechanical interlock between the ribs of the (deformed) reinforcing bar and the concrete. In doing so, pure slip along the steel-concrete contact surface has been neglected. In addition, possible local "crushing" in front of the steel ribs has been neglected, which seems to be justified since confinement in the form lateral compression was lacking. As a consequence, the computations aim at explaining bond-slip solely via elastic deformation, secondary cracking and longitudinal cracking.

The elastic properties of the concrete were assumed to be: Young's modulus  $E_c = 25000 \text{ N/mm}^2$  and Poisson's ratio  $\nu = 0.2$ . The crack parameters were taken as: tensile strength  $f_{ct} = 2.8 \text{ N/mm}^2$  (with a slight perturbation for the Gauss-points near midlength of the specimen), fracture energy  $G_f = 50 \text{ J/m}^2$ , crack band width  $h = 8.46 \text{ mm}$  and shear retention factor  $\beta = 0.5$ . The reinforcing bar was given a Young's modulus  $E_s = 200000 \text{ N/mm}^2$  and a yield stress  $\sigma_y = 400 \text{ N/mm}^2$ . The specimen was analysed under direct displacement control of the end-face of the reinforcing bar, using a modified Newton-Raphson incremental-iterative procedure.

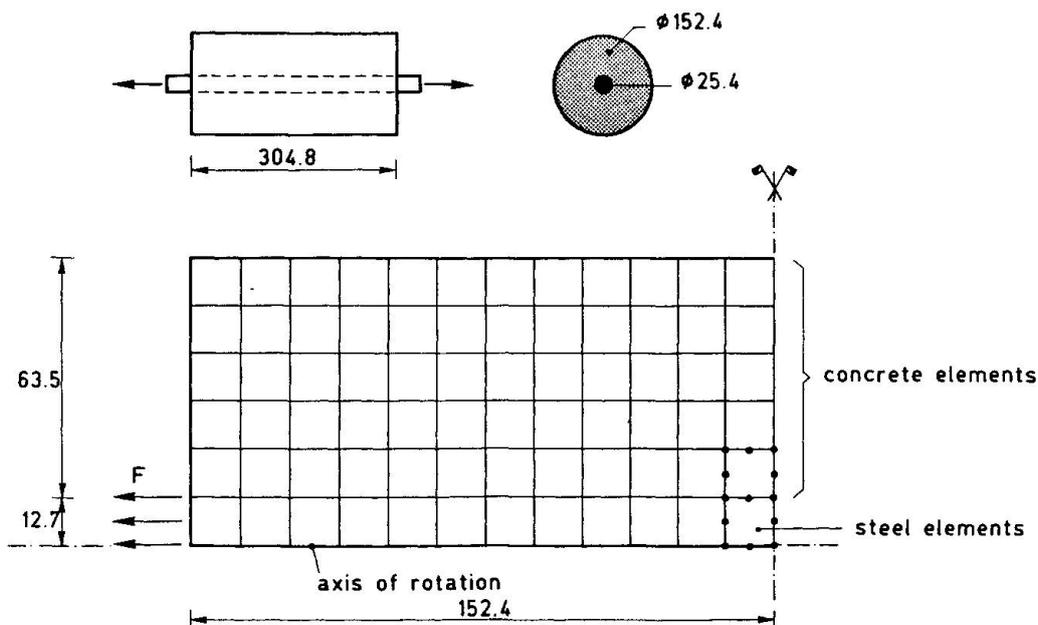
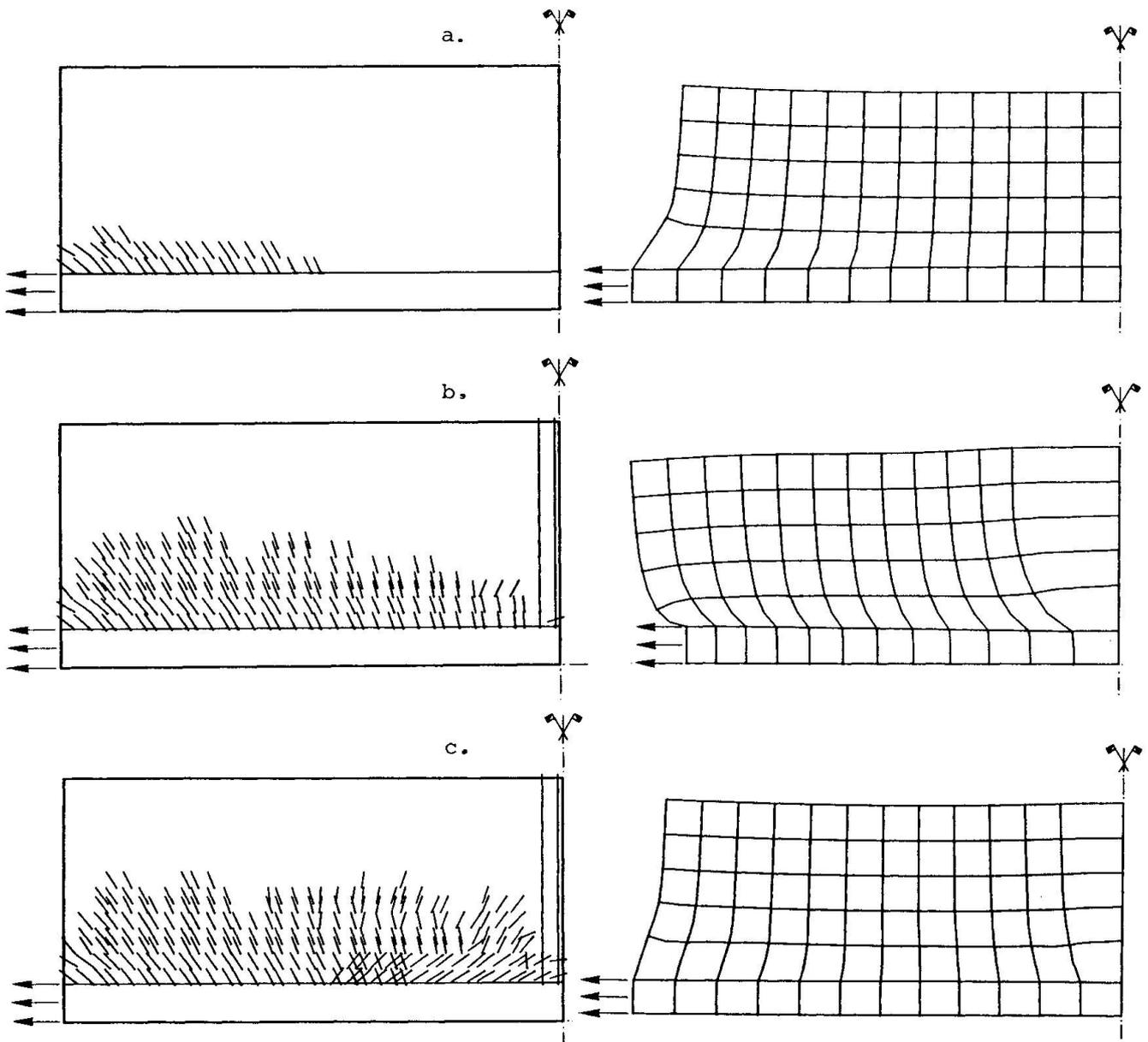


Fig. 2. Finite element idealization of tension-pull specimen.



#### 4. CONE-SHAPED SECONDARY CRACKS

The analysis progressed as shown in Figs. 3 and 4, giving the crack patterns and the corresponding incremental deformations at key-events. Initially, secondary cracks form at the location where the steel exits the concrete (Figs. 3a and 4a). On subsequent loading, these cracks propagate and additional secondary cracks nucleate further from the specimen end-face. At a certain load level also the tensile stress at midlength of the specimen reaches the tensile strength, which results in the unstable formation of a localized primary crack (Figs. 3b and 4b). Beyond this stage additional secondary cracks nucleate in response to the free surface provided by the primary crack, just like they did in response to the free end-face (Figs. 3c and 4c). Consequently, a number of Gauss-points display not only one but two secondary cracks, crossing each other. Formation of the "second" secondary crack was mostly accompanied by closing of the existing secondary crack.



