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Autor: Cohn, M.Z. / Riva, P.

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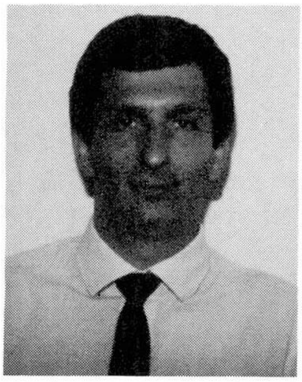
Constitutive Laws of Structural Concrete for Application to Ninlinear Analysis

Lois constitutives du béton armé et précontraint et application à l'analyse non-linéaire

Werkstoffgesetze des Stahlbetons für die Anwendung auf nichtlineare Berechnungen

M.Z. COHN
Prof. of Civil Eng.
University of Waterloo
Waterloo, ON, Canada



P. RIVA
Res. Assist.
University of Waterloo
Waterloo, ON, Canada

SUMMARY

This is the first of a two-part study on a practice-oriented approach to the non-linear analysis based on a lumped-plasticity concept and realistic constitutive laws for structural concrete. The approach consists of developing moment-curvature-rotation relations for reinforced, prestressed and partially prestressed concrete sections, starting from accurate descriptions of natural laws for concrete in tension and compression, as well as for ordinary and prestressing reinforcements. Resulting relations enable prediction of the section response under monotonic loading throughout all behaviour states up to failure.

RÉSUMÉ

Cette contribution est la première partie d'une étude orientée vers l'analyse non-linéaire basée sur la notion de 'concentration de la plasticité' et des lois constitutives réalistes pour le béton. L'approche consiste à développer des relations moment-courbure-rotation pour le béton armé, précontraint et partiellement précontraint, en partant des expressions correctes pour les courbes caractéristiques du béton en traction et compression, ainsi que pour les armatures ordinaires et précontraintes. Les relations qui en résultent permettent d'étudier le comportement des sections sous charges monotones en passant progressivement par tous les états jusqu'à la rupture.

ZUSAMMENFASSUNG

Dies ist der erste Teil einer zweiteiligen Studie über eine praxisorientierte Methode, die es erlaubt, auf Grund eines diskreten Plastizitätsmodells und realistischen Werkstoffgesetzes eine nichtlineare Berechnung von Betonelementen durchzuführen. Es werden Beziehungen zwischen dem Moment, der Krümmung und der Verdrehung bei bewehrten, vorgespannten und teilweise vorgespannten Betonquerschnitten entwickelt. Den Ausgangspunkt bilden dabei genaue Beschreibungen des Verhaltens von Beton unter Druck und Zug sowohl für eine schlaffe als auch für eine vorgespannte Bewehrung. Die vorgestellten Ergebnisse ermöglichen eine Vorhersage des Verhaltens des Querschnittes bei konstanter Last unter allen Bedingungen bis zum Versagen.

1. INTRODUCTION

This study is an extension of some earlier investigations on the nonlinear analysis of structural concrete sections [1,2,3,4], aiming at more reliable techniques for the nonlinear analysis of hyperstatic structures, with particular reference to the inelastic states. Parallel efforts are reported in [5,6,7,8] and elsewhere. Considerable work has been devoted to the section response at the cracking and post-cracking states, including the effect of tension-stiffening [9,10,11].

Recent studies on partial prestressing and nonlinear analysis [12,13] suggest that it is possible to develop flexural theories that realistically account for the material behaviour, equally apply to reinforced, prestressed and partially prestressed concrete (R.C., P.C., and P.P.C., respectively), and continuously describe all load-deformation states, including cracking, service, yielding, and the ultimate state.

The paper presents a constitutive model for structural concrete sections that has the above features and will serve as a basis for further developments in the nonlinear analysis of hyperstatic structures based on the "lumped-plasticity" model [14,15,16,17].

2. ANALYTICAL MODEL

2.1 Assumptions

(1) Quasi-static (monotonic, non-repeated, non-reversible) loading.
(2) Negligible shear effects.
(3) Planarity of sections (linear strain distribution).
(4) Known natural stress-strain relationships (analytical, experimental, etc.)
(5) Mono-axial stress-strain laws valid for section analysis.

For the cracked states the section considered (B) is the middle of an element (AC) of length ℓ_c, equidistant to the neighbouring cracks, as shown in Fig. 1.

Moments are assumed to be constant over the length of the element ℓ_c.

2.2 Material Constitutive Laws

For brevity, the adopted constitutive relations are summarized in Figs. 2 - 4, along with the relevant notation and equations.

