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Assessment of the Criticality of Bridge Components

Estimation de l'importance des éléments de ponts

Beurteilung der Wichtigkeit von Brücken-Komponenten

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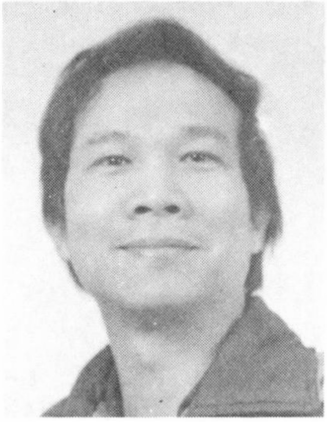

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SUMMARY

Subjective assessments are frequently used during bridge inspection. This paper introduces a method developed to improve the inspection procedures. The concept of fuzzy sets is used here to determine the condition and the urgency for preventive care of bridge components. Case studies are presented.

RÉSUMÉ

Une appréciation subjective est souvent à la base du contrôle des ponts. L'article présente une méthode destinée à améliorer les procédures d'inspection. Le concept de «fuzzy sets» est empoyé afin de déterminer les conditions et l'urgence d'un entretien préventif des éléments de ponts. Des exemples sont présentés.

ZUSAMMENFASSUNG

Bei der Inspektion von Brücken werden oft subjektive Massstäbe angewendet. Der Beitrag befasst sich mit einer Methode zur Verbesserung der Inspektions-Vorgänge. Mit Hilfe der «Fuzzy Set»-Theorie werden der Zustand von Brücken-Komponenten und die Dringlichkeit von Sanierungsmassnahmen beurteilt. Beispiele erläutern das Verfahren.

1. INTRODUCTION

Experience has shown that subjective notions are frequently used by an inspector when monitoring a bridge. Very often the inspector has to identify and deal with variables which are uncertain. Many of them can only be estimated subjectively and qualitatively through the engineer's experience, knowledge, and judgment.

This paper suggests a method for quantifying the subjective assessments of the condition and urgency for preventive care of bridge components based on visual inspections through the use of fuzzy set concept. It is expected that this procedure could be used for quality control purposes. Fuzzy set operations in the following sections are limited to only those pertinent to the study in this paper. The basics of fuzzy set theory can be found in References 1 and 5. For clarity, numerical examples and case studies are presented.

2. FUZZY SET OPERATIONS

Qualitative evaluation of certain variables may determine the criticality of bridge components. Three variables are considered in this study: 1. Condition (CON) determines the state of a component, 2. Importance (IMP) describes the importance of a component with regards to its use or structural integrity, and 3. Prevention urgency (PRE) indicates the urgency of the preventive care for the component. These variables have certain linguistic values which are expressed by the following fuzzy set:

$$[xi/f(xi)];\ i=1,2,..,5 \qquad (1)$$

where "/" is a delimeter. xi and f(xi) represent the element and the membership function of the fuzzy set, respectively. The element indicates the level of the variable which, in this study, ranges from 1 to 5. The membership value shows the degree of membership of the corresponding element in the fuzzy set and is a real number in the interval [0,1].

2.1 Composite Fuzzy Relation

A fuzzy relation is an operation used to relate different fuzzy sets. Let A and B be two fuzzy sets such that $A \in \phi(X)$ and $B \in \phi(Y)$, where X and Y are the nonempty sets and where $\phi(X)$ and $\phi(Y)$ denote the classes of all fuzzy sets of X and Y, respectively. The membership function of the fuzzy relation, R from A to B, or R=AxB, is expressed by:

$$f_R(xi,yj) = f_{AxB}(xi,yj) = \underset{\forall xi \in X,\ \forall yj \in Y}{\Lambda} [f_A(xi),f_B(yj)] \qquad (2)$$

which can also be represented by a matrix, where the membership value of each element contained in R is obtained from the minimum (Λ) of the membership values $f_A(xi)$ and $f_B(yj)$.