**Fig. 3.** Tangential crack pattern.

(a)  $u_A = 0.02\text{mm}$ , prior to primary cracking, (b)  $u_A = 0.06\text{mm}$ , at primary cracking,  
 (c)  $u_A = 0.10\text{mm}$ , beyond primary cracking.

**Fig. 4.** Incremental deformations.

The above mechanisms turn out to be in line with the experimental findings by Goto [13]. A discrepancy is that the computational result exhibits a rather diffuse pattern of secondary cracks, while in the experiments the tiny secondary cracks localized behind each rib of the steel bar. This discrepancy is due to the mesh adopted, which is too coarse to model the action of individual ribs. Further computational details, such as the impact of the various crack parameters on secondary crack formation are given in [19]. It appeared that not only the mode-I fracture parameters, but also the shear retention factor  $\beta$ , controlling the amount of stress rotation after crack initiation, and the inter-crack threshold angle [8], controlling the inclination between multi-directional cracks, played a crucial role. Improvements of the crack model regarding these issues are currently worked out, e.g. [20].

**5. PRIMARY CRACKS**

Fig. 4b indicates that primary cracking occurs in the form of a sudden, brittle type of fracture which is highly localized. The primary crack involves a drastic redistribution of the stresses within the specimen. In fact, the specimen is predicted to be halved into two sub-specimens, the behaviour of each of which is a copy of the original specimen.

This sudden transition from the one equilibrium state to a different equilibrium state is not free from complications. This becomes manifest if we plot the load-elongation response, which shows a local limit point followed by a sudden drop of the load, as shown in Fig. 5. Obviously, we are dealing with a "snap-back" [5,7,8], which can also be deduced from Fig. 4b revealing relaxation of the steel bar upon primary crack formation. In the present study the snap-back was passed by temporarily switching-off the iterations, allowing the load to drop suddenly. For a more elegant way to solve this problem one should resort to advanced solution techniques which control the opening displacement of the primary crack rather than the end-displacement of the reinforcing bar (de Borst [7,8]).

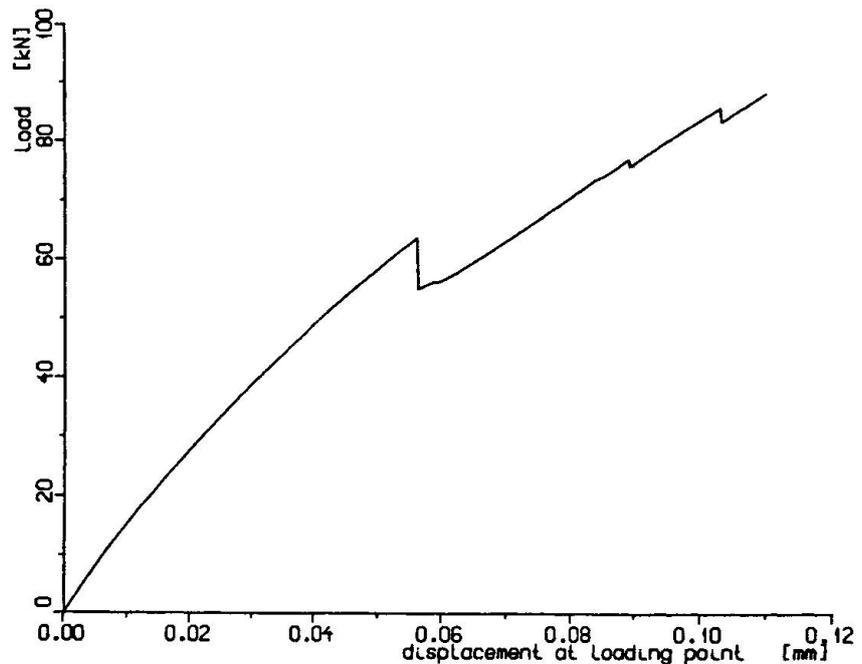
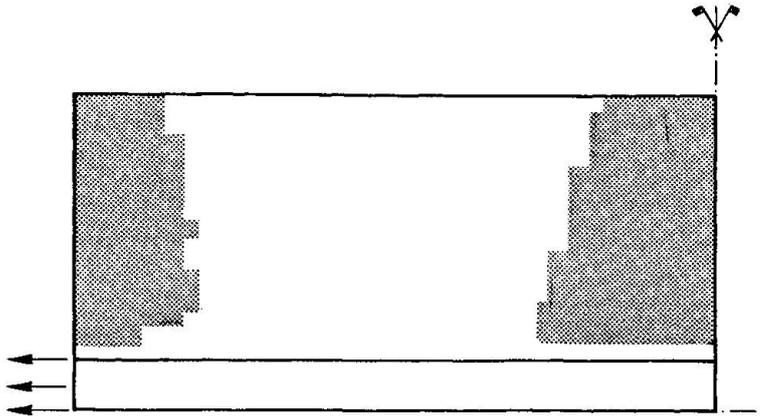


