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Material Model for Cracked Reinforced Concrete

Modèle du matériau pour le béton armé fissuré

Ein Materialmodell für gerissenen Stahlbeton

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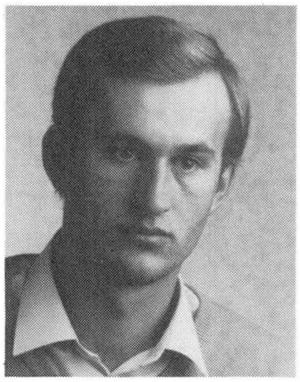

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SUMMARY
A material model for the analysis of cracked reinforced concrete surface structures is developed based on recent experimental work. The constitutive model employs the smeared crack concept, i.e. only average material stresses are considered at an integration point. A new formulation for the reduction of the compressive strength of cracked concrete is proposed. A refined procedure for the evaluation of tension stiffening is also presented.

RÉSUMÉ
Sur la base de nouveaux essais un modèle du matériau est proposé pour le calcul des structures bidimensionnelles en béton armé. Dans ce modèle du matériau le concept de fissuration homogénéisée est utilisé, c'est à dire que seules les tensions et torsions moyennes au point d'intégration sont considérées. Une définition nouvelle pour la réduction de la résistance à la compression du béton fissuré est proposée. Une méthode très élaborée pour déterminer les tensions de traction dans le béton fissuré est aussi présentée.

ZUSAMMENFASSUNG
Auf der Grundlage neuer experimenteller Untersuchungen wird ein Materialmodell für die Berechnung von Stahlbetonflächentragwerken entwickelt. Im Materialmodell wird das Konzept verschmierter Rißbildung verwendet, d.h. es werden nur mittlere Spannungen und Verzerrungen am Integrationspunkt berücksichtigt. Zur Reduzierung der Druckfestigkeit gerissenen Betons wird eine neue Formulierung vorgeschlagen. Eine verfeinerte Methode zur Ermittlung der Zugspannungen im gerissenen Beton wird ebenfalls vorgestellt.

1. INTRODUCTION

In the analysis of reinforced concrete surface structures cracking of concrete usually causes the main nonlinearities in the structural response. Many different formulations for the numerical treatment of cracked reinforced concrete have been presented in the literature. Differences exist in the treatment of

- the reduction of the concrete compressive strength after cracking (reduction according to Vecchio and Collins [12, 13] versus no reduction),
- the tension stiffening effect (as a property of concrete or as a property of reinforcement),
- the crack direction (fixed cracks versus rotating cracks), and of
- the magnitude of the shear modulus of cracked concrete.

In order to gain more experimental data on the behavior of cracked reinforced concrete under plane stress loading conditions, experiments have been carried out at the University of Kassel [6, 7] and at the University of Toronto [5]. All panels were subjected to uniform stress states. The strains in the panels were measured over lengths which included several cracks. The stress-strain relationships from these tests represent the average behavior of reinforced concrete specimens. Thus they are directly applicable to Finite Element analyses where the smeared crack concept is employed. The results of our experimental investigation led to improvements in the computational treatment of cracked reinforced concrete, which will be presented in this paper.

2. CRACKED CONCRETE

2.1 Reduction of concrete compressive strength

2.1.1 Description of the problem

The biaxial strength envelope by Kupfer, Hilsdorf and Rüsch [8] is often used as failure surface for uncracked concrete. The biaxial strength envelope is shown in Fig. 1 and regions of compressive failure (concrete crushing) and tensile failure (tensile splitting) are indicated. If a reinforced concrete panel, which is reinforced in direction of the applied tensile stress f_1 only, as shown in Fig. 1, is subjected to load path 1, failure will be due to crushing of concrete when the applied compressive stress reaches the concrete compressive strength, which is a function of the simultaneously acting tensile stress in concrete. If the applied load follows load path 2 of Fig. 1 the concrete will crack once the failure surface is reached. Upon cracking the average concrete tensile stress f_{c1} decreases. After cracking tensile concrete stresses exist only in the concrete struts between the cracks. The released tensile stresses are taken up by the reinforcement.

If it is assumed that the panel of Fig. 1 is sufficiently reinforced so that a failure of the reinforcement is prevented, crushing of the cracked concrete will govern the failure. But which criterion for the compressive strength of cracked concrete should be used? Is the strength of the cracked concrete equal to the cylinder crushing strength f_c', as for example assumed by Milford and Schnobrich [9]? Or is the strength of cracked concrete a function of the transverse strain ε_1 as proposed by Vecchio and Collins [12, 13]? Or should the concrete compressive strength of cracked concrete be generally reduced by 20 % as recommended by Schlaich and Schäfer [11]?

