An Analytical Model for Frictional Shear Slip of Cracked Concrete

Modèle analytique pour l'étude du glissement par friction et cisaillement de béton fissuré

Ein Rechenmodell für die Rissreibung in Beton

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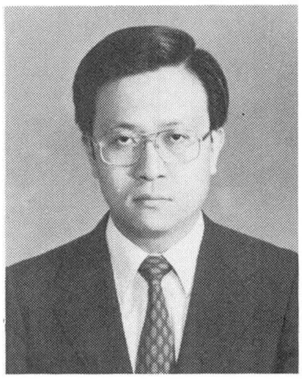

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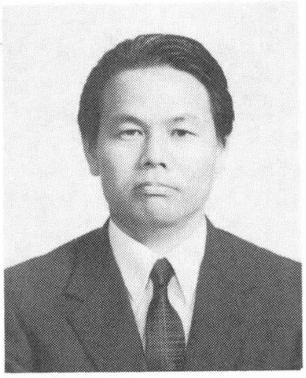

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SUMMARY

Analytical formulation of a constitutive equation for a single crack is made and modelling of four basic coefficients are discussed. Numerical simulation and comparisons with test results indicate that the proposed model can represent essential characteristics such as nonlinear shear transfer, aggregate interlock and crack dilatancy, and satisfactorily predicts experimentally observed results. Finally, the constitutive matrix is presented for concrete containing regularly distributed cracks.

RÉSUMÉ

La formulation analytique d'une équation constitutive pour une fissure simple est proposée et un modèle basé sur quatre coefficients est présenté. Une simulation numérique et des comparaisons avec des résultats d'essais montre que le modèle proposé peut représenter les caractéristiques essentielles telles que le transfert d'efforts tranchants non-linéaires, l'influence des agrégats et l'évolution des fissures. Il prédit de façon satisfaisante les résultats expérimentaux observés. Finalement la matrice constitutive est présentée pour un béton contenant des fissures distribuées de façon régulière.

ZUSAMMENFASSUNG

Ein Vierparameter-Werkstoffgesetz für den Einzelriss wird analytisch formuliert. Mittels numerischer Berechnungen und Vergleichs mit Versuchsergebnissen wird gezeigt, dass das Werkstoffgesetz die wichtigsten Merkmale wiedergibt, wie nichtlineare Schubübertragung, Rissdilatanz und Kornverzahnung. Zum Schluss wird das Werkstoffgesetz für Beton mit regelmässig verteilten Rissen gegeben.

1. INTRODUCTION

It has been long recognized that cracks in concrete have a significant effect on the mechanical response of reinforced concrete. This requires construction of an analytical model representing a single crack for nonlinear analysis of reinforced concrete members. The authors develop a constitutive equation that relates relative discontinuous displacements (shear slip and crack opening) and applied shear and normal stresses on crack surfaces of concrete in a state of plane stress.

In this paper, the formulation of a basic constitutive equation and modeling of the four coefficients contained in the constitutive matrix are described. Numerical examinations demonstrate that the shear stress-shear displacement relation exhibits highly nonlinear behavior and is quite sensitive to vertical constraint such as the normal displacement (crack width) and extensional stiffness of reinforcement across a crack. Numerical comparisons indicate satisfactory agreement with available test data.

The authors propose a constitutive model that can reflect such typical characteristics as shear transfer due to aggregate interlock, coupling effect (crack dilatancy and frictional contact slip) and path-dependence. Moreover, this model can easily be incorporated into conventional finite element codes without a great deal of revision by means of damage mechanics as well as crack strain method, which have been developed by Tanabe and his coworkers ([1]-[4], [17]-[19]).

2. BASIC EQUATIONS FOR STRESS-DISPLACEMENT RELATIONS

When expressing the mechanical behavior on the crack surface in a state of plane stress (Fig. 1), the relation between the shear stress τ_{nt}^c, the normal stress σ_n^c, the relative shear displacement (slip) δ_t and the relative normal displacement (crack opening) δ_n may be generally assumed in the form:

$$\begin{Bmatrix} d\tau_{nt}^c \\ d\sigma_n^c \end{Bmatrix} = \begin{bmatrix} B_{tt} & B_{tn} \\ B_{nt} & B_{nn} \end{bmatrix} \begin{Bmatrix} d\delta_t \\ d\delta_n \end{Bmatrix} , \quad \{d\sigma_c\} = [B]\{d\delta\} \tag{1}$$

in which B_{tt} and B_{nn} are the crack stiffness, and B_{tn} and B_{nt} function as off-diagonal terms ([5], [6]). Identifying all the terms in the matrix of [B] is a major objective of this paper, although no fully established formulation for Eq.1 has been attained yet.

