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Focal Points Model for Uniaxial Cyclic Behaviour of Concrete

Modèle des points focaux pour le comportement du béton sous charge cyclique uniaxiale

Brennpunktmodell für das einachsige zyklische Verhalten von Beton

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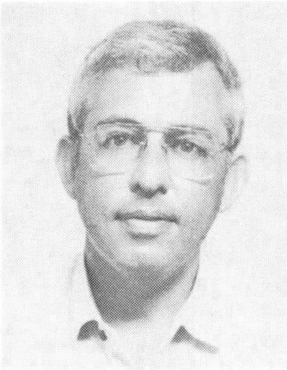

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SUMMARY
A new one-dimensional model for random cyclic compression and tension in plain concrete is developed. The model determines several geometrical loci in the uniaxial stress-strain plane that govern the unloading-reloading curves in the softening range. The model allows the prediction of unloading-reloading curves by simple graphical means without further calculations.

RÉSUMÉ
Un modèle nouveau est développé pour des cas de charges aléatoires répétés à la compression et tension du béton non armé. A l'aide du modèle, quelques lieux géométriques sont déterminés dans le plan des contraintes et déformations uniaxiales qui servent à la construction des courbes de déchargement et rechargement dans la région de dégradation. Le modèle permet la prédiction simple de courbes de déchargement et rechargement avec des moyens graphiques et sans calcul supplémentaire.

ZUSAMMENFASSUNG
Ein neues eindimensionales Modell für willkürliche wiederholte Druck- und Zugbelastungen von unbewehrtem Beton wurde entwickelt. Mit Hilfe des Modells werden einige geometrische Örter in der einachsigen Spannungsdehnungsebene bestimmt, die zur Konstruktion der Entlastungs-Wiederbelastungskurven im Entfestigungsbereich dienen. Das Modell erlaubt somit die einfache Vorhersage von Entlastungs-Wiederbelastungskurven mit graphischen Mitteln ohne weiteren Rechenaufwand.

1. INTRODUCTION

An unconfined concrete element which is subjected to random cyclic uniaxial compression or tension loading is considered. Quite a large variety of models are available for compressive loading, being based on the theory of elasticity [e.g. 5,7,9,13] , theory of plasticity [e.g. 6,7,18], plastic fracturing approach [e.g. 2] and the endochronic theory of plasticity [e.g. 3]. There also exist simplified models some of which are mathematical descriptions of test results e.g.[9,12,17].

Considerably less effort has been given to model the relatively new experimental results of tensile loading [e.g. 11,14.15,16,20]. Most of the models propose a description for the envelope curve and only a few introduce a simplified formulation for the unloading-reloading cycle [10,16,20].

Examination of many test results on concrete samples subjected to monotonic and especially cyclic loading, both in compression and in tension, has clearly shown that there exist some common geometrical properties in the uniaxial strain plain. Several fixed points are determined, and denoted as focal points, with aid of which the complete loading-unloading history may be reproduced. The approach has been examined with respect to many test data and shows very good agreement. It may be formulated as a material law and be implemented in a computer code. The geometrical interpretation yields a further advantage that the cyclic loading-unloading history may be reproduced graphically without any computations.

2. EXPERIMENTAL STRESS-STRAIN CURVES

2.1 Compressive Loading

The monotonic stress-strain curve shows linear behaviour up to about 30 % of the strength f_c, and nonlinear behaviour at higher stresses. Concrete softens until a peak stress is reached at a strain ε_o as a result of microcracks propagation. At larger strains a descending part of the stress-strain curve is observed.

The envelope curves for different cyclic loading histories have been found to fit, with a reasonably small scatter,with the monotonic curve.The unloading curve from that envelope gradually softens with continuing unloading and changes in strain are more pronounced at low stress levels. The reloading curve reverses curvature during increasing stresses and intersects with the unloading curve at the "common point limit" [12]. Cycling within a certain bounding loop lowers down the common point and within several cycles it stabilizes at the lower "stability limit". Starting unloading at larger strains shows smaller stiffnesses and larger strain changes.

2.2 Tensile Loading

The concrete sample behaviour under tensile loading is usually expressed in a stress vs. displacement relationship.The displacement is the total elongation of the microcracked zone, as measured by extensometers of certain length. The monotonic stress-displacement curve in tension behaves linearly up to about 80 % of the tensile strength f_t, with a tangent modulus of elasticity similar to that in compression. At higher stresses concrete softens considerably until the tensile strength f_t is reached at a displacement δ_o. At larger displacements a wide softening range is observed, which is characterized by a descending branch.