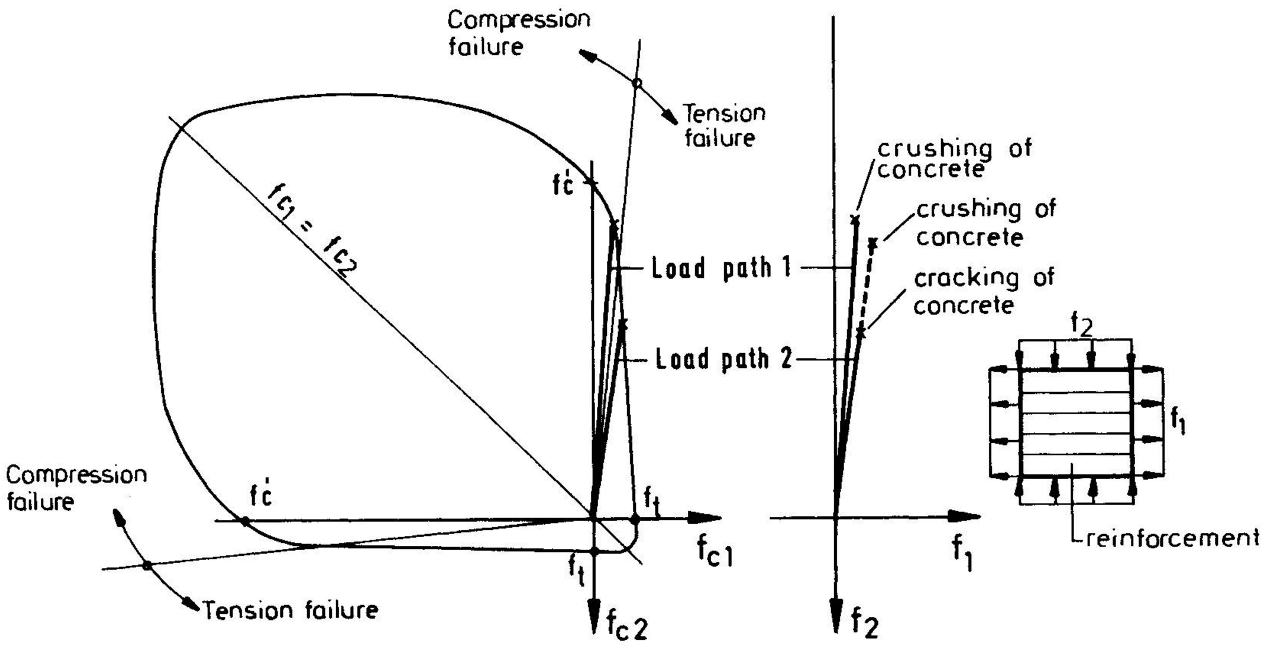


Fig.1 Biaxial concrete strength and RC panel subjected to tension and compression

2.1.2 Experimental results

In order to answer these questions a series of fifty panels is being tested at the University of Kassel and eight panels were tested at the University of Toronto. The detailed experimental results will be given in [5, 7]. Here, only the major conclusions regarding the compressive strength of cracked concrete will be summarized.

The maximum reduction of the concrete compressive strength in both series was around 20 to 25 %. The results of the Toronto tests [5] are compared with the phenomenological relationship for the reduction of the concrete compressive strength based on experimental results obtained by Vecchio and Collins [12] in Fig. 2. The relationship suggested by Vecchio and Collins considerably underestimates the observed concrete strengths of this series, as is shown in Fig. 2.

In cracked concrete transverse stresses exist in the concrete struts, which will be further investigated in section 2.2. The biaxial failure stresses of the reinforced concrete panels [5] are compared in Fig. 2 with the biaxial strength envelope of Kupfer et al. [8]. Three panels tested by Vecchio and Collins [12], which failed due to concrete crushing, have also been included in Fig. 2. Considering that average stresses in reinforced concrete panels are compared with results obtained on plain concrete specimens (200 mm x 200 mm x 50 mm) by Kupfer et al. in Fig. 2, good agreement of the biaxial failure stresses can be noted.

Kupfer et al. [8] report that specimens subjected to combined tension and compression behaved similarly to the specimens loaded in biaxial compression as long as the ratio of the applied stresses $f_1/-f_2$ was less than 1/15; with a stress ratio $f_1/-f_2$ equal to 1/10 tensile splitting failures occured (see Fig.2). These observations are also in good agreement with the results of the panel test series; the panels failed due to concrete crushing and the corresponding stress ratios at failure are in the compression failure region of the tests by Kupfer et al. If the reinforcement does not fail tension failure is not possible in a reinforced concrete panel. If the tensile stress in the concrete exceeds the cracking strength, a crack forms and the tensile stress is released. This explains why failures of reinforced concrete panels have to occur in the compression failure region of Fig. 2, unless the ultimate strength of the panel is governed by the load carrying capacity of the reinforcement in tension.