To obtain the intersection of fuzzy relations, the max-min Composite Fuzzy Relation (CFR) is used. Suppose X, Y and Z are three nonempty sets, and A, B and C are their fuzzy sets, respectively, such that $A \in \phi(X)$, $B \in \phi(Y)$, and $C \in \phi(Z)$. Suppose

R1=AxB and R2=BxC. Then the CFR of R1 and R2 is defined by T=R1oR2, where $T \in \phi(X \times Z)$. Its membership function is expressed by

$$f_T(xi,zk) = f_{R1oR2}(xi,zk) = \bigvee_{\forall x \in X,\ \forall y \in Y,\ \forall z \in Z} [f_{R1}(xi,yj) \wedge f_{R2}(yj,zk)] \quad \ldots\ldots\ (3)$$

whose operation is similar to matrix multiplication except that multiplication is replaced by minimum ($\wedge$) and addition by maximum ($\vee$).

2.1.1 Numerical Example

If, for illustration purposes, there were a specification that relates the condition and urgency measure of a bridge component. Let's say, the urgency for preventive care of a "very good" (VG) component is "very unnecessary" (VUN). Now suppose that we are interested in the urgency measure of "fairly good" (FG) condition. VG, VUN, and FG are linguistic values which, in this example, are defined by using Eq. 1 as follows:

$$\begin{aligned} VG = VUN &= [1/0.0,\ 2/0.1,\ 3/0.5,\ 4/0.9,\ 5/1.0] \\ FG &= [1/0.3,\ 2/0.7,\ 3/1.0,\ 4/0.7,\ 5/0.3] \quad \ldots\ldots\ (4) \end{aligned}$$

Through the use of Eqs. 2 and 3, the relation R=VUNxVG, where $R \in \phi(PRE \times CON)$, can be found as shown in the first matrix of Eq. 5. Using Eq. 3, the fuzzy composition of this matrix and FG results in the membership values of the urgency measure PRE1:

$$T = RoFG = \overset{R=VUNxVG}{\begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.0 & 0.1 & 0.5 & 0.5 & 0.5 \\ 0.0 & 0.1 & 0.5 & 0.9 & 0.9 \\ 0.0 & 0.1 & 0.5 & 0.9 & 1.0 \end{bmatrix}} \circ \overset{FG}{\begin{bmatrix} 0.3 \\ 0.7 \\ 1.0 \\ 0.7 \\ 0.3 \end{bmatrix}} = \overset{PRE1}{\begin{bmatrix} 0.0 \\ 0.1 \\ 0.5 \\ 0.7 \\ 0.7 \end{bmatrix}} \quad \ldots\ (5)$$

The corresponding value of the urgency measure is obtained by transposing PRE1:

$$PRE1 = [1/0.0,\ 2/0.1,\ 3/0.5,\ 4/0.7,\ 5/0.7] \quad \ldots\ldots\ (6)$$

which may be interpreted as close to "unnecessary."

2.2 Inverse Composite Fuzzy Relation

The inverse composite fuzzy relation (ICFR) operation was developed by Sanchez [4] to obtain the greatest membership values in an unknown relation R2 in T=R1oR2, if T and R1 are known. Sanchez defined the ICFR of R2 as $R1^T @\ T$ where $R1^T$ is the transpose of R1. The membership function of R2 is expressed by

$$f_{R2}(xi,zk) = f_{R1^T @ T}(xi,zk) = \bigwedge_{\forall x \in X,\ \forall y \in Y,\ \forall z \in Z} [f_{R1^T}(xi,yj)\ \alpha\ f_T(yj,zk)] \quad \ldots\ldots\ (7)$$

The operation is the same as matrix multiplication except that multiplication is replaced by α operation. This requires that each membership value, r1, in matrix R1, be compared with the membership value, t, in T such that $r1\,\alpha\,t = 1$ if $r1 \leq t$ and $r1\,\alpha\,t = t$ if $r1 > t$. The addition is replaced by taking the minimum of the results of α operation.

