

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 51 (1986)

**Artikel:** Monitoring of a historical wall painting  
**Autor:** Migliacci, Antonio / Mirabella Roberti, G. / Monti, Carlo / [s.n.]  
**DOI:** <https://doi.org/10.5169/seals-39589>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 03.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

**Monitoring of a Historical Wall Painting**  
**Surveillance d'une peinture murale historique**  
**Überwachung eines historischen Freskos**

**Antonio MIGLIACCI**

Prof. of Civil Engineering  
Technical University  
Milan, Italy

**G. MIRABELLA ROBERTI**

Engineer, doctoral student  
Technical University  
Milan, Italy

**Carlo MONTI**

Prof. of Surveying  
Technical University  
Milan, Italy

**Luigi MUSSIO**

Prof. of Applied Geodesy  
Technical University  
Milan, Italy

**SUMMARY**

A continuous check has been made on the masonry wall on which Leonardo painted the «Last Supper», recording measurements of temperatures, displacements, forces, etc. over about two years. Some of these quantities were chosen to perform a statistical analysis of relevant physical relationships; in particular the time dependence of the displacements has been carefully studied. The present work illustrates the statistical methods and the relationships found and gives an assessment on the safety of the painting on the basis of the examined data.

**RÉSUMÉ**

Un contrôle systématique a été fait sur la paroi en maçonnerie sur laquelle Leonardo a peint «L'Ultima Cena» à Milan. Des mesures de température, de déplacement, de force, etc. ont été enregistrées pendant deux ans. Certaines de ces valeurs ont été choisies pour effectuer une analyse statistique des relations physiques existant entre elles; l'évolution des déplacements dans le temps a été étudiée soigneusement. Le texte explique les méthodes statistiques employées et les relations physiques trouvées, et formule des jugements sur la sûreté de l'œuvre d'art à partir des données examinées.

**ZUSAMMENFASSUNG**

Während zweier Jahre wurden am Mauerwerk, welches Leonardo da Vinci's berühmtes Bild «Das letzte Abendmahl» trägt, kontinuierlich Temperatur-, Verschiebungs- und Kräftemessungen durchgeführt. Einige dieser Grössen sind einer statistischen Untersuchung unterzogen worden, um physikalische Beziehungen zwischen ihnen aufzeigen zu können. Insbesondere wurde die Zeitabhängigkeit der Verschiebung aufmerksam beobachtet. Der vorliegende Beitrag erklärt die angewandten statistischen Methoden und die Beziehungen zwischen den verschiedenen physikalischen Grössen und äussert sich auf dieser Basis zur Sicherheit des Kunstwerks.



## 1. THE PROBLEM

The structural safety of existing structures may be a problem of great interest, even if it does not menace human lives directly, when the survival of the building is to be assured for his special historical or artistical renown.

These buildings are somehow a really big patrimony of the whole society, and its safeguard is a duty that a civil society will consider of great importance.

In 1979 some alarming crack opening on the wall on which Leonardo da Vinci painted the celebrated "Last Supper" caused the intervention of the "Ministero dei Beni Culturali" by means of "Soprintendenza ai Beni Architettonici" of Milan.

Besides the different works made, some of with concerning the immediate safeguard of the monument, an automatic data acquisition and recording system was designed and set up, in order to collect a continuous checking of 76 instruments measuring temperatures, relative and absolute displacements, forces and settlements in some suitable points of the structure.

The collected data were represented in plots which showd, though in an empirical way, some presumable relationships existing among the observed quantities: in particular a very clear dependence of the displacements and of the forces on the temperature was noted, and an alarming time dependence was also suspected.

Some of these quantities were chosen to represent in the best way the phenomenon and to perform further analysis in order to more accurately describe the functional relationships among them.

## 2. THE METHOD AND THE STRATEGY

### 2.1 Basic hypothesis

The quantities selected are:

- 3 series of temperatures (namely T03, T07, T09),
- 4 series of "absolute" displacements of the wall and of the retaining steel frame installed on the backside of the wall (namely D4T, D4M, D7T, D7M);
- 2 series of forces interacting between the wall and the steel frame (namely F05 and f08).