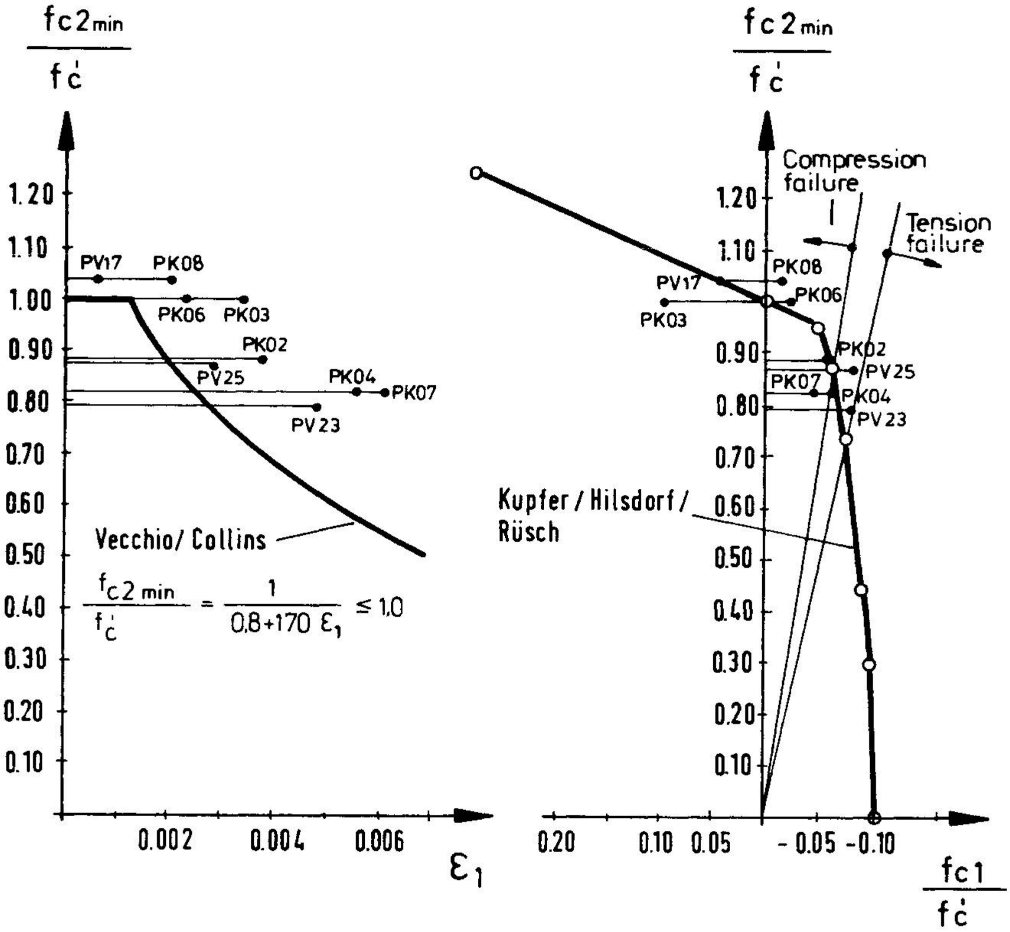


Fig.2 Compressive strength of concrete as a function of transverse strain and transverse stress

2.1.3 Proposed formulation

Based on the experimental results of Fig. 2 which are in agreement with the findings reported in [7] it is proposed to reduce the concrete compressive strength as a function of the simultaneously acting transverse stress. Fig. 3 shows how for a given stress f_{c1} the minimum compressive stress f_{c2} is found from the biaxial failure envelope. The strain corresponding to the peak stress $f_{c2\,min}$ is determined from the equation $\varepsilon_{c2\,min} = \varepsilon_c' * f_{c2\,min}/f_c'$.

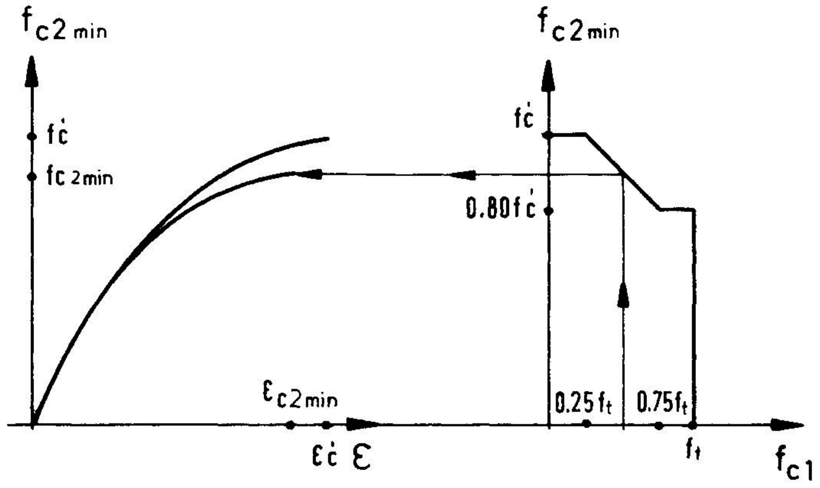


Fig.3 Proposed reduction of compressive strength for cracked concrete

2.2 Tension stiffening

2.2.1 General considerations

When formulating a material model for reinforced and prestressed concrete structures for either loading conditions, the realistic modelling of the stiffness after cracking deserves special attention. The inclusion of a realistic tension stiffening model is generally very important in the analysis of reinforced concrete shell structures which might be endangered by a stability failure. If the minimum concrete compressive stress is a function of the transverse stress, as outlined in section 2.1, a careful evaluation of the tensile concrete stresses is even more crucial.

2.2.2 Tension stiffening for coinciding principal tensile strain and reinforcement directions

The results of a tension test [5] are shown in Fig. 4. The response of the bare bar and the reinforced concrete specimen are compared in this figure. Mulitplying the stress difference of the two curves times the reinforcement ratio yields the tension stiffening curve of Fig. 5. While the concrete is free of stress at the cracks, between the cracks tensile stresses are transferred from the steel to the concrete by bond action. The tensile stress shown in Fig. 5 is the average concrete stress of the specimen. Experimental data on tension stiffening of panels for varying reinforcement ratios and different reinforcement properties is given in [5, 6]. All tension stiffening curves for coinciding principal tensile strain and reinforcement direction are of a shape similar to Fig. 5. After cracking the average concrete tensile stress drops gradually to a certain stress level. Tension stiffening vanishes once the yield strength of the reinforcement at a crack is reached. However, even if the reinforcement yields at a crack, between the cracks the concrete still carries tensile stresses.