Figs. 2a and 2b illustrate the $\sigma_c - \varepsilon_c$ and $\sigma_{ct} - \varepsilon_{ct}$ equations for the concrete in compression [1] and tension [10], respectively.

Fig. 3 shows the stress-strain curves and their analytical expressions for ordinary and prestressing steels, adapted from [1].

Figs. 4a and 4b illustrate the assumed bond-slip relations $\tau_{bs} - s_s$ and $\tau_{bp} - s_p$ for ordinary and prestressing steels, respectively [18].

2.3 Compatibility Equations

We consider a typical element AB with cross section strain, stress distribution and notation in Fig. 5. Since for the cracked section the neutral axis and rotation axis do not coincide, the compatibility conditions are expressed in terms of the latter, with corresponding ordinates related by $z = y - (y_G - z')$:

$$u_{cB} = \phi(z+z_0)/2 \ ; \quad u_{sB} = \phi(d_s-z'-z_0)/2 \ ; \quad u_{pB} = \phi(d_p-z'-z_0)/2 \qquad (1)$$

Strains in the reinforcing and prestressing steels are expressed with consideration of the bond effects as:

$$\varepsilon_{sB} = 2u_{sB}/\ell_c + \Delta\varepsilon_s \ ; \quad \varepsilon_{pB} = f_{pe}/E_p + 2u_{pB}/\ell_c + \Delta\varepsilon_p \qquad (2)$$