Only a limited amount of test data has been reported in literature on deformation controlled tensile tests in general and on cyclic tests in particular. The work that has been carried out at the Stevin laboratory [8,14,15] has covered various load histories through which stresses during unloading varied between the envelope to either low tensile stress level, low compression or higher compression, as may be seen in Fig. 1. For all load histories the envelopes were found to be similar to the monotonic curve in tension, and a unique envelope curve may be assumed.

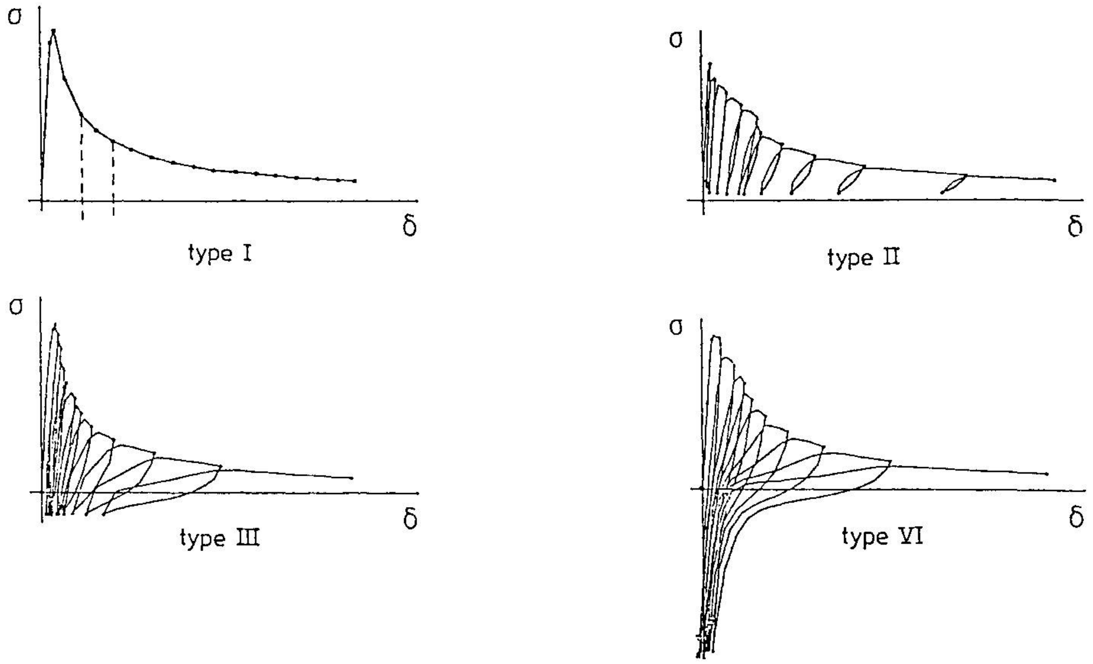

Fig. 1: Experimental Load Histories in Cyclic Tension.

During unloading the curve softens and around a zero stress level stiffness becomes very small and large displacements are involved.When the compressive stress increases, at further unloading, the curve stiffens up again (see Fig.1).

3. THE FOCAL POINTS MODEL

3.1 Model Concept

The cyclic stress-strain or stress-displacement curves, both in compression and in tension, exhibit a decreasing stiffness with unloading.The curve softens considerably whenn stresses drop close to zero and large plastic strains are developed. If unloading starts at a larger strain, the softening will be more pronounced. That trend resembles to rays originated at a low stress and strain level, on which lie those unloading curves. A similar observation relates to the reloading curves.

The model defines several geometrical loci in the uniaxial stress-strain or stress-displacement plane. These points are defined as "focal points" [19,20]. The focal points coordinates are independent on a specific cycle and are given as functions of the envelope parameters.

3.2 Model Rules for Compressive Loading

To obtain any cycle in compression, six focal points C1-C6 are defined (Fig. 2). Five of them are placed along the tangent to the envelope curve at the origin, and the sixth is placed on the strain axis.