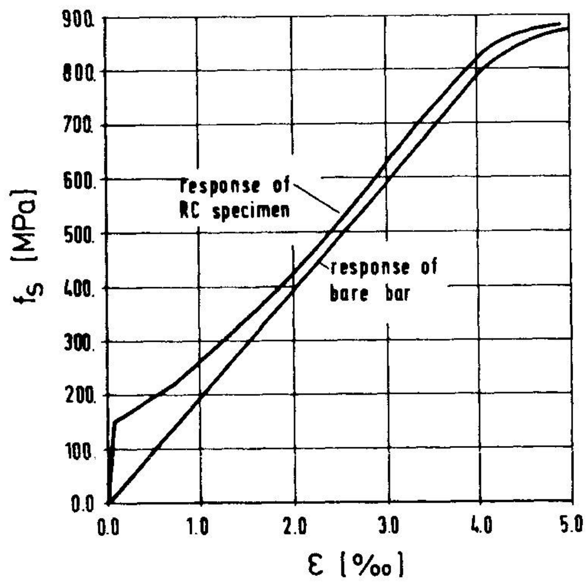


Fig.4 Results of tension test

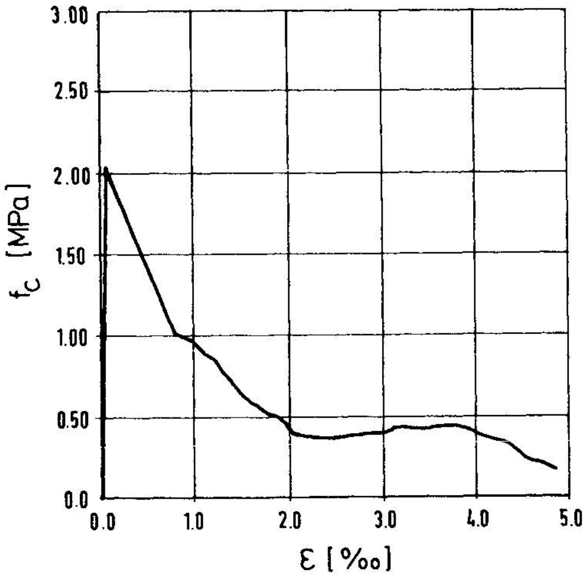


Fig.5 Tension stiffening curve

2.2.3 Tension stiffening for arbitrary angles between principal tensile strain and reinforcement directions

An evaluation of test results reported by Röder [10] and Vecchio and Collins [12] showed that the angles between the principal tensile strain and reinforcement direction do not have a noticeable influence on the tension stiffening, if tension stiffening is considered in the reinforcement direction [6]. This observation proved to be in good agreement with panel tests [5], where it was also noticed that the crack spacing did not depend on the angles between principal tensile strain and reinforcement directions, if the crack spacing was measured in the reinforcement directions.

It is suggested to evaluate the tension stiffening in the reinforcement directions and then to transform the concrete tensile stresses to the principal strain direction. A numerical example for this procedure is given in section 4.1.

2.2.4 Additional transverse stresses in concrete struts

Additional transverse tensile or compressive stresses may exist in the concrete struts due to strain compatibility of reinforcement and concrete. On precracked panels subjected to compressive stresses f_2 the formation of these transverse stresses will be explained. Fig. 6 shows a panel where the reinforcement is inclined by 45° with respect to the applied stress. In a similar experiment [5] compressive stresses developed in the reinforcement when the loading was applied. Compressive steel stresses can only exist if they are balanced by tensile concrete stresses. Since the panel is precracked tensile stresses cannot be transferred across the cracks. But between the cracks tensile stresses exist as is indicated in Fig. 6.

Ir the reinforcement is oriented orthogonal to the applied stress (Fig. 7) compressive transverse stresses develop in the concrete struts, because the transverse deformation of the concrete struts, caused by Poisson's ratio and dilatation effects, is restrained by the reinforcement.

In order to evaluate the transverse stresses in the concrete struts the stresses in the reinforcement within a strut are determined from the principal compressive strain ε_2, the transverse strain in the strut $\nu \cdot \varepsilon_2$ and the angle Θ. The steel stresses are then transformed to the direction 1. Because of equilibrium in the transverse direction, the concrete stress has to be qual to the transformed steel stress. The transverse concrete stresses are neither uniform over the width nor over the height of the strut (Figs. 6 and 7). In order to account for this fact the transverse concrete stresses have to be scaled down by a factor, which depends on the reinforcement properties. This transverse stress in the concrete strut is added to the stress caused by tension stiffening. This stress is then used to determine the compressive strength of cracked concrete as described in section 2.1.