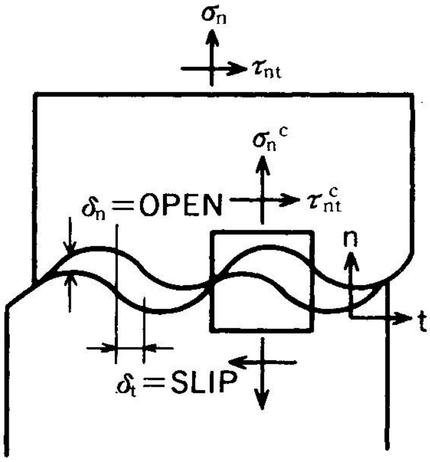


Fig. 1 Stresses and Discontinuous Displacements in a Single Crack

Examining experimental observations offered by many researchers, the following relations may be deduced. The shear displacement (shear slip) is affected by not only the applied shear stress but also the normal displacement (crack opening). The normal stress (always compressive) is of course determined by the normal displacement and also induced by the application of the shear stress especially when the crack opening is restrained. Consequently, the relations among these four variables can be expressed as follows:

$$\begin{aligned} \delta_t &= \delta_t(\tau^c_{nt}, \delta_n) \\ \sigma^c_n &= \sigma^c_n(\delta_n, \tau^c_{nt}) \end{aligned} \tag{2}$$

Differentiating each of Eq. 2 by making use of the chain rule, then,

$$d\delta_t = \frac{\partial \delta_t}{\partial \tau^c_{nt}} d\tau^c_{nt} + \frac{\partial \delta_t}{\partial \delta_n} d\delta_n \, , \quad d\sigma^c_n = \frac{\partial \sigma^c_n}{\partial \delta_n} d\delta_n + \frac{\partial \sigma^c_n}{\partial \tau^c_{nt}} d\tau^c_{nt} \tag{3}$$

is obtained. Here, we introduce the four coefficients defined by the differential derivatives appeared in Eq. 3 such that:

$$K_t \equiv \left(\frac{\partial \delta_t}{\partial \tau^c_{nt}}\right)^{-1}, \quad k_n \equiv \frac{\partial \sigma^c_n}{\partial \delta_n}, \quad \mu_f \equiv \left(\frac{-\partial \sigma^c_n}{\partial \tau^c_{nt}}\right)^{-1}, \quad \beta'_d \equiv \left(\frac{\partial \delta_t}{\partial \delta_n}\right)^{-1}, \quad \beta_d \equiv \frac{\partial \delta_n}{\partial \delta_t} \tag{4}$$

in which k_t = the shear stiffness, k_n = the normal stiffness, β_d , β'_d = the dilatancy ratio and μ_f = the frictional coefficient, respectively, all of which are defined in the tangential (incremental) form. Then, substituting the four coefficients into Eq. 3 leads to the following constitutive relation in the matrix form.

$$\begin{Bmatrix} d\delta_t \\ d\sigma^c_n \end{Bmatrix} = \begin{bmatrix} \frac{1}{k_t} & \frac{1}{\beta'_d} \\ -\frac{1}{\mu_f} & k_n \end{bmatrix} \begin{Bmatrix} d\tau^c_{nt} \\ d\delta_n \end{Bmatrix} \tag{5}$$

Note that in both vectors in Eq. 5 stress components and their corresponding displacement components are mixed, which may stem from the coupling effect between opposite crack surfaces. This important but cumbersome mechanical behavior can be represented by the above derived equations, which seems quite different from conventional types of stress$\sim$strain relations for continuum solid mechanics.

By modifying the form of Eq. 5 into the usual form as Eq. 1, the following equation is obtained.

$$\begin{Bmatrix} d\tau^c_{nt} \\ d\sigma^c_n \end{Bmatrix} = k_t \begin{bmatrix} 1 & -(1-\xi)\frac{1}{\beta_d} \\ -\frac{1}{\mu_f} & \frac{1}{\mu_f \beta_d} \end{bmatrix} \begin{Bmatrix} d\delta_t \\ d\delta_n \end{Bmatrix} \tag{6}, \qquad \begin{Bmatrix} d\delta_t \\ d\delta_n \end{Bmatrix} = \frac{1}{\xi k_t} \begin{bmatrix} 1 & (1-\xi)\mu_f \\ \beta_d & \mu_f \beta_d \end{bmatrix} \begin{Bmatrix} d\tau^c_{nt} \\ d\sigma^c_n \end{Bmatrix} \tag{7}$$

where ξ is a nondimensional parameter calculated from the four coefficients such that:

$$\xi = \mu_f \beta_d \frac{k_n}{k_t} \tag{8}$$

Eqs. 6 and 7 appear to be asymmetric and the matrix [F] becomes singular when $\xi=0$ and is guaranteed positive definite under the condition that $\xi>0$.
It should be noted that the proposed model, Eq. 7, includes Heuze and Barbour's formula [8] (called "uncoupled approach") as a special case when $\xi=1$ and that ASCE's comment (made in Chapter 5 of Ref. [5]) that the term F_{tn} in the matrix [F] is very unlikely, corresponds to Eq. 7 assuming $\xi=1$. Eq. 7 when $\xi=0$ is reduced to slip-dilatancy model by Bažant and Tsubaki [7] if solid concrete between cracks is assumed rigid.

3. MODELING OF THE FOUR COEFFICIENTS

It is expected that the four coefficients introduced in the previous section are not constant quantities but vary depending on applied stresses and displacements as well as concrete strength, size of aggregate and roughness of crack. Thus, sophisticated modeling for k_t, k_n, β_d and μ_f is required, being represented as nonlinear functions of state-variables and concrete properties. This is described subsequently, making use of experimental data presently available from existing literature.