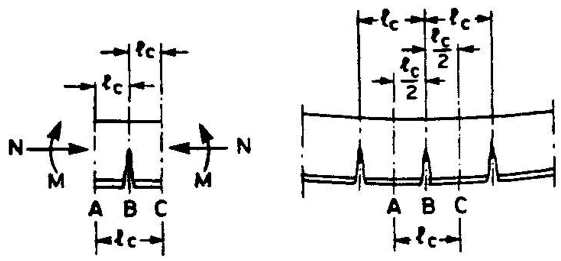


Fig. 1

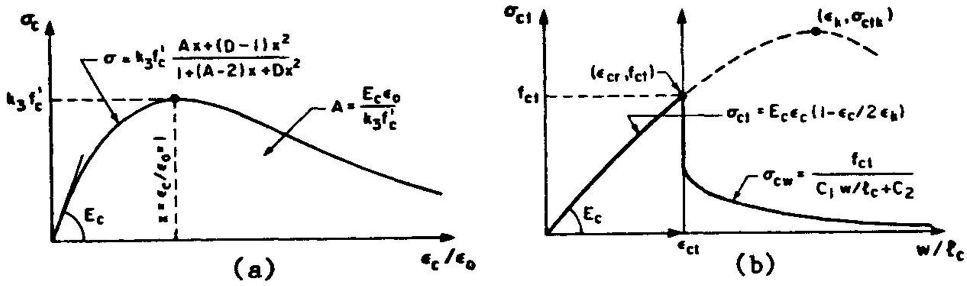


Fig. 2

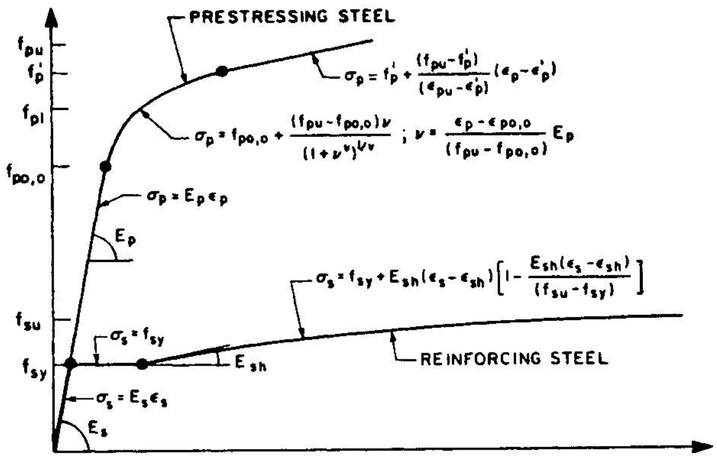


Fig. 3

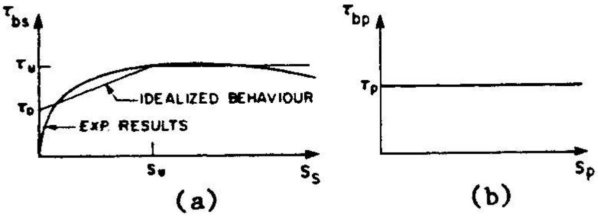


Fig. 4

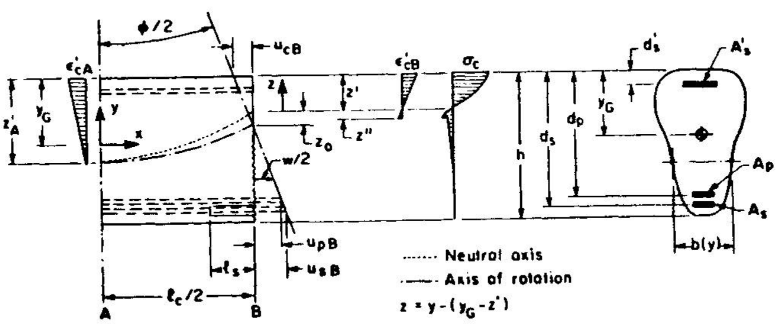


Fig. 5

where ℓ_c is the crack spacing, $2u_{sB}/\ell_c$ and $2u_{pB}/\ell_c$ are the average deformations of A_s and A_p, respectively, and $\Delta\varepsilon_s$, $\Delta\varepsilon_p$ are the strain contributions of the tension-stiffening effect.

With the bond-slip constitutive laws in Fig. 4, expressions for $\Delta\varepsilon_s$ and $\Delta\varepsilon_p$ may be obtained for various behaviour states.

(a) Partial bar slip ($\ell_s < \ell_c/2$): Denoting as ℓ_s the bar length over which slip takes place, it can be shown that [19]:

$$\Delta\varepsilon_s = -2u_{sB}/\ell_c + \alpha\left[\frac{z}{t}\,\frac{2\alpha\ell_s}{\ell_c} + \frac{\varepsilon''}{\alpha}\right]\frac{\sinh(2\alpha\ell_s/\ell_c)}{2\alpha\ell_s/\ell_c} \tag{3}$$

where $\alpha = \sqrt{\tau_1\ell_c^2/E_sD_s}$; $t = \tau_1\ell_c/\tau_0$; $\varepsilon'' \approx \varepsilon_{sA} = \varepsilon_{cr}(h - z_A')/(d_s - z_A')$; and a_s is related to u_{sB} by the equation:

$$\cosh(2\alpha\ell_s/\ell_c) = 1 + [2u_{sB}/\ell_c - \varepsilon''(1-\ell_s/\ell_c)]/[2/t+\varepsilon''\ell_c/2\alpha^2\ell_s] \tag{3'}$$

(b) Full bar slip and moderate cracking ($\ell_s = \ell_c/2$ and $w_s \leq 2s_u$): Denoting as w_s the crack width at the reinforcing steel level and the slip limit (Fig. 4) as s_u, $\Delta\varepsilon_s$ may be expressed as [19]:

$$\Delta\varepsilon_s = \left(2/t+\varepsilon''/\alpha^2\right)\left[\alpha\,\frac{\cosh\alpha-1}{\sinh\alpha}\right] - \frac{\varepsilon''}{2}\,\alpha\,\frac{\cosh\alpha}{\sinh\alpha} + \frac{2u_{sB}}{\ell_c}\left(\frac{\alpha\cosh\alpha}{\sinh\alpha} - 1\right) \tag{4}$$

(c) Extensive cracking ($w_s > 2s_u$):

$$\Delta\varepsilon_s = (\tau_{su}/E_s)(\ell_c/D_s) \tag{5}$$

(d) Prestressing steel contribution:

$$\Delta\varepsilon_p = (\tau_p/E_p)(\ell_c/D_p) \tag{6}$$

where D_s and D_p are the diameters of a reinforcing steel bar and of a prestressing tendon, respectively.

Consideration of the strain distributions at sections A and B in Fig. 5 enables the longitudinal displacement in the cracked section to be expressed as [9]:

$$u_{cB}(z) = \int_0^{\ell_c/2} \varepsilon_c(x,z)\,dx = \varepsilon_{cB}(z)\ell_c/2\gamma(z) \tag{7}$$

where ε_{cB} is the concrete strain at a fibre z of the cracked section and $\gamma(z)$ is the ratio between the concrete strain and the average deformation, i.e., $\gamma(z) = (\ell_c/2)\,\varepsilon_{cB}(z)/\int_0^{\ell_c/2}\varepsilon_c(x,z)\,dx$.