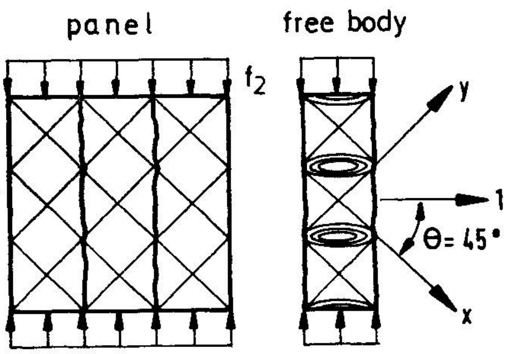


Fig.6 Transverse tensile stresses in concrete struts

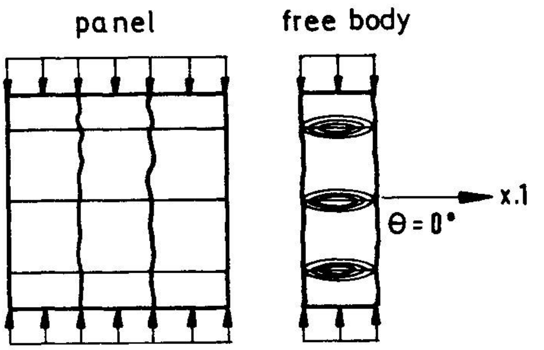


Fig.7 Transverse compressive stresses in concrete struts

2.3 Reorientation of principal strain direction

The numerical algorithm used to account for a reorientation of the principal strain direction is the "rotating crack model" by Akbar and Gupta [1]. Progressive cracking, or changes in the crack direction are considered in this model assuming that the crack direction is always normal to the direction of the maximum principal tensile strain. This assumption is in good agreement with experimental results as is shown by a sequence of pictures (Figs. 8 to 11) of a panel test, carried out by the first named author. The panel, which was reinforced in the x-direction only, was subjected to tensile stresses in the x-direction and shear stresses. Initially the cracks formed in a direction normal to the principal tensile stress in concrete (Fig. 8). Upon increased loading the tips of the cracks turned into a new direction, the original cracks became smaller and new cracks opened (Fig. 9). When the ultimate strength of the panel was reached (Fig. 10), the original cracks were closed and the failure of the panel was governed by an uncontrolled opening of the new cracks. The rotating crack model would describe the behavior of this panel by a rotation of the original concrete struts (Fig. 8) by approximately 45° into the final position. With the rotating crack approach the overall behavior of reinforced concrete panels can be captured better than with any fixed crack model. How complicated the actual behavior of the panel was, is shown in the close-up picture in Fig. 11. The crack pattern of Fig. 11 could probably not even be reproduced by the most sophisticated discrete crack models. But for the global analysis of shell structures the rotating crack model is a simple and efficient method to account for changes in the crack direction.

Using the rotating crack model the principal strain directions are updated in each iteration, i.e. the maximum principal strain is always orthogonal to the crack. From this follows that the shear strain is always zero. Therefore, no shear modulus has to be retained when the rotating crack model is used.

