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Active Control of Bridge Structures Based on Modal Analysis

Concept of digital type active optimal control of suspension bridge is provided. One-to-one accurate control against an unsteady wind force caused by typhoon is realized by the use of step-by-step digital control, and it will possibly lead to a dual design, for which the bridge is designed only for the ordinary self-weight and traffic load, and for which the anti-wind capacity of the bridge is mainly supported by controlling.

State equation of 'equation of motion' under the action of control and external forces is described as follows in a continuous-time system:

$$\mathbf{A}\dot{\mathbf{X}}(t)+\mathbf{B}\mathbf{X}(t)=\mathbf{C}\mathbf{U}(t)+\mathbf{D}\mathbf{F}(t)$$

in which $\mathbf{X}^t=[\dot{x}_1,\cdots,\dot{x}_n|x_1,\cdots,x_n]$ (x: displacement), $\mathbf{U}$ is control force and $\mathbf{F}$ is external (wind) force. When the equation is represented in a discrete-time system, matrix exponential function appears and it gives a large defect on the numerical calculation. The efficiency of the iterative calculation is hugely improved by introducing an orthogonal technic based on the complex modal analysis into the state equation. Discrete type state equation is represented as follows:

$$\mathbf{x}(t+\Delta t)=\mathbf{B}^*\mathbf{x}(t)+\mathbf{C}^*\mathbf{U}(t)+\mathbf{D}^*\mathbf{F}(t)$$

Optimal value of control force $\mathbf{U}(t)$ is obtained, in a real time, based on the Pontrjagin's maximum principle. Let $\mathbf{U}(t)$ be positive (tensile only).

Suspension bridge is modelled as shown in Fig. 1, in which girder of the bridge is discretized into isoparametric shell elements in order to represent a twisting motion, which will produce the most disadvantageous vibration mode to the suspension bridge. External forces used in the numerical examples are also shown in Fig. 1. Deformation of the bridge is shown in Fig. 2 and Fig. 3, which correspond to the bending and twisting vibration, respectively. Thick lines implies the vertical displacements d_A and d_B (Fig. 2) and torsional angles r_A and r_B (Fig. 3) at points **A** (solid lines) and **B** (break lines). Thin solid lines (point **A**) and break lines (point **B**) imply the deformation without controlling. Control forces u_A and u_B in each case is indicated in the lower parts of Figs. 2 and 3, in which right-side forces (u_A^2 and u_B^2) are shown by thin lines. Deformation d is non-dimensional value divided by the maximum deformation without controlling. Control state varies according to the selection of parameters specifying the balance between the allowable deformation and control capacity.

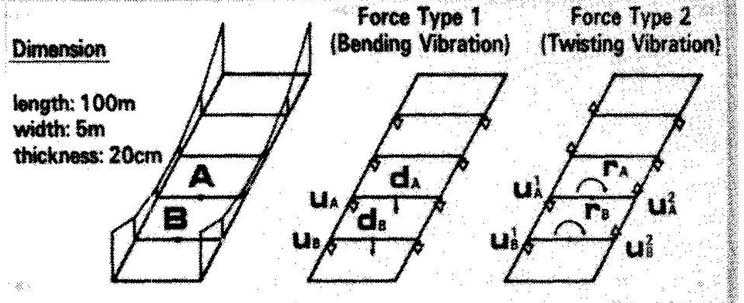

Fig. 1 Suspension Bridge.

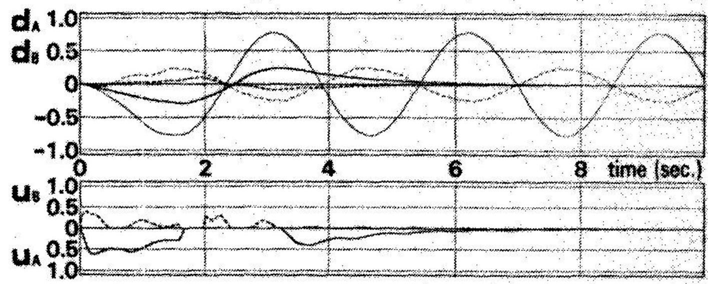

Fig. 2 Control of Bending Vibration Caused by Force Type 1.