Fig. 5. Load versus end-displacement reinforcing bar.

It is interesting to note that the above phenomena not only adhere to computational predictions, but also to experimental research, where primary cracks have been repeatedly referred to as being "unstable" (e.g. Goto [13]).

**6. LONGITUDINAL SPLITTING CRACKS**

In addition to the tangential primary and secondary cracks the specimen is predicted to have longitudinal splitting cracks, as shown in Fig. 6. These cracks occur near the free surfaces of the specimen. Our experience is that the inclusion of a longitudinal cracking option is essential because this type of crack formation correctly limits the compressive strut action in the concrete cones radiating from the steel ribs.

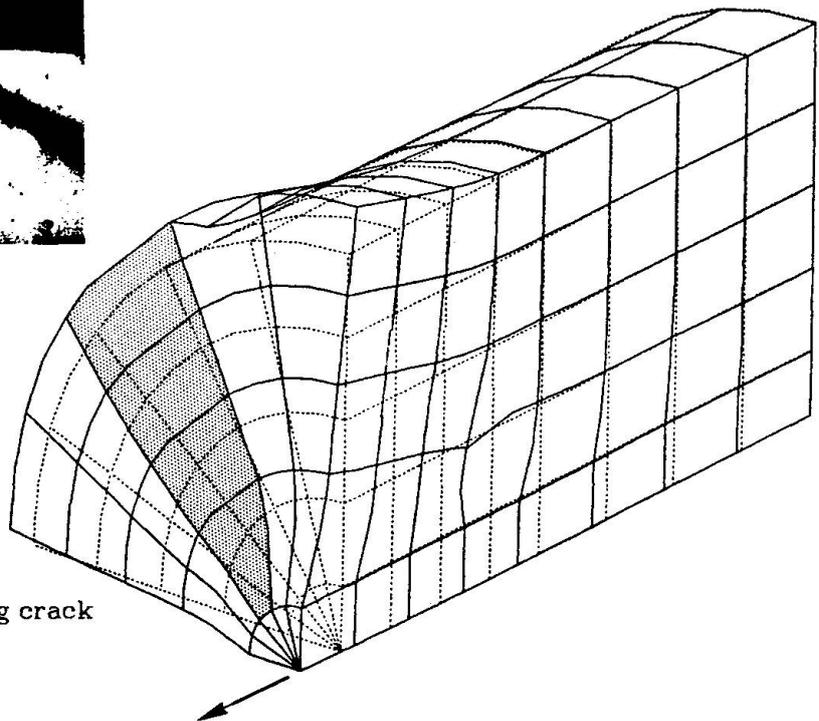


**Fig. 6.** Region in which longitudinal (radial) splitting cracks have developed. (axi-symmetric analysis).

Because of the axi-symmetric idealization, the longitudinal cracks in Fig. 6 are smeared out over the circular cross-section. As a consequence, it was impossible to model localization of these cracks. From experiments it is known that such localizations occur, as shown in Fig. 7 which was obtained using holographic interferometry [3]. To circumvent this deficiency of the model, the specimen was re-analysed using a fully 3-dimensional configuration with a 3D-variant of the crack model. One quarter of the cross-section was modelled and perturbations of strength properties were used. Indeed, this analysis turned out to be capable of predicting a localized longitudinal splitting crack, as shown in Fig. 8.