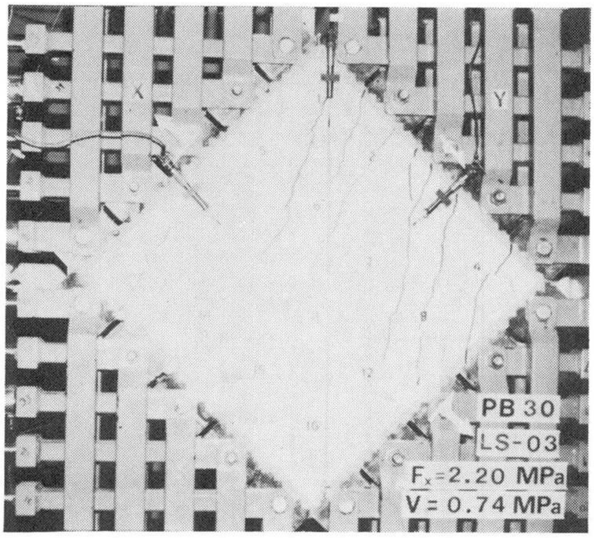


Fig.8 Panel PB30 LS-03

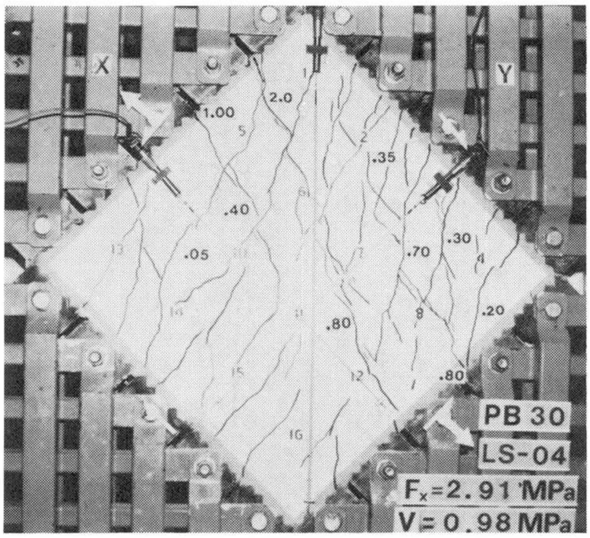


Fig.9 Panel PB30 LS-04

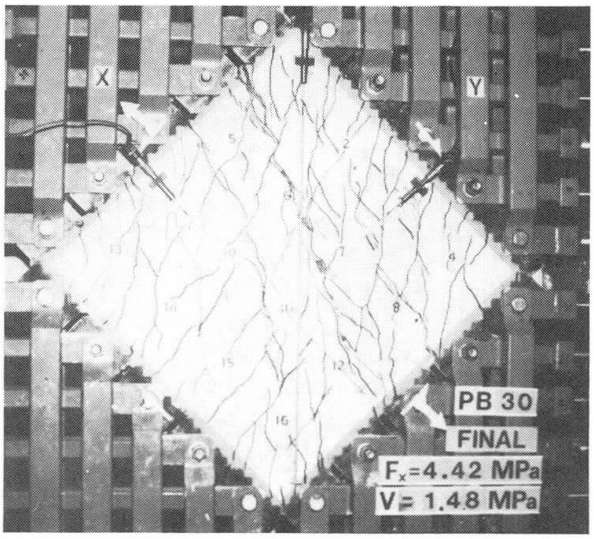


Fig.10 Panel PB30 FINAL

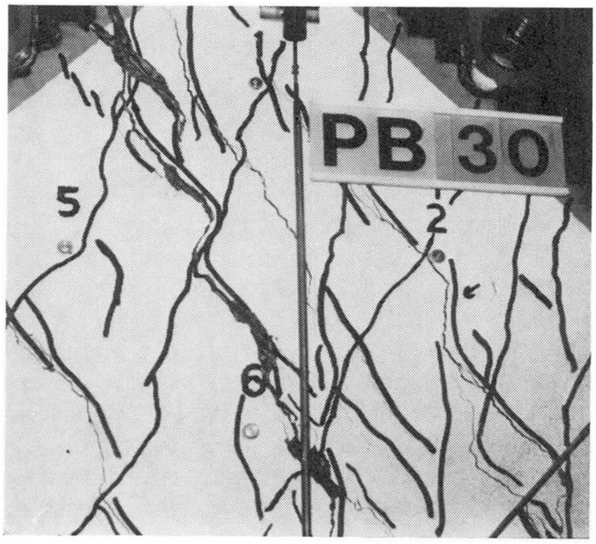


Fig.11 Detail of panel PB30

3. UNCRACKED CONCRETE

Uncracked concrete is modelled by Figueiras' plasticity model [4]. For the geometrically and physically nonlinear analysis of reinforced concrete shell structures the finite element code SEGNID [2] is used. The solution capabilities of the program as well as the available element library are described in [3].

4. EXAMPLES

4.1 Panels subjected to tension

The response of a panel subjected to tensile stresses in direction of the reinforcement is shown in Fig. 12. The stress-strain relationship of this example is qualitatively similar to the actual experiment shown in Fig. 4. The same response is obtained for tension stiffening models which evaluate concrete stresses as a function of the principal tensile strain ε_1 and as a function of the strain in the reinforcement direction ε_x. Tension stiffening vanishes when the yield strain of the reinforcement ε_x = 0,002 is reached.

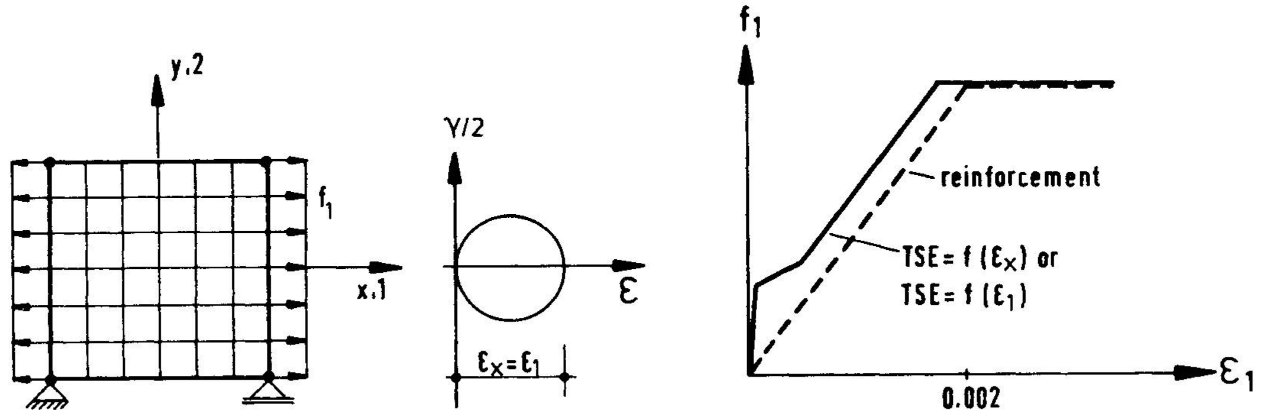


Fig.12 Panel subjected to tension in reinforcement direction

If the reinforcement is rotated by 45 degrees with respect to the applied tensile stress then different responses are obtained for tension stiffening formulations as a function of ε_x and ε_1. This is shown in Fig. 13, where it has been assumed that the concrete is infinitely stiff in compression. In the case that the tension stiffening depends on the strain orthogonal to the crack, the tensile stresses vanish when a strain of ε_1 = 0.002 is reached. If tension stiffening is considered in the reinforcement directions, the stiffening effect of the tensile stresses in the concrete would be noticeable until the reinforcements will yield at $\varepsilon_x = \varepsilon_y$ = 0.002. A strain of 0.002 in the reinforcement directions corresponds to a principal tensile strain ε_1 = 0.004, as is indicated by the Mohr's circle of strain in Fig. 13. This example shows that the actual tension stiffening is considerably underestimated, if tension stiffening is associated with the principal tensile strain direction.