2.2.1 Numerical Example

If R ϵ ϕ(PRExCON) and PRE1 ϵ ϕ(PRE) are again expressed by the first and third matrices in Eq. 6, using Eqs. 7, the membership values of the condition CON1 ϵ ϕ(CON) yields:

$$\mathrm{CON1} = \overset{R\,\epsilon\,\phi(\mathrm{CONxPRE})}{\begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.0 & 0.1 & 0.5 & 0.5 & 0.5 \\ 0.0 & 0.1 & 0.5 & 0.9 & 0.9 \\ 0.0 & 0.1 & 0.5 & 0.9 & 1.0 \end{bmatrix}} \; @ \; \overset{\mathrm{PRE1}}{\begin{bmatrix} 0.0 \\ 0.1 \\ 0.5 \\ 0.7 \\ 0.7 \end{bmatrix}} = \overset{\mathrm{CON1}}{\begin{bmatrix} 1.0 \\ 1.0 \\ 1.0 \\ 0.7 \\ 0.7 \end{bmatrix}} \quad \ldots\ldots (8)$$

The result of Eq. 8 shows the following condition value:

$$\mathrm{CON1} = [1/1.0,\ 2/1.0,\ 3/1.0,\ 4/0.7,\ 5/0.7] \quad \ldots\ldots\ldots\ldots\ldots (9)$$

which is different from the original condition value in Eq. 4. This difference is expected since the ICFR produces the greatest membership values for CON1 (Note that the fuzzy set for FG in Eq. 4 is the subset of CON1 in Eq. 9.).

2.3 Polynomial Fuzzy Sets

The CFRs described in the foregoing section were of monomial forms. However, if the fuzzy relation R1 in the CFR have multiple values, the fuzzy set composition takes the following polynomial form:

$$T = \bigvee_{i=1}^{n} (R1^{(i)} \circ R2) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

T in Eq. 10 will become a constraint. Given this constraint, the problem to solve for unknown R2, if $R1^{(i)}$ is known, becomes one of how to simplify this polynomial form. Ohsato and Sekiguchi [3] transformed this form into i monomial forms through decomposition procedures such that $T=(R1^{(1)} \circ R2^{(1)})$, $T=(R1^{(2)} \circ R2^{(2)})$, ..., $T=(R1^{(n)} \circ R2^{(n)})$.

Hence, each monomial form is now solved using the previously described ICFR procedure to obtain the unknown $R2^{(i)}$. The solution of R2, that incorporates all $R2^{(i)}$ is

$$R2 = \bigwedge_{i=1}^{n} R2^{(i)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where Λ is the conjunction (or intersection, in probability theory) of all $R2^{(i)}$.

3. CASE STUDIES

This study involved the evaluation of a bridge deck which consists of seven components as listed in column 2 of Table 1. As mentioned earlier three factors, IMP, CON, and PRE, (also known as linguistic variables) are considered for the analysis. The fuzzy set model developed by the author is used for the variables. The model, shown in Figure 1, encompasses seven linguistic values whose relations between the membership function and fuzzy set element are represented in Table 2.

Fuzzy relations between the variables are presented in Table 3. R1 defines the relation between IMP and PRE which indicates that the more important a component the more susceptible it is towards the urgency for the preventive care. Therefore, important components are given negative values and are related to the negative quality of the urgency care. R2, relating CON to PRE, shows that the better the condition of a component, the less urgent (unnecessary) it is for its preventive care. These relations were used for the following case studies.

3.1 Case A

In this case, an inspector monitored a bridge deck and assigned values for the variables IMP and CON for each deck component. He/she used the linguistic values in Table 2 for the assessments, which were then entered in columns 3 and 5 of Table 1. Suppose that the summary of his/her assessments on the bridge deck condition as a whole was required and a decision about the urgency care for the bridge deck has to be made.

First, we should find the fuzzy relations R1 and R2 for each bridge deck components. For example, for the bridge deck floor, the value for IMP is VI (Table 1, column 3), which is related to VUR in Table 3; therefore, R1=VIxVUR. Suppose that the inspector rated the condition of this floor as poor, or P (Table 1, column 5); then, from Table 3, R2=URxP. A similar procedure was applied to obtain R1 and R2 of the other bridge deck components. The total effect on the bridge deck was determined by taking the disjunction (or union, in probability theory) of membership values of all R1's and R2's:

$$\text{R1Tot} = \begin{bmatrix} 1.00 & 0.92 & 0.83 & 0.75 & 0.00 \\ 0.92 & 0.92 & 0.83 & 0.75 & 0.00 \\ 0.83 & 0.83 & 0.83 & 0.75 & 0.00 \\ 0.75 & 0.75 & 0.75 & 0.75 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \quad \text{R2Tot} = \begin{bmatrix} 1.00 & 0.92 & 0.83 & 0.83 & 0.00 \\ 0.92 & 0.92 & 0.83 & 0.83 & 0.75 \\ 0.83 & 0.83 & 0.83 & 0.83 & 0.83 \\ 0.75 & 0.75 & 0.83 & 0.92 & 0.92 \\ 0.00 & 0.75 & 0.83 & 0.92 & 1.00 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ (12)$$