The concrete strain ε_{cB} may be obtained from (1) and (7):

$$\varepsilon_{cB}(z) = \phi z f/\ell_c \tag{8}$$

where ϕ/ℓ_c is the average compression of the element and $f = \gamma(z)(z+z_0)/z$ is a parameter related to the strain distribution variation between sections A and B [9]. These distributions also allow the definition of the distance z_0 betwen the neutral axis and the rotation axis of the cracked section (Fig. 5). The resulting expressions are:

$$f = 2z_A'^2/(z_A'^2+z'^2) \quad \text{and} \quad z_0 = z'^2(z_A'-z')/(z_A'^2+z'^2) \tag{9}$$

Relation (8) also allows the strain in the compression reinforcement to be expressed as:

$$\varepsilon_{sB}' = (\phi/\ell_c)(z'-d_s')f \tag{10}$$

2.4 Equilibrium Equations

Expressing the strain equations (7), (8) and (10) as functions of the average curvature $\bar{\phi} = \phi/\ell_c$ through relations (1), and adopting the appropriate $\sigma - \varepsilon$ constitutive laws for concrete and steels (Section 2.2), the equilibrium equations and $M - \bar{\phi}$ constitutive law may be formulated.

The equilibrium equations, referred to the centroid of the gross cross section, are:

$$N = \int_0^{z'} \sigma^+_{cB}\, b\,dz + A'_s \sigma'_{sB} + \int_{-z''}^{0} \sigma^-_{cB}\, b\,dz + \int_{-(h-z')}^{-z''} \sigma^-_{cB}(w)\, b\,dz - A_s \sigma_{sB} - A_p \sigma_{pB} \tag{11}$$

$$M = \int_0^{z'} \sigma^+_{cB}[z+(y_G-z')]\,b\,dz + A'_s \sigma'_{sB}[y_G-d'_s] + \int_{-z''}^{0} \sigma^-_{cB}[z+(y_G-z')]\,b\,dz + \int_{-(h-z')}^{-z''} \sigma^-_{cB}(w)[z+(y_G-z')]\,b\,dz + A_s \sigma_{sB}[d_s-y_G] + A_p \sigma_{pB}[d_p-y_G] \tag{12}$$

In equations (11) and (12) $b = b(z)$, $\varepsilon = \varepsilon(z)$, $\sigma = \sigma(\varepsilon)$ and $w = w(z)$; the term in w is due to the constitutive law adopted for the concrete in tension, based on experimental results reported in [20]. Further tests in progress at Politecnico di Milano, Milano, Italy, seem to confirm this law and the values of related constants [10].

Finally, the crack width may be expressed as:

$$w(z) = w_s(z-z'')/(d_s-z'-z'') \tag{13}$$

where the crack width at the reinforcing steel level is:

$$w_s = [(\phi/\ell_c)(d_s-z'-z_0)-\varepsilon_{sA}(1-\ell_s/\ell_c)-\varepsilon_{sB}\ell_s/\ell_c]\,\ell_c \tag{14}$$

We note that the adopted constitutive law for concrete in tension, Fig. 2a, is appropriate for predicting the overall behaviour of cracked sections, but is not suitable for the accurate investigation of the local response around the crack tip.

2.5 Computational Features

A general computer program, MOCURO (MOment-CUrvature-ROtation) has been developed in order to automatically handle the governing conditions of section response at all loading states. Any symmetrical concrete section with up to fifteen layers of mild and/or prestressing steel, under either positive or negative bending, may be analyzed. The program accepts any experimental, analytical or assumed point-by-point constitutive laws.

Each point of the $M - \bar{\phi}$ constitutive law is determined by imposing the curvature and solving the equilibrium equation (11) by an iterative procedure. During the phase of partial slip, the length ℓ_s is imposed and the curvature is given by equation (3').

3. PARAMETRIC STUDY

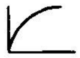
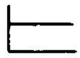
3.1 Governing Parameters

A parametric study has been conducted in order to ascertain the influence of the basic parameters that govern the $M - \bar{\phi}$ constitutive law.

Material characteristics have been assumed constant throughout the study and their influence on the $M - \bar{\phi}$ relation has not been investigated.

By reference to material laws and definitions in Figs. 2, 3 and 4, the numerical constants used in computations are identified in Table 1.