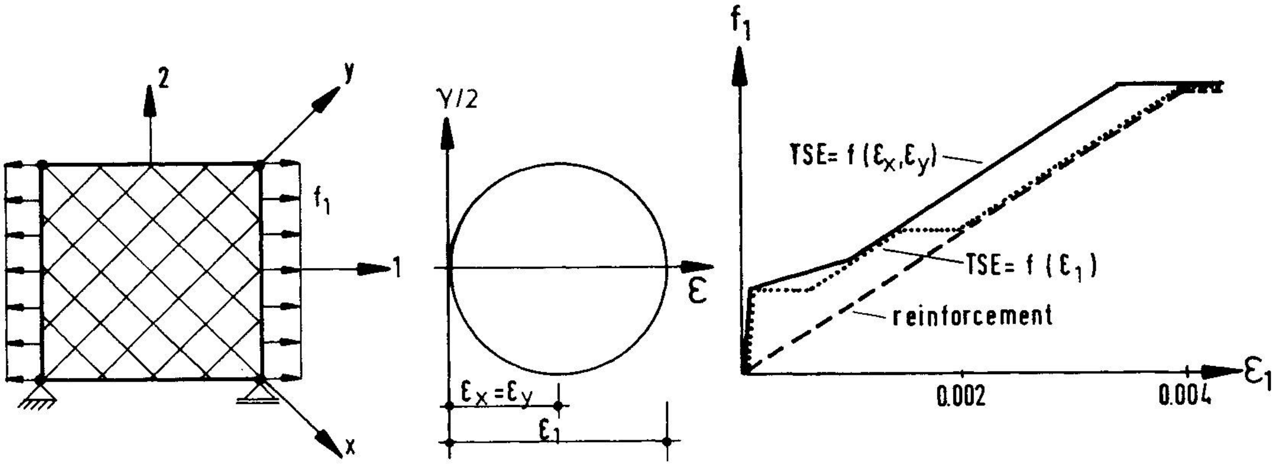
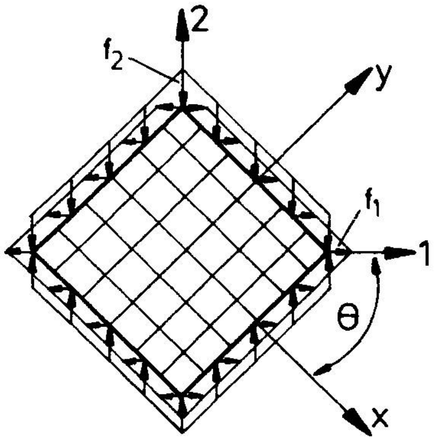


Fig.13 Panel subjected to tension at an angle of 45° with respect to reinforcement

4.2 Panel subjected to tension and compression

The properties of panel PK04 [5] which was subjected to biaxial tensile and compressive stresses are shown in Fig. 14. In the experiment the tensile stresses were applied first up to a stress of f_1 = 5.4 MPa. The tension was kept at this level when the compression was applied. The principal strains of experiment and analysis as a function of the applied compressive stress are compared in Fig. 15.

In the analysis one four node plane stress element was used as shown in Figs.12 and 13. The concrete compressive strength was reduced according to Fig. 3. At failure the effective concrete strength of the analysis was 0.80 f_c'. This compares well with the maximum concrete strength of the experiment which turned out to be equal 0.82 f_c' (Fig. 2). The reduction of the concrete compressive strength as suggested by Vecchio and Collins (Fig. 2) would yield an effective concrete strength of only 0.56 f_c'.



Concrete: $f_c' = -20.2$ MPa, $\varepsilon_c' = -0.002$, $f_t = 1.7$ MPa

Steel in x-and y-dir.: $f_y = 700$ MPa, $\rho = 0.0106$, $E_s = 200\ 000$ MPa

Tension stiffening effect (x-and y-direction):