where R1Tot ϵ ϕ(IMPxPRE) and R2Tot ϵ ϕ(PRExCON). Subsequently, the composition R1Tot o R2Tot = Ra, where Ra ϵ ϕ(IMPxCON), can be found as follows:

$$\begin{array}{rcc} & & \text{IMP(a)} \\ \text{Ra} = & \begin{bmatrix} 1.00 & 0.92 & 0.83 & 0.83 & 0.83 \\ 0.92 & 0.92 & 0.83 & 0.83 & 0.83 \\ 0.83 & 0.83 & 0.83 & 0.83 & 0.83 \\ 0.75 & 0.75 & 0.75 & 0.75 & 0.75 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} & \begin{array}{c} 1.00 \\ 0.92 \\ 0.83 \\ 0.75 \\ 0.00 \end{array} \\ \text{CON(a)} & \begin{array}{ccccc} 1.00 & 0.92 & 0.83 & 0.83 & 0.83 \end{array} & \end{array} \quad \ldots\ldots\ldots\ (13)$$

Then, projection on variable space CON, by taking the maximum membership value in each column of matrix Ra, yields the membership value of the bridge deck total condition: "close to fairly poor." Projection on variable space IMP yields the total importance of the bridge deck which indicates "fairly important." These values are illustrated in Figure 2.

3.2 Case B

This case is concerned with the assessment of two inspectors whose consensus in determining the preventive care of a bridge deck is needed in addition to a certain existing maintenance policy. Suppose that the ratings of the inspector in Case A are used here as well as the ratings of another inspector. Their assessments on the variables IMP and CON of the bridge deck components are listed in Table 1. A similar procedure used in Case A is performed here to obtain the matrix $Rb \in \phi(IMPxCON)$ for the assessment of the second inspector. The result of the the total matrix Rb is shown in Eq. 14.

$$
\begin{array}{rcc}
 & & \text{IMP(b)} \\
Rb = & \begin{bmatrix} 1.00 & 0.75 & 0.50 & 0.75 & 0.75 \\ 0.92 & 0.75 & 0.50 & 0.75 & 0.75 \\ 0.83 & 0.75 & 0.50 & 0.75 & 0.75 \\ 0.75 & 0.75 & 0.50 & 0.75 & 0.75 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} & \begin{array}{c} 1.00 \\ 0.92 \\ 0.83 \\ 0.75 \\ 0.00 \end{array} \\
\text{CON(b)} & \begin{array}{ccccc} 1.00 & 0.75 & 0.50 & 0.75 & 0.75 \end{array} &
\end{array}
\qquad \text{.........} \quad (14)
$$

The projection on spaces IMP and CON summarizes the bridge deck condition and is shown in Figure 2. Comparison between the two cases shows a more conservative assessments for CON from the second inspector but a similar result for IMP.

Let us suppose that the importance of the bridge deck as a whole was determined based on a certain policy (e.g., from the Department of Transportation) and is considered as "fairly important," or FI. Based on Table 3, the relation, T, between IMP and PRE can be obtained and becomes a constraint to reach the consensus for the urgency care. Now the relations between the IMP, CON, and PRE are shown in Figure 3 and can be expressed as $T = (Ra \circ X) \vee (Rb \circ X)$, or through decomposition process:

$$Ra^T @ T = Xa \text{ and } Rb^T @ T = Xb \quad \text{.............................} \quad (15)$$

where $Ra, Rb \in \phi(IMPxCON)$; $Ra^T, Rb^T \in \phi(CONxIMP)$; $T \in \phi(IMPxPRE)$; and $Xa, Xb \in \phi(CONxPRE)$.