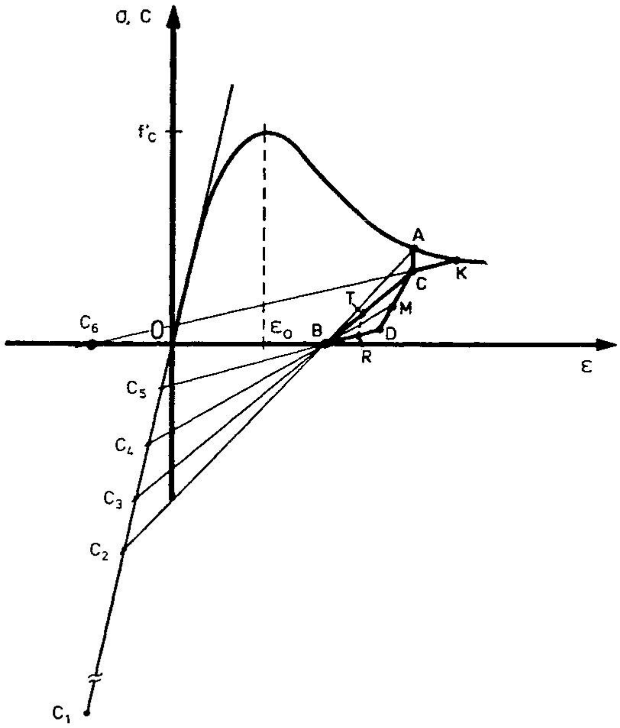

Fig. 2: Focal Point Model for Cyclic Compression

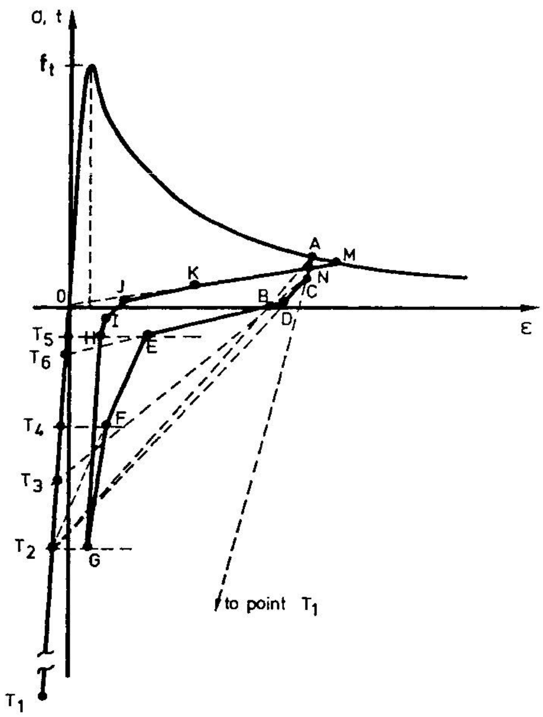

Fig. 3: Focal Point Model for Cyclic Tension

The focal points stress coordinates are expressed as function of the uniaxial compression strength (negative stresses mean compression):

$c_1 = 3*f_c'$		$c_4 = 0.47*f_c'$	
$c_2 = f_c'$		$c_5 = 0.2*f_c'$	
$c_3 = 0.75*f_c'$		$c_6 = 0.0\,;\varepsilon_{C6} = -\varepsilon_o$	

The unloading curve, starting from point A on the envelope, is idealized by the piecewise linear curve A-C-D-B (Fig.2). Point B is the intersection of the line connecting point A and focal point C_2. Line A-C is parallel to the stress axis, and point C is obtained by the intersection of this line with the line connecting focal point C_3 and point B. Point D is the intersection of lines C-C_1 and C -B.

The reloading curve is idealized by B-C-K. Point K is the intersection of line C -C with the envelope.
Point C is the common point and point M, which is the intersection of C_4-B with C-D, is the stability limit.

3.3 Model Rules For Tensile Loading

An average tensile strain is defined as the tensile elongation per unit gage length, and the experimental stress-displacement curve may be transformed into a stress-strain curve in tension. In this coordinate system any cycle may be obtained with aid of seven focal points: point O (the coordinate origin), points T_1-T_5, which are placed along the tangent to the envelope curve at the origin, and point T_6, the coordinates of which depend on the strain at which unloading starts (Fig. 3).