Table 1

CONCRETE COMPRESSION		f'_c = 40 MPa ; E_c = 29 930 MPa ; ϵ_o = 0.00264 ; A ≅ 2.5 ; 0 ≃ 0.362 ; k = 0.8 [1]
CONCRETE TENSION		f_{cr} = 4.5 MPa ; c_2 = 2 ; c_1/e_c = 12 000 [10]
MILD STEEL		f_{sy} = 400 MPa ; E_s = 200 000 MPa ; f_{su} = 600 MPa ; ϵ_{su} = 7 % [22]; ϵ_{sh} = 1% ; E_{sh} = 6500 MPa [21]
MILD STEEL BOND		τ_o = 3 MPa ; τ_{su} = 10 MPa ; s_u = 0.5 mm [18]
PRESTRESSING STEEL		$f_{po,p}$ = 1 300 MPa ; E_r = 190 000 MPa [STELCO]; f_{p1} = 1580 MPa, ϵ_p = 1% ; f_{pu} = 1860 MPa, ϵ_{pu} = 3.5% [22]; f'_p = 17.40 MPa ; ϵ'_p = 1.9 %
PRESTRESSING STEEL BOND		τ_p = 4 MPa [23]

Major parameters are related to the problem geometry and include (1) the net reinforcement index, $\bar{\omega}$; (2) the mixed reinforcement index, $\bar{\gamma}$; (3) the degree of prestressing, κ; (4) the lateral steel percentage, ρ ; (5) the crack spacing, ℓ_c; and (6) the section shape.

Parameters ω and γ essentially reflect the ultimate flexural behaviour of concrete sections [4] and are redefined accordingly as follows:

$$\omega = (A_s f_{su} + A_p f_{pu} - A'_s f'_{su})/(bdf'_c) \qquad (15)$$

$$\gamma = (A_p f_{pu})/(A_p f_{pu} + A_s f_{su}) \qquad (16)$$

These definitions ensure that the ultimate moments of R.C. and P.C. sections with the same ω are identical; if strain-hardening of the reinforcing steel is negligible, definitions (15) and (16) coincide with those given in [4].

Parameter $\bar{\kappa}$ is defined as [4]:

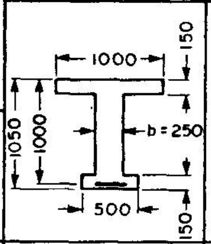
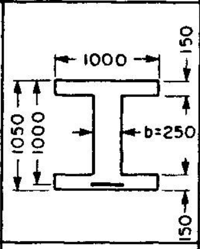
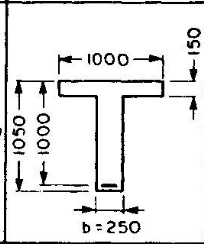
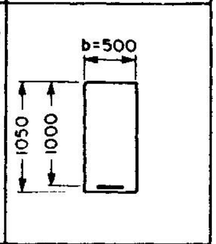
$$\bar{\kappa} = f_{pe}/f_{pa} \qquad (17)$$

where f_{pe} and f_{pa} are the effective and maximum allowable prestressing stresses, respectively.

3.2 Numerical Simulation Program

A set of 24 numerical tests on four section shapes, identified as A, B, C and D in Table 2, has been performed in order to study the influence of the aforementioned geometric parameters. The stirrups area and spacing, as well as the degree of prestressing κ (κ = 1), have been assumed constant in all the tests. For sections A and C both positive and negative moments have been considered. In all cases studied the yielding point is assumed to correspond to $\varepsilon_s = \varepsilon_{sy} = 0.2\%$ or $\Delta\varepsilon_p = \varepsilon_{sy} = 0.2\%$.

Table 2

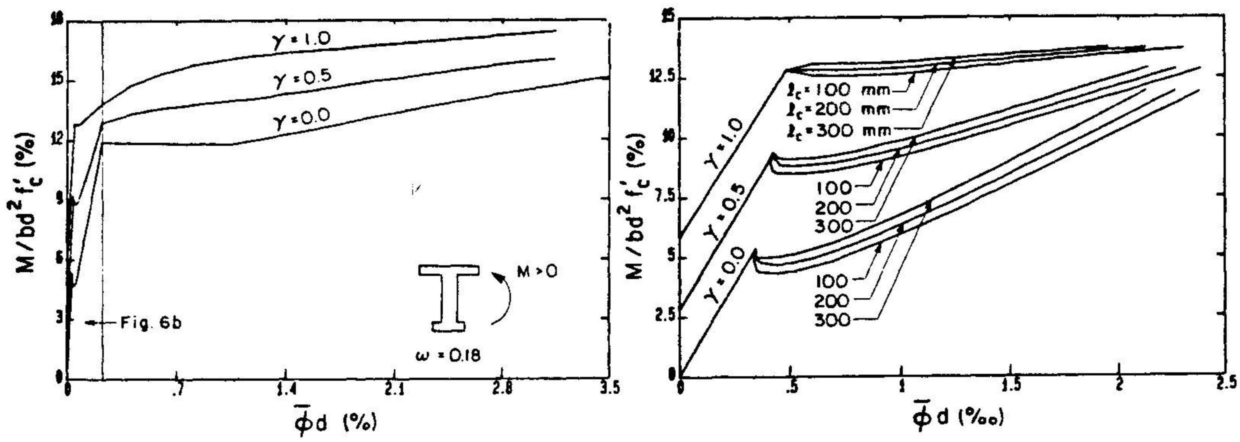
$\bar{K} = 1$			SECTION A	SECTION B	SECTION C	SECTION D
ω	γ	ℓ_c				
0.12	0.5	200	•	•	•	•
0.18	0.0	100	•			
		200	•			
		300	•			
	0.5	100	•			
		200	•	•	•	•
		300	•			
	1.0	100	•			
		200	•			
		300	•			
0.24	0.5	200	•	•	•	•
0.30	0.5	200	•	•	•	•