Shear Stiffness k_t: According to experimental observations, in the initial shear load, rather free slippage occurs on the cracked plane which is not in close contact, and further application of the shear stress makes the cracks stiffer due to firm contact (aggregate interlock). Finally, the shear stress levels off approaching the ultimate shear strength. This behavior can be represented by a hyperbolic curve such that:

$$\tau_{nt}^{c} = \tau_u \cdot \frac{\tanh\{\frac{K_o}{\tau_u}(\delta_t - \delta_{t1})\} + q}{1 + q} \tag{9}$$

$$k_t = K_{IST} \cdot sech^2\{\frac{K_o}{\tau_u}(\delta_t - \delta_{t1})\} \tag{10}$$

$$\text{where, } K_o = K_{IST}(1 + q), \quad q = \tanh(\frac{K_o}{\tau_u}\delta_{t1}) \tag{11}$$

in which K_{IST} denotes the maximum shear stiffness (expressed in MPa/mm) in the shear displacement δ_{t1} (in mm) and τ_u is the maximum shear strength (in MPa), which are illustrated in Fig. 2. The values of K_0 and q can be calculated from K_{IST}, δ_{t1} and τ_u by Eq. 11. Thus, the shape of a curve for shear slippage is characterized by means of these three values, which are considered to be significantly influenced by the normal displacement δ_n and material property. The following expressions are identified by regression analysis from experimental data ([9]-[13]).

$$K_{IST} = 3.74\,(\frac{f_c}{25})^{0.60}\,\delta_n^{-0.96} \tag{12}$$

$$\delta_{t1} = 1.42\,(\frac{D_a}{16})^{-1.20}\,\delta_n^{1.31} \tag{13}$$

$$\tau_u = \frac{0.01}{0.01 + (\frac{\delta_n}{D_a})^2}\,\tau_o, \quad \tau_o = (0.2 \sim 0.3) f_c \tag{14}$$

$$(f_c{:}MPa,\ D_a{:}mm,\ \delta_n{:}mm)$$

where f_c is the compressive strength of concrete (in MPa), D_a is the maximum aggregate size (in mm) and τ_0 is the shear strength of uncracked concrete (in MPa) interpreted as limiting value of asymptote for τ_u when $\delta_n \to 0$. Empirical formula for τ_u is taken from Ref.[6].

Normal Stiffness k_n: The relation between the normal stress and the normal displacement exhibits the simpler behavior as indicated in Fig. 3. Thus, the $-\sigma_n^c \sim \delta_n$ relation and the normal stiffness k_n can be represented in the form:

$$(15) \quad -\sigma_n^c = b_1(\delta_n - \beta_d\delta_t)^{-b_2} \tag{15}$$

$$(16) \quad k_n = b_1 b_2(\delta_n - \beta_d\delta_t)^{-(b_2+1)} \tag{16}$$

in which b_1 and b_2 are material constants. Although these constants are supposed to be dependent on concrete strength and aggregate size, this is not clarified due to the scarcity of test data. According to the authors' experiment, b_1=0.0082 and b_2=0.878 were obtained [14].

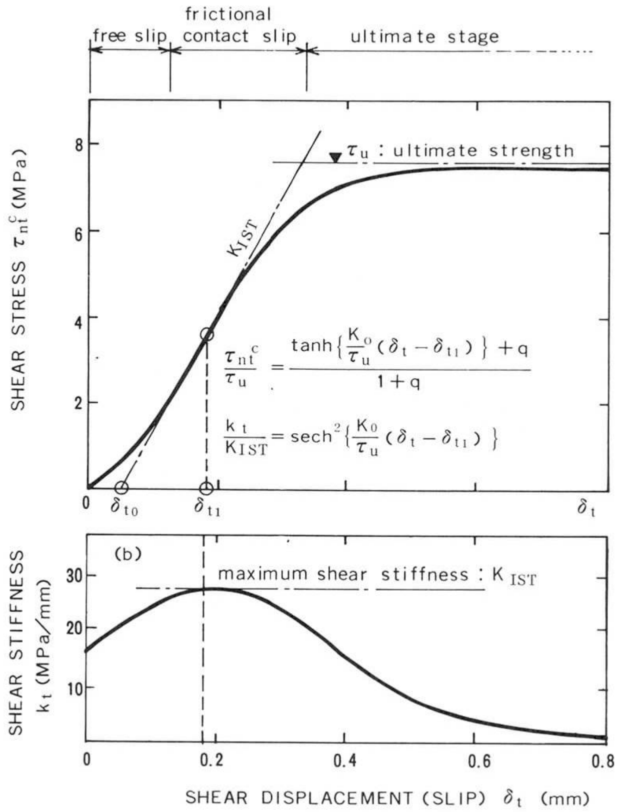


Fig. 2 Idealization for Shear Behavior, (a) Shear Stress ~ Shear Displacement Relation, (b) Tangential Shear Stiffness (An Example for a Case of δ_{t1}=0.18 mm, K_{IST}=27 MPa/mm, τ_u =7.5 MPa)