Using Eq. 15, Xa and Xb are found as shown below:

$$
\underset{Ra^T \in \phi(CONxIMP)}{\begin{bmatrix} 1.00 & 0.92 & 0.83 & 0.75 & 0 \\ 0.92 & 0.92 & 0.83 & 0.75 & 0 \\ 0.83 & 0.83 & 0.83 & 0.75 & 0 \\ 0.83 & 0.83 & 0.83 & 0.75 & 0 \\ 0.83 & 0.83 & 0.83 & 0.75 & 0 \end{bmatrix}}
@
\underset{T \in \phi(IMPxPRE)}{\begin{bmatrix} 1.00 & 0.92 & 0.83 & 0.75 & 0 \\ 0.92 & 0.92 & 0.83 & 0.75 & 0 \\ 0.83 & 0.83 & 0.75 & 0.75 & 0 \\ 0.75 & 0.75 & 0.75 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}
=
\underset{Xa \in \phi(CONxPRE)}{\begin{bmatrix} 1.00 & 0.92 & 0.75 & 0.75 & 0 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0 \end{bmatrix}}
$$

$$
\underset{Rb^T \in \phi(CONxIMP)}{\begin{bmatrix} 1.00 & 0.92 & 0.83 & 0.75 & 0 \\ 0.75 & 0.75 & 0.75 & 0.75 & 0 \\ 0.50 & 0.50 & 0.50 & 0.50 & 0 \\ 0.75 & 0.75 & 0.75 & 0.75 & 0 \\ 0.75 & 0.75 & 0.75 & 0.75 & 0 \end{bmatrix}}
@
\underset{T \in \phi(IMPxPRE)}{\begin{bmatrix} 1.00 & 0.92 & 0.83 & 0.75 & 0 \\ 0.92 & 0.92 & 0.83 & 0.75 & 0 \\ 0.83 & 0.83 & 0.75 & 0.75 & 0 \\ 0.75 & 0.75 & 0.75 & 0.75 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}}
=
\underset{Xb \in \phi(CONxPRE)}{\begin{bmatrix} 1.00 & 0.92 & 0.75 & 0.75 & 0 \\ 1.00 & 1.00 & 1.00 & 1.00 & 0 \\ 1.00 & 1.00 & 1.00 & 1.00 & 0 \\ 1.00 & 1.00 & 1.00 & 1.00 & 0 \\ 1.00 & 1.00 & 1.00 & 1.00 & 0 \end{bmatrix}}
\quad \text{...................} \quad (16)
$$

Through the use of Eq. 11 the conjunction of Xa and Xb yields:

$$
X \in \phi(CONxPRE) = \begin{bmatrix} 1.00 & 0.92 & 0.75 & 0.75 & 0.00 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0.00 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0.00 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0.00 \\ 1.00 & 1.00 & 0.75 & 0.75 & 0.00 \end{bmatrix} \quad \text{............} \quad (17)
$$

Finally, projection on space PRE leads to the value of the prevention urgency as shown below:

$$X \in \phi(PRE) = [1/1, 2/1, 3/0.75, 4/0.75, 5/0] \quad \ldots\ldots\ldots\ldots \quad (18)$$

which can be represented graphically in Figure 2. The consensus of the assessors' rating, including the constraint, yields a measure of "close to fairly urgent" for the bridge deck repair.

4. CONCLUSIONS

In this paper, the procedures for assessing bridge condition and the urgency for its preventive care quantify the subjective judgments provided by the bridge inspectors. This quantification process can only be performed through the use of fuzzy set concept. The fuzzy set manipulations can be performed with the help of computer programs to solve complex polynomial problems. The procedures described in this study allow a great deal of flexibility in determining the basic information such as that provided in Tables 2 and 3. This information can be updated or modified accordingly, depending upon the users need. The procedure also incorporates graphical representations for the values of the variables studied.

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No. (1)	Component (2)	IMP (3)[a,b]	IMP (4)[b]	CON (5)[a,b]	CON (6)[b]
1	Floor	VI(-3)	I(-2)	P(-2)	VP(-3)
2	Wearing Surface	I(-2)	VI(-3)	G(+2)	P(-2)
3	Curbs and Walkways	FI(-1)	FI(-1)	VG(+3)	VG(+3)
4	Median	FI(-1)	FI(-1)	FP(-1)	P(-2)
5	Railing	I(-2)	VI(-3)	VP(-3)	P(-2)
6	Drainage	I(-2)	I(-2)	FG(+1)	G(+2)
7	Expansion Joints	I(-2)	FI(-1)	P(-2)	P(-2)

[a]Rating of inspector for Case A; [b]Rating of inspector for Case B.