All focal points, except for focal points T_6, are fixed in the stress-strain coordinate system and independent on a certain cycle. Their stress coordinates are expressed as function of the uniaxial tensile strength f_t (negative stress means compression):

$t_o = 0.0$	$t_3 = -0.75*f_t$
$t_1 = -3*f_t$	$t_4 = -0.5*f_t$
$t_2 = -f_t$	$t_5 = -0.125*f_t$

The coordinates of focal point T_6 are $T_6\ [\epsilon_A/2, -0.075*f_t]$, where ϵ_A is the strain at point A on the envelope, at which unloading starts.

The unloading curve in tension, which starts at point A on the envelope, is idealized by the piecewise linear curve A-C-D-B and continues in the compression range along the curve B-E-F-G. Point B is the intersection with the strain axis of line A-T_2. Point C is the intersection of lines T_1-A and T_3-B. Point D is the intersection of lines T_2-C and T_6-B. Point E has the stress level as focal point T5. Point F has the stress level of focal point T_4 and is placed along T_2-E. Point G lies along T_{1-F} and has the stress level of focal point T_2.

Reloading will follow the elastic stiffness (slope O-T_1) as long as unloading has not reached the stress level of focal point T_5 (line G-H in Fig. 3). If reloading starts at point G, the reloading curve will be idealized by the piecewise linear curve G-H-I-J-K-M. The segment I-J is parallel and equal in length with segment C-D.Segment J-K is parallel to D-E where point K lies on O-N. Point N lies von A-C and its stress level is 85% of the stress at point A. Line O-N intersects with the envelope at M. If reloading starts at a lower compressive stress than $-f_t$, then that point on the unloading curve will be denoted H.

4. COMPARISONS WITH TEST RESULTS

At the present stage, the model assumes a given envelope, which coincides with the monotonic curve.The cycle starts from and returns to the given envelope.Once the envelope is known to coincide with the experimental one, comparisons may be made between experimental cycles and the focal point model cycles.

4.1 Tests of Cyclic Compression

The focal points model has been compared with various test results in which the sample is loaded in uniaxial cyclic compression. Fig. 4 shows comparison with a test performed by Karsan & Jirsa [12], and Fig. 5 compares the model prediction with test results of Okamoto [1]. In these figures the predictions have been obtained graphically and good correspondence is obtained.

More comparisons of the focal points model with characteristics of the cyclic behavior in compression have been made [19]. The model predictions of the common point limit, the stability limit, the residual plastic strain and the point at which the reloading curve meets with the envelope, have been compared with both various test data and empirical expressions. Those comparisons show very good agreement.

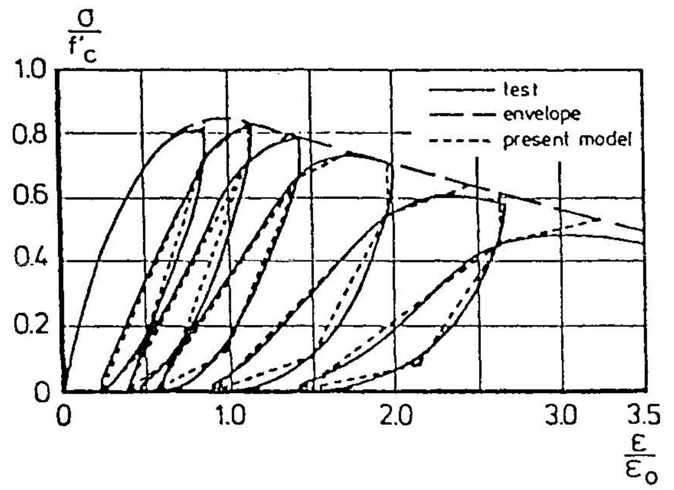

Fig. 4: Comparison with Test Results by Karsan & Jirsa [12]

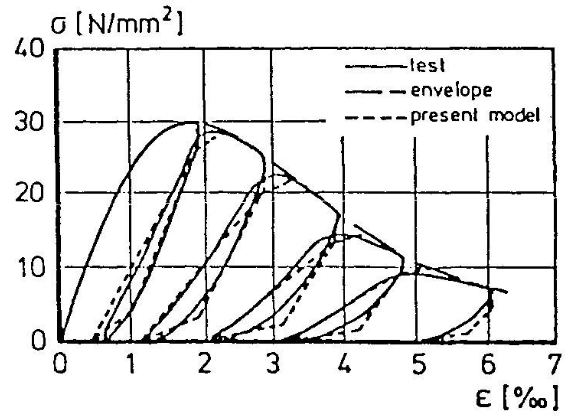

Fig. 5: Comparison with Test Results by Okamoto [1]