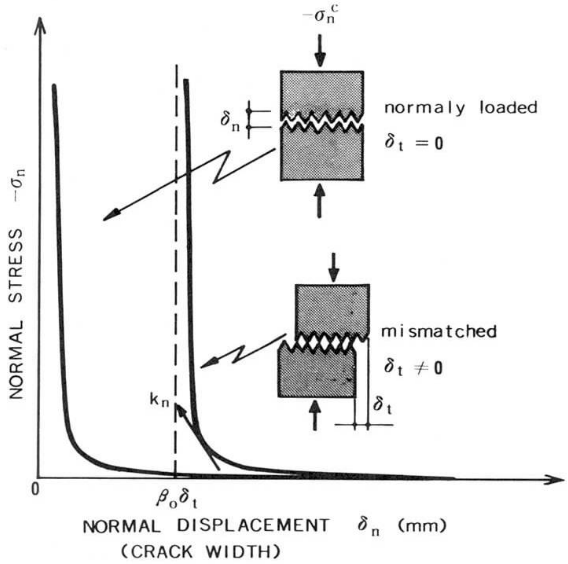


Fig. 3 Idealization for Relation of Normal Stress and Normal displacement

Dilatancy Ratio β_d and Frictional Coefficient μ_f: Even though crack dilatancy and frictional slip have been experimentally recognized and pointed out as important characteristic acompanying discontinuities, no fully successful model seems to have been attained in the past. The dilatancy ratio is defined as the ratio of δ_n and δ_t at constant σ_n^c (< 0), the following expression is obtained using experimental data (Yoshikawa [14]), as shown in Fig. 4.

$$\mu_f = 1.16\ exp\ (0.61\ \delta_n) \tag{17}$$

The frictional coefficient μ_f is, on the other hand, described as the ratio of $-\sigma_n^c$ and τ_{nt}^cat constant δ_n , which is found to be:

$$\beta_d = 1.64\ exp(-6.42\ |\frac{\sigma_n^c}{f_c}|) \tag{18}$$

from test results ([9], [14], [15]) as shown in Fig. 5.

Figs. 4 and 5 indicate that experimentally obtained results display large fluctuations of up to ±50%. Hence, constants of c_1 for μ_f and c_3 for β_d, both of which vary 0.5 to 1.5, are introduced for the present to compensate for the uncertainty of these two coefficients. This implies a need for further refinement of proposed expressions, Eqs. 17 and 18, as well as for carrying out more extensive experimental works.

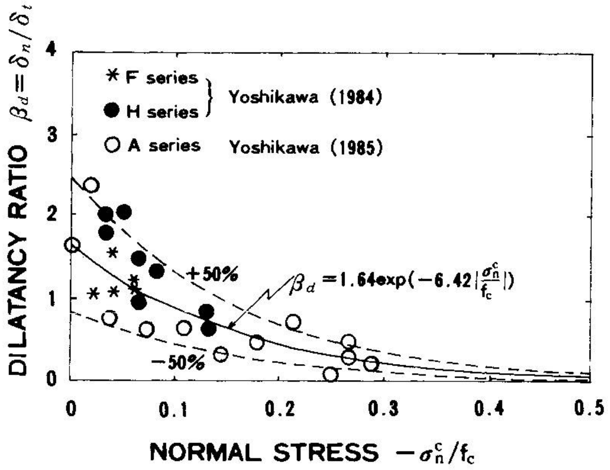


Fig. 4 Experimental Data for Dilatancy Ratio β_d and Obtained Regression Curve

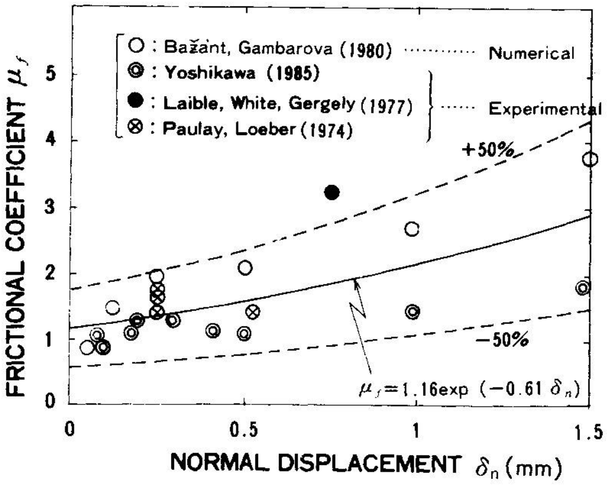


Fig. 5 Experimental Data for Frictional Coefficient μ_f and Obtained Regression Curve

All of the expressions for the four coefficients by the authors are summarized in Table-1. The shear stiffness k_t among others reveal the most complicated expressions requiring twelve material constants, $a_1 \sim a_{12}$, which seem to be inevitable because of the complexity of shear transfer mechanism.