Table 1 Bridge deck components and ratings for IMP and CON

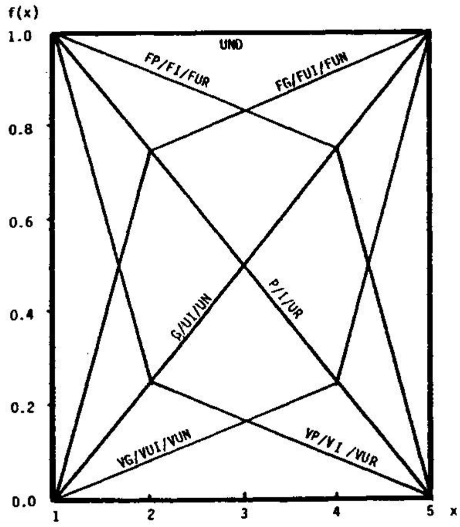

Fig. 1 Fuzzy set model H

Linguitic values (1)	Notation (Rating) (2)	f(x) (3)
Very-Good/Unimportant/Unnecessary	VG/VUI/VUN (+3)	(x-1)/12; 1≤x≤4 (3x-11)/4; 4<x≤5
Good/Unimportant/Unnecessary	G/UI/UN (+2)	(x-1)/4
Fairly-Good/Unimportant/Unnecessary	FG/FUI/FUN (+1)	(3x-3)/4; 1≤x≤2 (x+7)/12; 2<x≤5
Undecided (Fair:Btw FG-FP etc.)	UND (0)	1
Fairly-Poor/Important/Urgent	FP/FI/FUR (-1)	(13-x)/12; 1≤x≤4 (15-3x)/4; 4<x≤5
Poor/Important/Urgent	P/I/UR (-2)	(5-x)/4
Very-Poor/Important/Urgent	VP/VI/VUR (-3)	(7-3x)/4; 1≤x≤2 (5-x)/12; 2<x≤5

Table 2 Fuzzy set model H

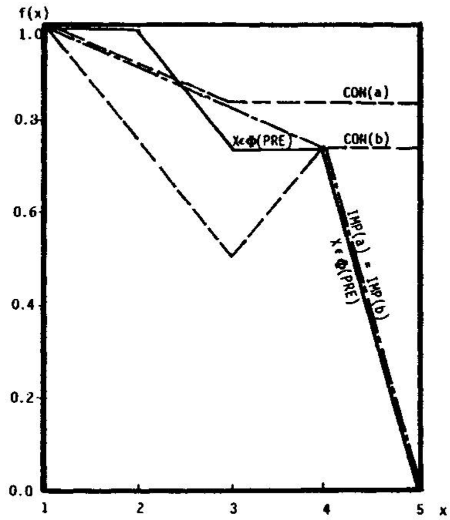

Fig. 2 Results from Cases A and B

IMP (1)	PRE (2)	CON (3)	PRE (4)
VI(-3)	VUR(-3)	VG(+3)	VUN(+3)
I(-2)	UR(-2)	G(+2)	UN(+2)
FI(-1)	FUR(-1)	FG(+1)	FUN(+1)
UND(0)	UND(0)	UND(0)	UND(0)
FUI(+1)	FUN(+1)	FP(-1)	FUR(-1)
UI(+2)	UN(+2)	P(-2)	UR(-2)
VUI(+3)	VUN(+3)	VP(-3)	VUR(-3)

Table 3 Relations between IMP, CON, and PRE

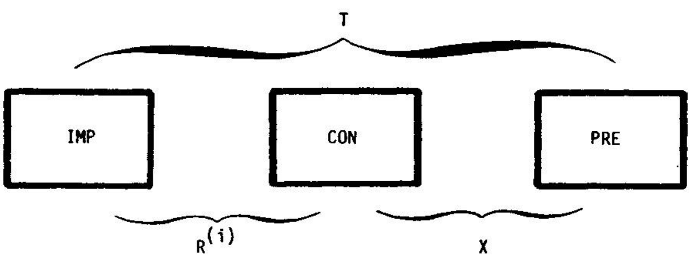

Fig. 3 Polynomial relations between IMP, CON, and PRE