**Fig. 7.** Localized splitting cracks at end-face, experimentally [3].



**Fig. 8.** Localized splitting crack for 3D analysis.

## 7. TAU-DELTA CURVES

Bond-slip research has been primarily directed towards the experimental determination of "tau-delta" curves, with tau being the bond shear traction and delta being the slip between concrete and reinforcement (e.g. [1,10,11]). In a similar way, the computational outcome allows such curves to be recorded, which is exemplified by Fig. 9 giving the tau-delta curve for point A some distance away from the reinforcing bar. The curve shows fair qualitative agreement with experimental measurements, which indicates that bond-slip in reinforced concrete can be at least partially explained from the underlying concrete and crack properties.

It should be noted that the location of the measuring point A does not seem to have been uniquely defined in literature, which explains some of the scatter encountered in tau-delta curves. We have also monitored the bond shear traction and the bond-slip for different points further away from the specimen end-face than point A, and the resulting curves indeed turned out to differ markedly from Fig. 9. It is therefore concluded that a tau-delta curve is a non-unique *structural* property rather than a unique *material* property. It is a challenge to investigate these effects by means of additional computational research. Together with the available experimental data this may contribute to the development of a sound bond-slip model which is independent of boundary conditions, specimen dimensions and so on. At present, attempts in this direction are being made (e.g. [12]).

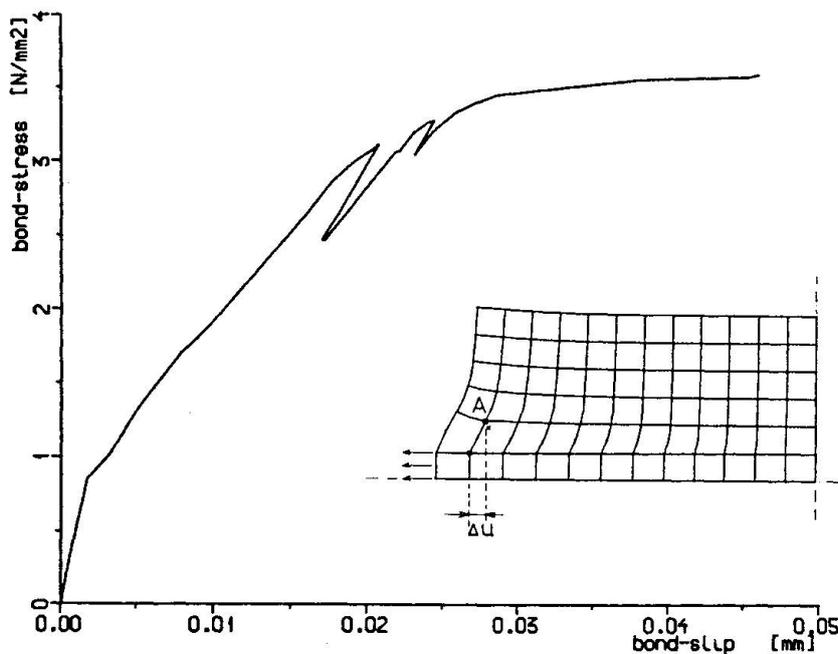


Fig. 9. Local bond-stress vs. bond-slip curve for point A.

## 8. BOND-SLIP ELEMENTS

Although bond-slip is known to be one of the dominant factors governing the non-linear behaviour of concrete structures, it appears that most practical finite element computations are still being performed under the assumption of overall perfect bond [16]. However, considering the above analyses it seems natural to lump the bond-slip contribution due to cracking into an interface element, which can be subsequently inserted for general purposes. This idea was followed by a number of researchers [16], and a particular result obtained before [19] will be reviewed here. It concerns a reinforced concrete beam which fails in bending. A bilinear bond stress-slip law was used for the interface elements and an elastic-softening model for the concrete. For details of the mesh and the material properties the reader is referred to [19].



Fig. 10 gives a comparison between the crack pattern obtained with perfect bond and the pattern obtained with bond-slip elements. The inclusion of bond-slip not only appears to enhance strain-localization, but it also yields a smooth and undistorted crack pattern in the neighbourhood of the reinforcement. In contrast, the perfect bond assumption may give rise to distortions and spurious checkerboard patterns, as was demonstrated before in e.g. [5,18]. Our experience is that the inclusion of bond-slip is essential for finding a detailed resolution of stress and strain fields in the vicinity of the reinforcement. Perfect bond tends to homogenize stress and strain fields and therefore conflicts with the delicate fracture mechanics treatment of individual cracks. In this case it may be better to resort to global tension-stiffening techniques rather than tension-softening techniques.