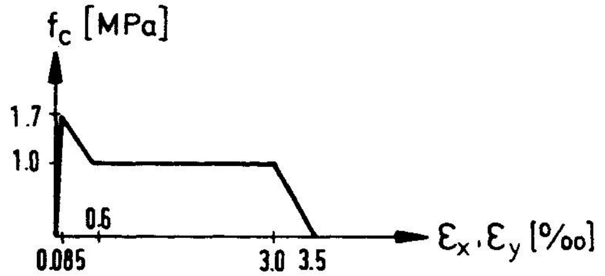


Fig.14 Loading and properties of panel PK04

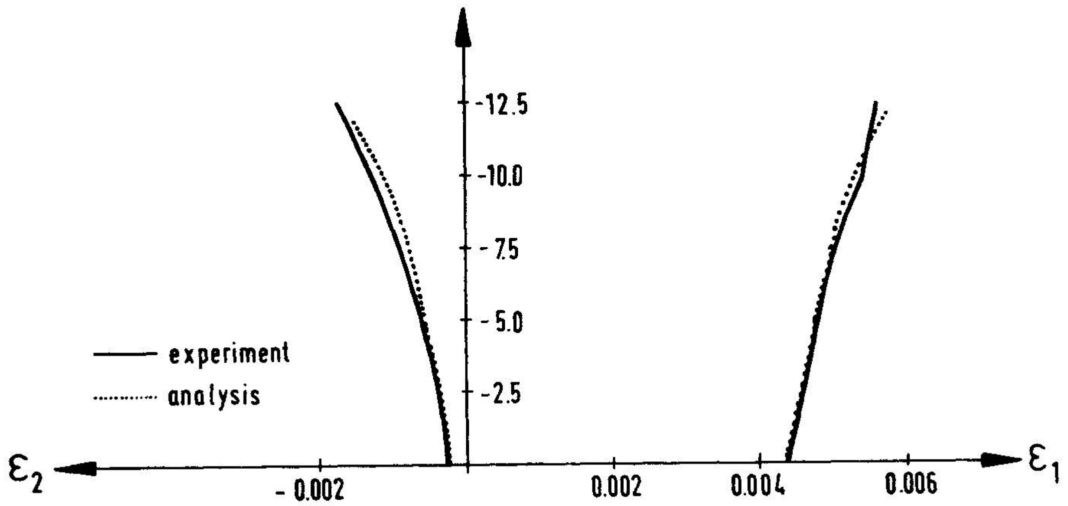


Fig.15 Stress-strain response of panel PK04

4.3 Panel subjected to shear

Panel PV19 was tested by Vecchio and Collins in pure shear. The reinforcement of the panel in the y-direction was weaker than in the x-direction (Fig. 16). Therefore the principal strain direction Θ, which should be 45 degrees before cracking, changes after cracking as a function of the applied load. Fig. 17 shows that the rotating crack model is able to reproduce the stress-strain response of the test specimen. The failure load of the rotating crack model is 5 % higher than in the experiment. An analysis with a fixed crack model would overestimate the experimental failure load by 30 % (Fig. 17). In [12] the failure of the panel is attributed to concrete crushing. But the analysis reveals that the failure occurs, when the second reinforcement starts to yield.

Assuming an elastic-perfectly plastic stress-strain relationship for the reinforcement, the determinant of the stiffness matrix becomes zero at this load stage.

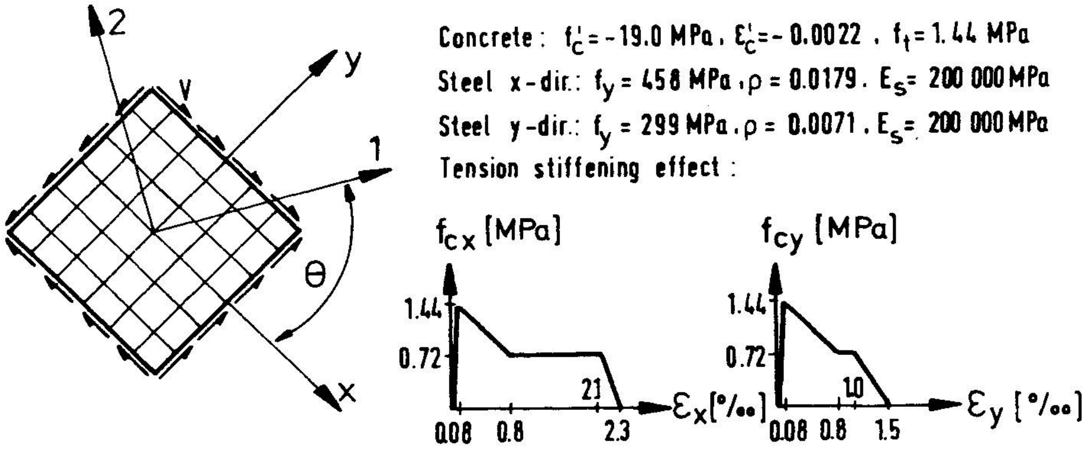


Fig.16 Loading and properties of panel PV19

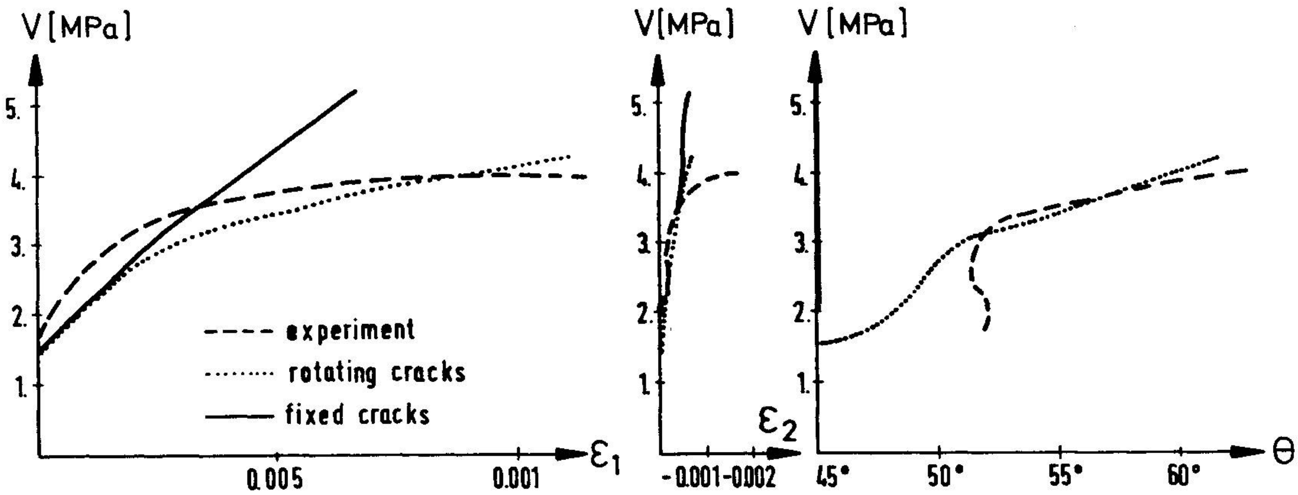


Fig.17 Stress-strain response of panel PV19

5. CONCLUSIONS

In the proposed material model the concrete compressive strength is a function of the simultaneously acting transverse stress. Therefore, the accurate evaluation of tensile stresses in cracked concrete is important. Based on experimental research a realistic procedure for the determination of tensile stresses in cracked concrete has been presented. It is believed that the presented formulation for cracked reinforced concrete will prove to be very effective in the nonlinear analysis of complex shell structures.

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