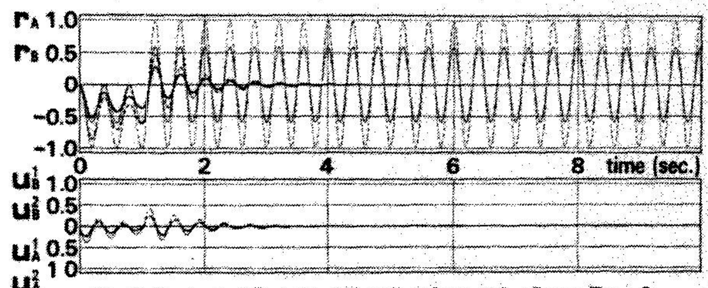

Fig. 3 Control of Twisting Vibration Caused by Force Type 2.

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ABSTRACT

Digital type active optimal control of suspension bridge is provided. One-to-one accurate control against an unsteady wind force caused by typhoon is realized by the use of step-by-step digital control.

State equation represented in a digital form contains a matrix exponential function, and it gives a large defect on the calculation of control force. Efficiency of the calculation is hugely improved by using an orthogonal technic based on the complex modal analysis. Control of suspension bridge, for which the bending and twisting vibration is induced, is conducted as numerical example.

INTRODUCTION

Active optimal control, purposing to prevent a structural vibration, has been applied to the civil engineering field in the 1970's by Abdel-Rohman, Carotti, Leipholz, Lin, Yahagi and Yang. However, in those cases, the state equation has been formulated in the analogue type control system. The system is inappropriate to one-to-one accurate control, which is the most reliable control method against an unsteady state. They have restricted the problem in a steady state as a regulator problem, or they have re-formed the problem into a quasi-steady state using the frequency response functions.

The writers have worked on the digital type control[1)]. The digital control is realized by calculating the optimal control forces, step-by-step, against the unsteady external forces. The advantageous point of the system is the one-to-one correspondence between external and control forces. The disadvantageous point is the numerical calculation which needs a high-speed, large-capacity computer[1)], and it can be improved by the use of complex modal analysis. The digital control, therefore, can be applied practically to the control of the multi-nodes consistent-mass structure.

The digital control is applied not only as a supplementary tool to decrease an unpleasant vibration, but also as a principal mechanism to prevent a structural collapse due to strong wind. The digital control of suspension bridge is provided as numerical example, where isoparametric shell element is employed.

OBSERVATION AND CONTROL

The following two assumptions are used:

1. Magnitude of the future wind force is assumed as zero. It implies only the present value of wind force is reflected on the control.

2. Present state of the structural deformation is substituted by the simulative value obtained numerically by solving the equation of motion.

STATE EQUATION

Equation of motion under the action of control force $\mathbf{u}(t)$ and wind force $\mathbf{f}(t)$ is represented as follows, if the viscous damping is ignored:

$$\mathbf{M}\ddot{x}(t) + \mathbf{K}x(t) = \mathbf{u}(t) + \mathbf{f}(t) \qquad (1a)$$

in which $x(t)$ is a nodal displacement vector, $\mathbf{K}$ is a stiffness matrix and $\mathbf{M}$ is a consistent mass matrix. The second order differential equation in Eq.(1a) is modified to a form of equation of evolution, for which the Pontrjagin's maximum principle is applicable, by introducing the following two relations:

$$\mathbf{M}\dot{x}(t) - \mathbf{M}\dot{x}(t) = 0 \qquad (1b)$$

$$X(t)^T = \{\dot{x}_1(t) \cdots \dot{x}_n(t) \mid x_1(t) \cdots x_n(t)\} \qquad (2)$$

in which $X(t)$ is a state variable vector. State equation of control is expressed as shown in the first equation in our poster; that is,

$$\mathbf{A}\dot{X}(t) + \mathbf{B}X(t) = \mathbf{C}U(t) + \mathbf{D}F(t) \qquad (3)$$

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{M} \\ \mathbf{M} & 0 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 0 \\ \mathbf{I}_u \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 0 \\ \mathbf{I}_f \end{bmatrix}$$

in whixh $\mathbf{I}_u$ and $\mathbf{I}_f$ are transformation matrices which represent the direction of control and wind forces, respectively.

Let the nodal displacement x be expressed as

$$x(t) = \phi \exp(\omega t)$$

and therefore $X(t)$ can be represented as the product of eigen matrix Φ ($\Phi^T = [\phi\omega \mid \phi]^T$) and generalized displacement vector $\mathbf{Q}(t)$ as follows:

$$X(t) = \Phi\, \mathbf{Q}(t) \qquad (4)$$

in which ϕ is an eigen vector, ω is a diagonalized eigen value matrix.