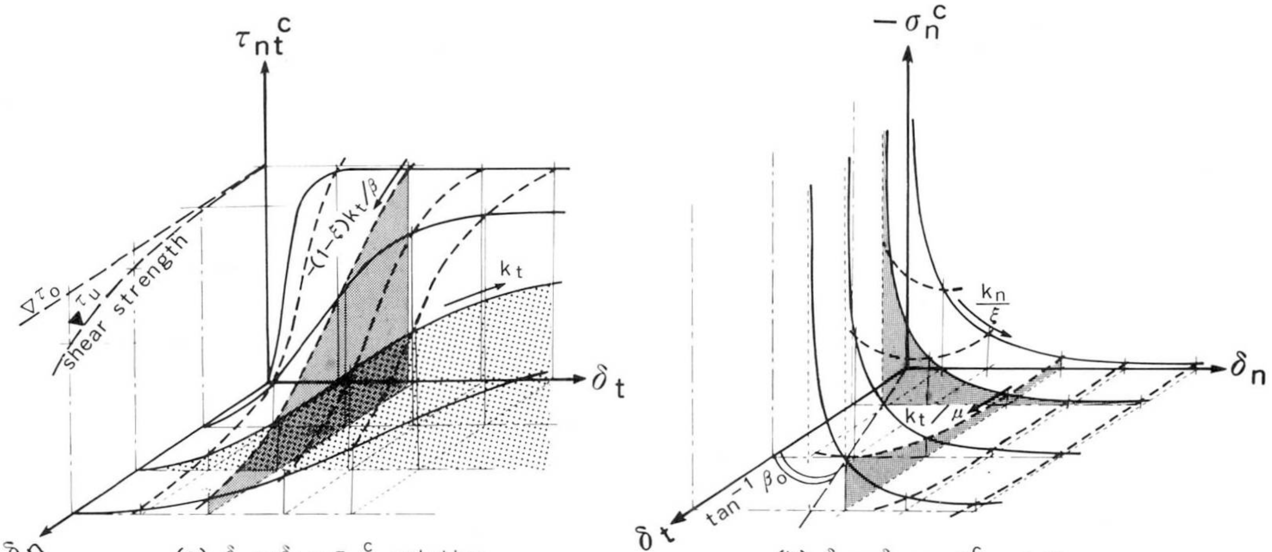
Table 1 Proposed Expressions for the Four Basic Coefficients and Experimentally Obtained Constants

SHEAR STIFFNESS : k_t [MPa/mm]	CONSTANTS
$k_t = K_{IST}\,\mathrm{sech}^2\left\{\frac{K_0}{\tau_u}(\delta_t - \delta_{t1})\right\}$	
$K_{IST} = a_1\left(\frac{f_c}{25}\right)^{a_2}\left(\frac{D_a}{16}\right)^{a_3}\delta_n^{-a_4}$	$a_1 = 3.74$, $a_2 = 0.60$ $a_3 = 0$, $a_4 = 0.96$
$\delta_{t1} = a_5\left(\frac{f_c}{25}\right)^{a_6}\left(\frac{D_a}{16}\right)^{-a_7}\delta_n^{a_8}$	$a_5 = 1.42$, $a_6 = 0$ $a_7 = 1.20$, $a_8 = 1.31$
$\tau_u = \tau_0 \frac{a_9}{a_{10} + (\delta_n / D_a)^{a_{11}}}$, $\tau_0 = a_{12} f_c$	$a_9 = a_{10} = 0.01$ $a_{11} = 2$
$K_0 = K_{IST}(1 + q)$, $q = \tan h\left(\frac{K_0 \delta_{t1}}{\tau_u}\right)$	$a_{12} = 0.2 \sim 0.3$ (0.245)

NORMAL STIFFNESS : k_n [MPa/mm]	CONSTANTS
$k_n = b_1 b_2 (\delta_n - \beta_d \delta_t)^{-(b_2+1)}$	$b_1 = 0.0082$ $b_2 = 0.878$
FRICTIONAL RATIO : μ_f	**CONSTANTS**
$\mu_f = c_1 \mu_0 \exp(c_2 \delta_n)$	$\mu_0 = 1.16$ $c_1 = 0.5 \sim 1.5$ $c_2 = 0.61$
DILATANCY RATIO : β_d	**CONSTANTS**
$\beta_d = c_3 \beta_0 \exp\left(-c_4\left\|\frac{\sigma_n^c}{f_c}\right\|\right)$	$\beta_0 = 1.64$ $c_3 = 0.5 \sim 1.5$ $c_4 = 6.42$

4. NUMERICAL SIMULATION AND COMPARISONS WITH TEST RESULTS

Fig. 6 depicts schematic descriptions of shear stress and normal stress, being represented as functions of displacements, δ_t and δ_n , which are computed from the authors' proposed model. Shown in Fig. 7 are variations of the shear stress under the condition that D_a=25mm and f_c =25MPa or 40MPa. It may be concluded that these figures reveal the realistic nonlinear relations among the four state-variables, $\tau_{nt}{}^c$, $-\sigma_n{}^c$, δ_t and δ_n , and that obtained numerical results well conform to actually observed behavior.



(a) $\delta_n \sim \delta_t \sim \tau_{nt}^c$ relation (b) $\delta_t \sim \delta_n \sim -\sigma_n^c$ relation

Fig. 6 Schematic Descriptions of Shear and Normal Stresses being Represented as Functions of Shear and Normal Displacements