STIRRUPS #10 AT 300 mm IN ALL CASES

3.3 Influence of γ and ℓ_c on the M – $\bar{\phi}$ Constitutive Law

The influence of parameters γ and ℓ_c for Section A and ω = 0.18 is shown in the diagrams of Fig. 6. The basic differences in the behaviour of R.C. (γ = 0), P.P.C. (γ = 0.5) and P.C. sections may be summarized as follows:

- the stiffness of the first state is practically constant, but the cracking moment and curvature increase with the amount of prestressing;
- the transition from the first to the second state is characterized by a sudden drop of the moment, which is larger for small γ values;
- for the second state, section stiffnesses decrease with higher γ values;
- increases of the crack spacings ℓ_c correspond to smaller moment drops at the onset of cracking and to increases of the tension stiffening effects;
- for the R.C. section (γ = 0), the transition from the second to the third state takes place at $\varepsilon_s = \varepsilon_{sy}$ = 0.2%, and is sharper than for the P.P.C. or P.C. sections ($\gamma > 0$); for the P.C. section the transition point is no longer well defined;
- the third state is essentially characterized by the strain hardening of the section, with a more pronounced initial yield plateau for the R.C. section (γ = 0);
- for the P.C. and R.C. sections, the ultimate moments are the same; for the P.P.C. section with the same ω, the ultimate moment is smaller because, as the prestressing still governs the behaviour, the mild steel reinforcement cannot develop its ultimate strength;
- the ultimate curvature is much larger for the R.C. than for the P.C. and P.P.C. sections because of the correspondingly higher maximum strain of the mild steel;
- as expected, the crack spacing ℓ_c has only a negligible influence on the third state.

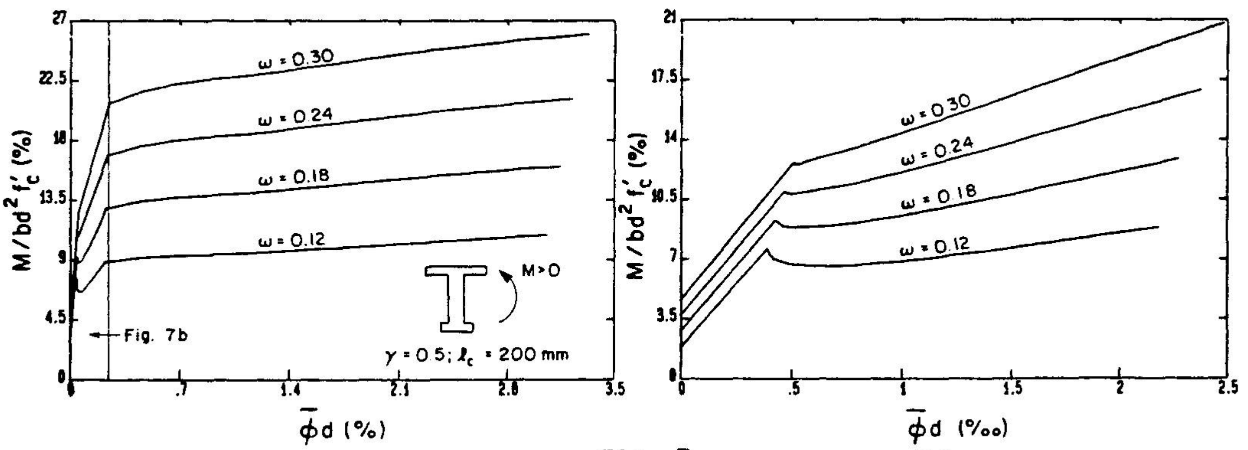


(a) **Fig. 6** (b)

3.4 Influence of ω on the M – $\bar{\phi}$ Constitutive Law

The influence of ω for Section A, with γ = 0.5, may be seen from Fig. 7:

- the stiffness of the first state is practically constant, while the cracking moment and curvature increase with ω;
- the drop of the moments at the onset of cracking increases for decreasing ω values;
- the stiffnesses of the second state increase with ω; for the lower ω value investigated (ω = 0.12) after the moment drop, the cracking moment value is again reached only for a curvature equal to more than three times the cracking curvature;
- the yielding moments and curvatures increase with ω;
- the stiffness in the third state are practically independent of ω, while the ultimate moments and curvatures increase with ω.