The state equation Eq.(3) is represented in the followings [the second equation in our poster] by introducing Eq.(4) and by assuming the 0th order hold, where let $\mathbf{U}(t)$ and $\mathbf{F}(t)$ be constant during the time interval Δt:

$$x(t+\Delta t) = \mathbf{B}^* x(t) + \mathbf{C}^* \mathbf{U}(t) + \mathbf{D}^* \mathbf{F}(t) \qquad (5)$$

$$\mathbf{B}^* = \phi A(\phi^T\phi)^{-1}\phi^T, \quad \mathbf{C}^* = \phi \mathbf{B} \phi^T \mathbf{I}_u, \quad \mathbf{D}^* = \phi \mathbf{B} \phi^T \mathbf{I}_f$$
$$A = \exp(\omega\Delta t), \quad \mathbf{B} = -(2\omega\phi^T\mathbf{M}\phi)^{-1}[\mathbf{I} - \exp(\omega\Delta t)]$$

OPTIMIZATION

Hamiltonian H is defined as follows:

$$\begin{aligned} H = {} & (1/2)[x^T(t+\Delta t)\mathbf{R}x(t+\Delta t) + x^T(t+2\Delta t)\mathbf{R}x(t+2\Delta t)] \\ & + (1/2)[\mathbf{U}^T(t)\mathbf{S}\mathbf{U}(t) + \mathbf{U}^T(t+\Delta t)\mathbf{S}\mathbf{U}(t+\Delta t)] \\ & + \lambda^T(t+\Delta t)[-x(t+\Delta t) + \mathbf{B}^* x(t) + \mathbf{C}^* \mathbf{U}(t)] \\ & + \lambda^T(t+2\Delta t)[-x(t+2\Delta t) + \mathbf{B}^* x(t+\Delta t) + \mathbf{C}^* \mathbf{U}(t+\Delta t)] \end{aligned} \qquad (6)$$

in which λ is a Lagrangean multiplier vector, $\mathbf{R}$ and $\mathbf{S}$ are weight matrices defined in diagonal forms as ($\mathbf{I}$: unit matrix)

$$\mathbf{R} = \alpha \mathbf{I} \quad , \quad \mathbf{S} = \beta \mathbf{I} .$$

Rate of α to β, α/β, is an important scalar index in the control problem; that is, if the ratio α/β is selected as small value, control force will be small and the control effect becomes also small, on the contrary if the ratio is large, strong control effect is expected.

Conditions for the optimization are as follows:

$$\partial H/\partial \mathbf{x}(t+\Delta t) = \mathbf{0} \;, \quad \partial H/\partial \mathbf{x}(t+2\Delta t) = \mathbf{0} \;, \quad \partial H/\partial \mathbf{U}(t \quad) = \mathbf{0} \tag{7}$$

NUMERICAL EXAMPLE

Digital control of suspension bridge, as shown in Fig.1, is provided. Control forces $\mathbf{U}(t)$ are applied to the structure as vertical pull forces generated by hydraulic power of the tendons, which is attached in parallel with vertical hanger cables. Compressive forces cannot be generated by the tendons, henceforth the following constraint is introduced in the numerical analysis:

$$\mathbf{U}(t) \geq \mathbf{0} \tag{8}$$

Let the vertical external force $P = 1$ be applied suddenly, at the time $t = 0$, to the equally divided five points as shown by arrow symbols in Fig.1, and let it be removed at $t = 1.56$ second (Force type 1) and $t = 1.00$ second (Force type 2). Force type 1 corresponds the bending vibration, and force type 2 is twisting vibration. Time interval Δt is selected as 0.02 second.

In the case of bending vibration [see Fig.2], vertical displacements at the points A and B of slab, indicated by d_A and d_B, are shown by a thick solid line and thick break line, respectively. Displacements are represented in non-dimensional quantities divided by the maximum displacement in the un-controlled case, which is shown by thin lines as a comparison. Control forces are also shown in Fig.2, in which forces u_A and u_B are shown by solid and break lines, respectively. The ratio α/β is selected as 10^{-3}.

In the case of twisting vibration [see Fig.3], axial rotation angles at the points A and B, indicated by r_A and r_B, are shown by a thick solid line and thick break line, respectively. Rotation angles are represented in non-dimensional quantities based on the un-controlled case, which is shown by thin lines. Control forces are also shown in Fig.3, in which forces u^1_A, u^1_B (left side) and u^2_A, u^2_B (right side) are shown by solid and break (A: thick, B: thin), respectively. The ratio α/β is selected as 10^{-4}.

CONCLUSION

Active optimal control in discrete-time system, diagonalized based on the complex modal analysis, becomes an efficient tool in the control problem of the suspension bridge under wind motion.

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