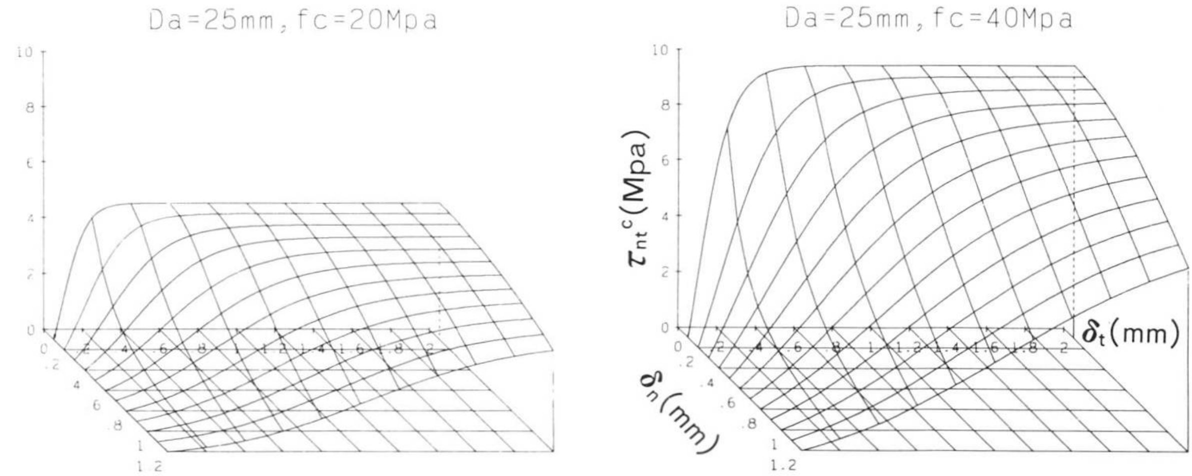


Fig. 7 Relationship between Shear Stress $\tau_{nt}{}^c$ and Displacements δ_n and δ_t (Effect of Concrete Strength on Shear Behavior)

Fig. 8 shows relations of two variables among τ_{nt}^c, $-\sigma_n^c$, δ_t and δ_n, in which fixed normal displacements (crack width) δ_{no} are chosen as a parameter in Fig. 8 a), b) and fixed normal stress $-\sigma_{no}^c$ is a parameter in Fig. 8 c).

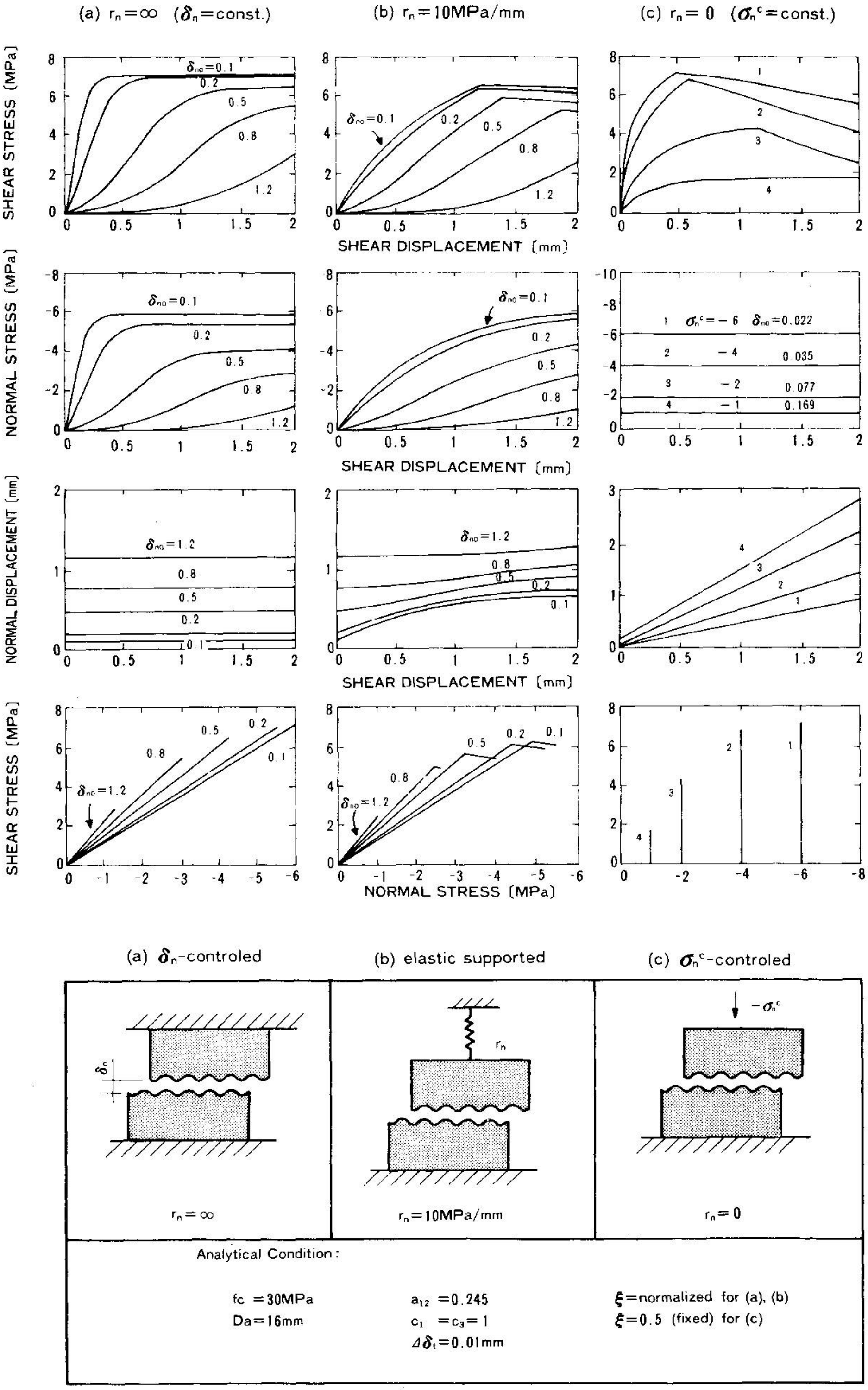
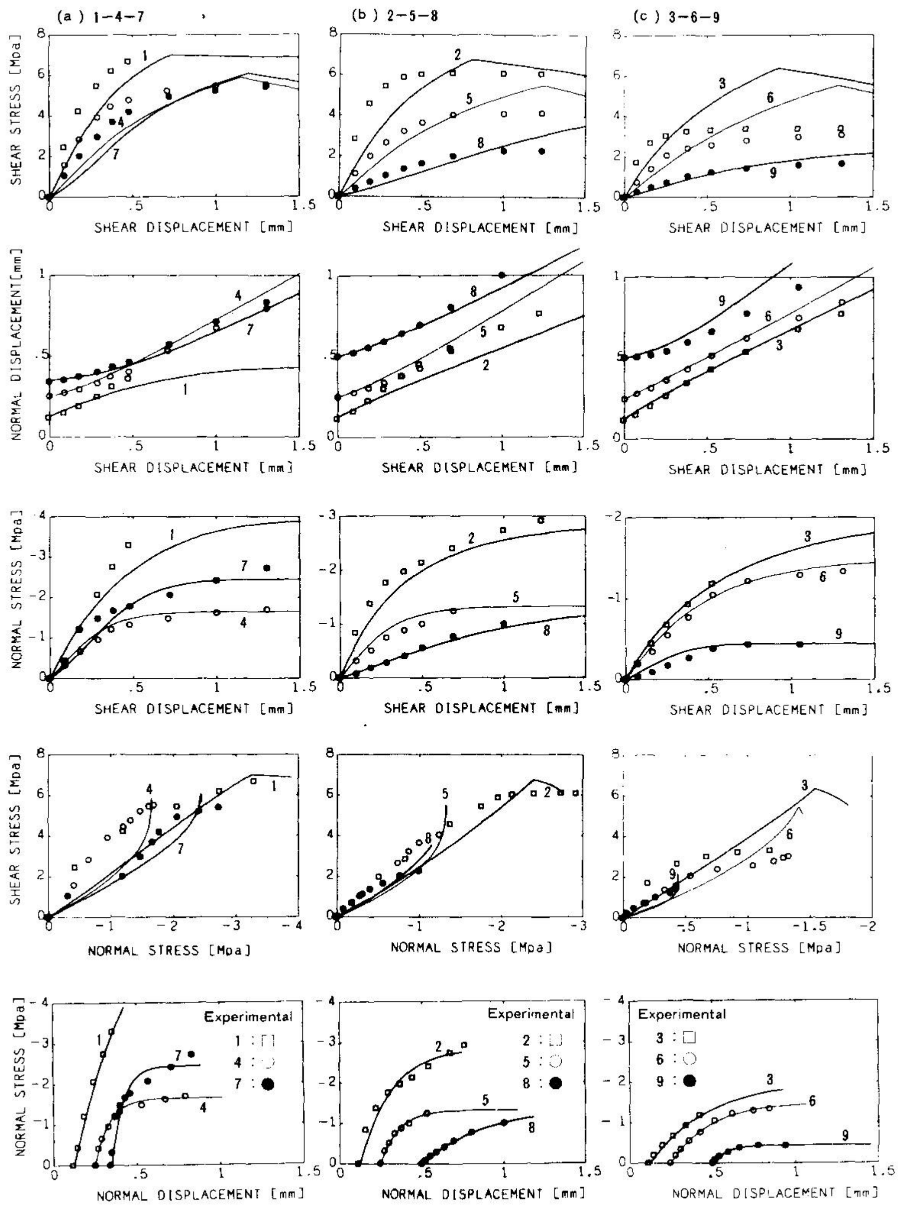