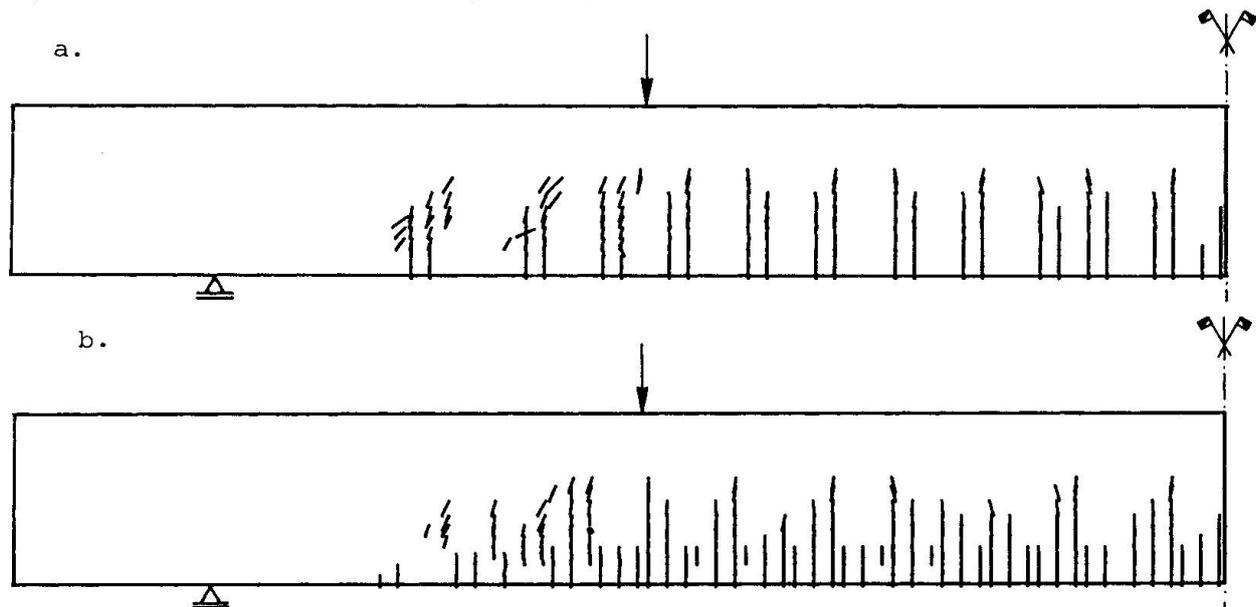


Fig. 10. Crack pattern for reinforced beam.  
(a) bond-slip included, (b) overall perfect bond.

## 9. CONCLUDING REMARKS

The bond-slip phenomenon in reinforced concrete has been explained by means of computational crack models. A finite element approach using elastic-softening material laws was shown to be capable of simulating primary cracks, secondary cracks and longitudinal cracks which accompany bond-slip. The method was demonstrated to be capable of predicting "tau-delta" relationships, which can be subsequently implemented into special bond-slip interface elements for predicting the overall response of reinforced concrete structures. An example thereof has been presented.

It is remarked that similar studies have been undertaken before in 1980 by De Groot, Kusters & Monnier [9], using a less far evolved crack model and more simple solution strategies. They repeatedly reported numerical divergence and related problems, which indicates that the increase of knowledge of concrete cracking and solution procedures over the past few years has significantly enhanced the possibilities of non-linear finite element codes. Bond-slip research may take advantage of that.

## ACKNOWLEDGEMENTS

Computational results for this paper were obtained with the DIANA finite element environment of the Institute TNO for Building Materials and Structures (TNO-IBBC). Support from Mr. R. de Borst, Mr. J.C.M. Jansen, Mr. P. Nauta, Mr. F.C. de Witte and other colleagues from the DIANA-group of TNO-IBBC is gratefully acknowledged. The research was supported financially by the Netherlands Technology Foundation (STW) and by CUR-committee A26 "Concrete Mechanics".

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