4.2 Tests of Cyclic Tension

Comparisons of the focal point model with test results obtained at the Stevin Laboratory, Delft University of Technology, are shown in Fig. 6-7. Fig. 6 shows the cyclic tensile tests in which unloading goes to slight compression and Fig. 7 shows the cyclic tensile tests, in which unloading reaches a compression level that is equal to the tensile strength f_t. The focal point model cycles are found to be in good agreement with the measured cycles, although their shape is rather complex. The predicted cycles may be obtained either graphically or through a mathematical subroutine which follows the model rules.More comparisons appear in [20] and all of them show good correspondence with test results.

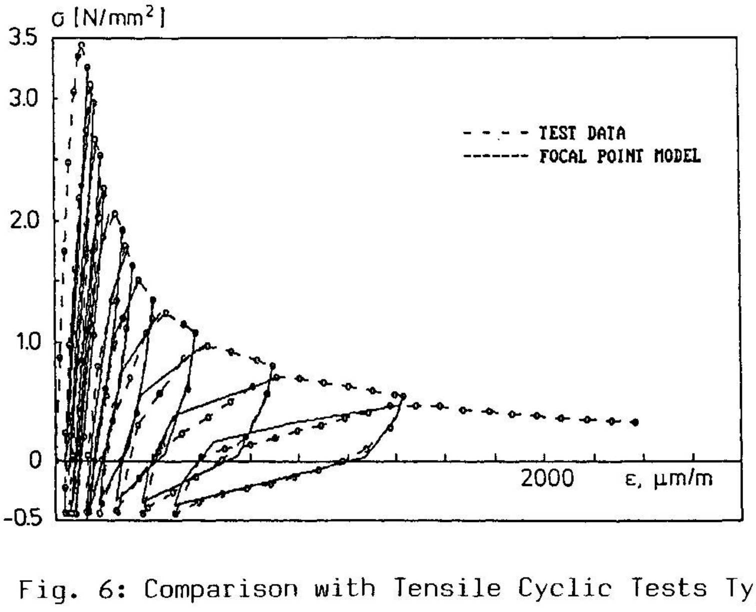

Fig. 6: Comparison with Tensile Cyclic Tests Type III

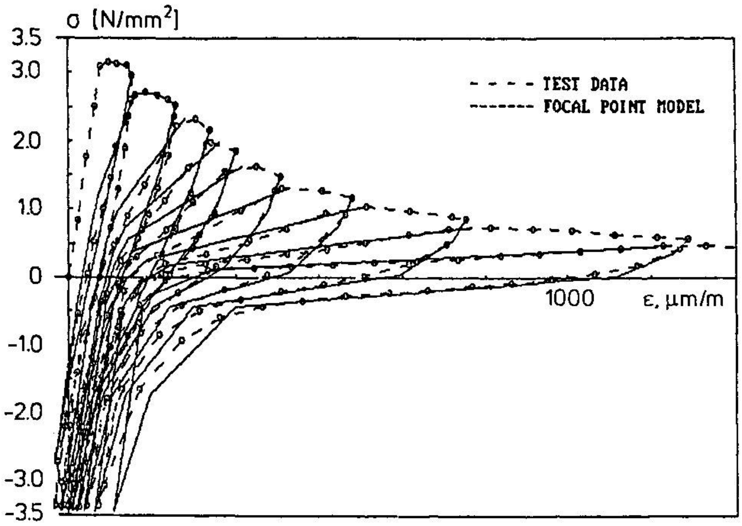

Fig. 7: Comparison with Tensile Cyclic Tests Type IV

5. SUMMARY AND CONCLUSIONS

A new one-dimensional model for random cyclic compression and tension is proposed. The model provides a set of rules to follow the cyclic uniaxial response of concrete once the envelope curves are given. The model determines a set of focal points with aid of which the complete piecewise linear unloading-reloading cycle, starting at a given point on the envelope, may be reproduced. The focal point model may be used graphically, with no accompanied calculations, or mathematically, following a subroutine in which the model rules are implemented.

The model has been compared with a variety of test results, in compression, in tension and in tension unloaded to compression and it is found to compare well with those tests. The model enables a more realistic representation of the complex behaviour of concrete in compression-tension and might be implemented in computer codes.

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