Fig. 8 Numerical Demonstration for Relations of Two Variables between τ_{nt}^c, $-\sigma_n^c$, δ_t and δ_n

In calculations, linear elastic spring is arranged in the direction normal to the crack surface, whose extensional stiffness is r_n (expressed in MPa/mm). The magnitude of the spring stiffness r_n determines the constraint condition such that δ_n remains to be constant when $r_n=\infty$ (Fig. 8 a)) and that $-\sigma_n^c$ is a constant value when $r_n=0$ (Fig. 8 c)), and r_n assumed in Fig. 8 b) exists in-between (where r_n=10 MPa/mm is assumed). A series of drawings in Fig. 8 indicates that

Direct Shear Tests by Millard, Johnson (1985)



Analytical Condition

ξ	a_{12}	c_1	c_3	Initial crack width δ_{no} (mm)			Concrete
0.5	0.245	1	0.5	□ 1 : 0.125	□ 2 : 0.125	□ 3 : 0.125	f_c=30Mpa
				○ 4 : 0.25	○ 5 : 0.25	○ 6 : 0.25	D_a=16mm
				● 7 : 0.35	● 8 : 0.50	● 9 : 0.50	

Fig. 9 Comparison of Calculated Values With Experimental Data (Millard and Johnson [16])

transmission of the shear stress along the crack surface is very sensitive to the constraint condition perpendicular to the crack surface. It is demonstrated that the shear stress is always accompanied by an increase of the crack opening ($d\delta_n > 0$), otherwise compressive normal stress is induced ($d\sigma_n^c < 0$) if the crack opening is restrained by the elastic spring.