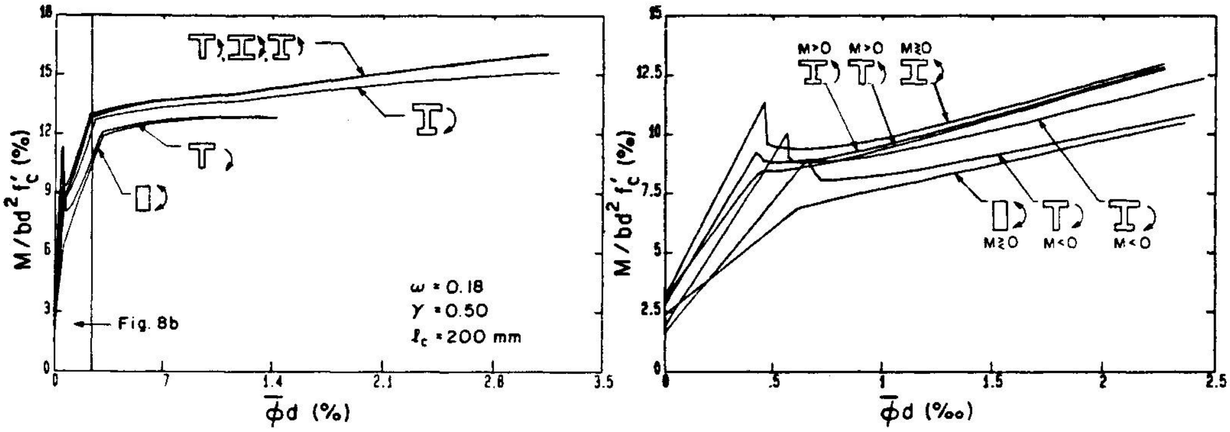
(a) **Fig. 7** (b)

3.5 Influence of the Section Shape on the M – $\bar{\phi}$ Constitutive Law

The influence of the section shape when ω = 0.18 and γ = 0.5 may be seen from Fig. 8:

- the stiffnesses in the first state and the cracking moment and curvature values are strongly affected by the section shape;
- sections with a flange in tension exhibit the highest moment drops at the onset of cracking because of the correspondingly largest losses of inertia; no discontinuities are noted for the rectangular and T sections under positive moments;

- for I sections the moment drop in the second state is as high as 15%, and the cracking moment value is again reached only at a curvature of about 3.5 times the cracking curvature;
- the slope of the curves in the second state is practically independent of the section shape, as it essentially depends on ω;
- the stiffnesses at the third state are also practically constant;
- sections with compression flanges exhibit higher ultimate moments and curvatures than the rectangular section and the T section under negative moment;
- the behaviour at the advanced second and third states is governed by flange-to-web-width ratios: a similar behaviour is noted for the T and I sections under positive moment (identical b_f/b_w), and for the rectangular and T section under negative moment; the non-symmetric I section under negative moment exhibits an intermediate behaviour.



(a) **Fig. 8** (b)

3.6 Ductility Factor $\bar{\phi}_u/\bar{\phi}_y$

The following remarks are made with reference to Fig. 9:

- for the wide-flanged sections, the ductility factor is practically independent of ω and has a value of about 14;
- the rectangular and T section under negative moment exhibit the lowest ductility, which is strongly affected by ω values;
- ductility factors of non-symmetrical I sections under negative moment assume intermediate values to the above cases;
- it is noted from Fig. 6 that ductility factors are reduced by increases of γ value and are almost independent of the crack spacings ℓ_c.

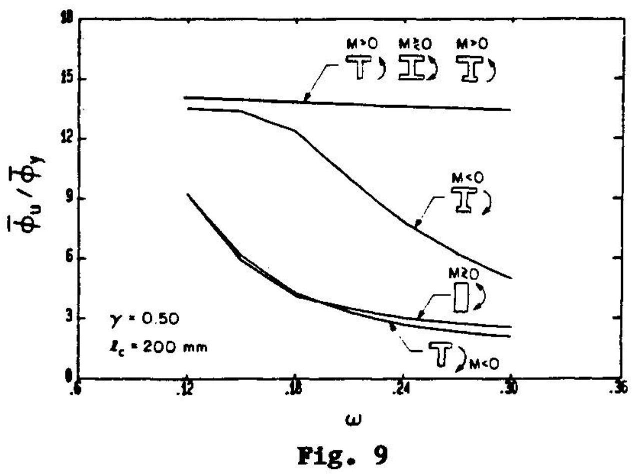


Fig. 9

4. CONSTITUTIVE LAW AND BEHAVIOUR OF STRUCTURAL CONCRETE

4.1 R.C. versus P.P.C. and P.C.

The basic differences between R.C., P.P.C. and P.C. sections can be summarized as follows:

- P.P.C. and P.C. sections are characterized by an increasingly higher cracking resistance;
- R.C. and P.C. sections can reach the same ultimate moment, whereas the same moment cannot be reached by P.P.C. sections (Section 3.3);
- R.C. sections are more ductile than P.P.C. and P.C. sections.

4.2 State I

- The behaviour is basically linearly elastic up to cracking. The cracking resistance increases, while the elastic stiffness slightly decreases, with increasing amounts of prestressing. This trend is noted for all section shapes.
- For high ω values, the real transition from the first to the second state occurs for moments slightly higher than the cracking moment, because of the resistance of the reinforcement to crack opening.
- For nonlinear analysis purposes, a perfectly elastic model up to the first cracking (neglecting the slight nonlinearity and the effective value of the transition moment) can be adopted without introducing appreciable errors and loss of generality or consistency.

4.3 State II

- The transition from the first to the second state is characterized by a sudden moment drop of up to 15% for I and T sections under negative moment. This drop becomes smaller for higher ω and γ, for sections under positive moment, and becomes negligible for rectangular and T sections.
- The stiffnesses at the second state increase with ω, decrease for larger γ, and are practically independent of the section shape.
- The tension stiffening effect increases with better bond behaviour of the reinforcement and with the crack spacing.
- Due to the high moment and curvature discontinuity at the onset of cracking and at the early second state, a reliable model for nonlinear analysis must account for the tension stiffening effect and the crack propagation.
- A linear approximation of the second state is valid only for rectangular or T sections under positive moment.

4.4 State III

- For R.C. and lightly prestressed P.P.C. sections, the transition from the second to the third state is clearly identified and corresponds to the yielding of the mild steel. For P.C. sections, the transition is characterized by a gradual change of stiffness and a yielding point is no longer clearly identified.
- The third state is characterized by strain-hardening, with the exception of an initial plastic behaviour for R.C. sections.
- R.C. and P.C. sections with the same ω may develop the same ultimate moment. In general, P.P.C. sections cannot reach the same moment.
- The ductility is mainly governed by ω and by the shape of the section. It is higher for flanged sections and lower for high ω values.
- A linear approximation of the third state appears reasonable for P.C. and highly prestressed P.P.C. sections. In the transition zone from the second to the third state, a nonlinear behaviour must be assumed if no arbitrary transition point is chosen.

5. CONCLUSIONS

This study has enabled the analytical derivation of the moment-curvature constitutive law for R.C., P.C. and P.P.C. flexural sections under quasi-static loading, starting with the description of stress-strain relations of constitutive materials. Consideration of tension-stiffening effects over elements of variable lengths (equal to the crack openings) allows a better understanding of the section behaviour at and after cracking. Generation of the constitutive law is automatically performed by a computer program (MOCURO) that combines the positive features of similar codes during the first and last behaviour states [10,4]:

(a) realistic prediction of material response;
(b) general application to R.C., P.C. and P.P.C.;
(c) historical, complete description through all loading states.

Parametric studies on the effects of the amount of prestressing, the total reinforcement index, the cross-section shape, and the crack spacing on the flexural response have yielded results within the expected ranges and tend to confirm the reliability of the model.

Further investigations, currently in progress, consist of applying the developed model to the analysis of hyperstatic structures of reinforced, prestressed and partially prestressed concrete. The purpose of these investigations is to achieve the highest possible accuracy for the lumped-plasticity approach to nonlinear analysis [17], and to assess its capabilities with regard to both structural engineering research and practice. Indeed, this approach may help clarify some oustanding problems such as: the hyperstatic effects of prestressing, moment redistribution in the cracked range, realistic stiffness evaluation at service and inelastic moment redistribution. Also, a relatively simple, automated code, that uses standard-type input and predicts the complete nonlinear response, may become a valuable tool for structural concrete designers.

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