Numerical comparison with observed data from Millard and Johanson's work [16] is shown in Fig. 9, where 9 specimens were tested with different initial crack width δ_{no} and different stiffness of reinforcement crossing a crack. Fig. 9 indicates that numerical values calculated from the proposed model are in relatively good agreement with experimental results.

5. CONCRETE WITH REGULARLY DISTRIBUTED CRACKS

We consider now a reinforced concrete panel (or wall, shell) that is carrying in-plane stresses, where regulaly distributed cracks are gradually generated due to excessive tensile stress. The proposed constitutive equation prepared for a single crack can be applied to such a cracked reinforced panel, utilizing the crack strain concept.

Letting Lc be crack spacing (a distance between two adjacent cracks), the expression for a crack strain vector in a state of plane stress can be made as follows:

$$\{d\varepsilon_{cr}\}^T = \{d\varepsilon_n^{cr} \quad d\varepsilon_t^{cr} \quad d\gamma_{nt}^{cr}\} = \frac{1}{L_c}\{d\delta_n \quad o \quad d\delta_t\} \tag{19}$$

assuming crack strains due to shear slip and crack opening are distributed evenly over the control area. Yoshikawa and Tanabe [4] mathematically derived the above equation using Dirac's delta function and its derivative.

The discontinuous displacements (shear slip and crack opening) given by Eq. 7 are substituted into Eq. 19, then, one has:

$$\{d\varepsilon_{cr}\} = \frac{1}{\xi k_t L_c}\begin{bmatrix} \mu_f \beta_d & o & \beta_d \\ o & o & o \\ (1-\xi)\mu_f & o & 1 \end{bmatrix}\begin{Bmatrix} d\sigma_n^c \\ d\sigma_t^c \\ d\tau_{nt}^c \end{Bmatrix} = [F]\{d\sigma_c\} \tag{20}$$

in which $\{d\sigma_c\}$ means applied stress vector having its three components, $d\sigma_n{}^c$, $d\sigma_t{}^c$ and $d\tau_{nt}{}^c$, and the matrix [F] denotes such that:

$$[F] = \frac{1}{\xi k_t L_c}\begin{bmatrix} \mu_f \beta_d & o & \beta_d \\ o & o & o \\ (1-\xi)\mu_f & o & 1 \end{bmatrix} \tag{21}$$

Here, it is assumed that the total strain $\{d\varepsilon\}$ is expressed by the sum of the crack strain, $\{d\varepsilon_{cr}\}$, and the elastic/plastic strain between cracks, $\{d\varepsilon_{sc}\}$, namely:

$$\{d\varepsilon\} = \{d\varepsilon_{sc}\} + \{d\varepsilon_{cr}\} \tag{22}$$

Letting [Dc] be the stiffness matrix of the uncracked part of concrete (solid concrete), then the constitutive equation of uncracked concrete is expressed by the following usual form:

$$\{d\sigma_c\} = [D_c]\ \{d\varepsilon_{sc}\} = [D_c]\ (\{d\varepsilon\} - \{d\varepsilon_{cr}\}) \tag{23}$$

Then, the relation of applied stress and total strain is obtained by eliminating crack strain from Eqs. 20 and 23. This is:

$$\{d\sigma_c\} = ([I] + [D_c][F])^{-1}\ [D_c]\{d\varepsilon\} \tag{24}$$

in which $[I]$ is a unit matrix.

Finally, the constitutive matrix $[D^*]$ expressing overall stiffness of cracked concrete is written as follows:

$$\begin{aligned} [D^*] &= [\psi][D_c] \\ \text{where, } [\psi] &= ([I] + [D_c][F])^{-1} \end{aligned} \tag{25}$$

In the above equation, the nondimensional matrix $[\psi]$ is considered to express the magnitude of degradation of cracked concrete due to discontinuous displacements (shear slip and crack opening) in addition to plasticity of solid concrete. This implies that nonlinear behavior of cracked concrete can be represented by means of a single constitutive matrix so-obtained, which may readily be applied to finite element analyses only if difficulty in treating unsymmetric matrix is overcome.

6. CONCLUDING REMARKS

The proposed model successfully reflects such typical characteristics of a crack in concrete as aggregate interlocking, crack dilatancy and frictional slip. Numerical simulation suggests that off-diagonal terms in the constitutive matrix characterized by dilatancy ratio and frictional coefficient play an important role in the shear transfer mechanism according to constraint conditions normal to the crack direction. Although this behavior has been recognized as coupling effect or cross effect by experimental works, very few analytical models were proposed.

One advantage of the authors' model is that the derived stiffness matrix of cracked concrete can readily be incorporated into existing finite element codes, requiring no additional isolated interelements. This decreases computational costs especially when large-scale reinforced concrete structures like nuclear facilities are dealt with.

The study of the tension stiffening effect is beyond the scope of the present paper, because this effect is observed in the completely different crack mode. A mathematical model for the tension stiffening based on analytical formulation taking bond-slip mechanism into consideration has been proposed in the authors' recent works ([17]-[19]).

We call the crack mechanism discussed in this paper the F-mode (frictional contact slip mode), while the tensile cracking mode where the tension stiffening effect must be taken into consideration is referred to as the S-mode (nonfrictional separation mode). In actual reinforced concrete structures, both of these crack modes are mixed. Hence, A further study is needed to combine both crack modes.

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