First of all, some hypotheses was made, by engineering judgement, on the relationships existing among the measurements of displacements ( $d$ ), temperatures ( $T$ ), forces ( $f$ ), and time ( $t$ ), namely:

$$\begin{aligned} d &= d(T, t), \\ f &= f(d). \end{aligned}$$

The method employed to find the "best" relationships is based on a multiple linear regression by means of least squares technique; the best estimate  $\hat{y}$  of a quantity  $y$  on the base of the quantities  $x_i$  ( $i = 1, \dots, r$ ) is found in the form:

---

(\*) See: Migliacci, A. et al., "Leonardo's Ultima Cena masonry wall", IABSE symposium Venezia 1983.

$$\hat{y} = a_0 + \sum_i a_i x_i$$

If we know the value of  $y$  and  $x_1, \dots, x_r$  in  $n$  points, in each of them we define a residual  $v$  as:

$$\hat{v}_{yi} = y_{o,i} - \hat{y}_i$$

and impose the condition:  $\sum_j \hat{v}_j^2 = \text{minimum}$ , to be realised among the possible choices of the coefficients  $a_i$ .

The autocorrelation function of the residuals  $\hat{v}_j$  (ordered on a time axis) was then analysed in order to separate the signal ( $s$ ) from the noise ( $n$ ) of the process giving a new estimate of  $y$  by means of the last square collocation technique:

$$\hat{v}_y = \hat{s}_y + \hat{n}_y$$

$$\hat{y} = \hat{y} + \hat{s}_y$$

$$y_o = \hat{y} + n_y$$

Similarly a study of the crosscorrelation functions of the residuals of different quantities was performed to control the relationships not explained by the chosen model.

## 2.2 Modelling the observed quantities

The measured temperatures have shown a typical periodical time dependence, related to seasonal variations: thus they were modelled by means of a Fourier-type series based on harmonics of the fundamental period of one year:

$$T_o = c_T + \sum_i (a_{Ti} \sin(i\pi t/365) + b_i \cos(i\pi t/365)) + \hat{v}_T = \hat{T} + \hat{v}_T$$

The second step was to explain the displacements by a linear regression on the explained temperatures  $\hat{T}_i$  and the time  $t$ :

$$d_o = c_d + \sum_i a_{d,i} + b_d t + \hat{v}_d = \hat{d} + \hat{v}_d$$

A particular study was performed to check the possible fitting of a less dangerous time dependence of the kind:  $y = c - a e^{-bt}$ , which supposes that the phenomenon is tending asymptotically to a constant.

These are the steps:

displacements on temperatures only:

$$d_o = c_d^* + \sum_i a_{d,i}^* \hat{T}_i + \hat{v}_d^*$$

time on temperatures:

$$t_o = c_t + \sum_i a_{Ti} \hat{T}_i + \hat{v}_t = \bar{t} + (t_o - \bar{t})$$

and so the comparison was made between:



$$\hat{v}_d^* = a^{**} \hat{v}_t + \hat{v}_{v_d}$$

and

$$\hat{v}_d^* = c^{**} - a^{**} \exp(-\hat{v}_t/365) + \hat{v}_{v_d}$$

the very little difference in the results between the two estimates of the displacements makes the choice between the two models impossible. So we can state that the phenomenon is possibly exhausting very slowly and a period of time of two years is not enough to predict his real behaviour.

The third step was then to explain the forces by the displacements (measured in correspondence of the force transducers)

$$f = c_f + \sum_i a_{fi} \hat{d}_i + \hat{v}_f = \hat{f} + \hat{v}_f$$

All the residuals  $v$  can then be broken in a signal  $s$  and a noise  $n$ , giving a new estimate of  $y$ .

### 2.3 The least squares collocation method.

The collocation method requires appropriate models to interpolate the empirical autocovariance function of the signal, obtained from the residuals of the linear regression. This interpolated function is then used (in addition to the model found by the linear or non-linear regression) to predict the value of the studied quantities at times where on observations are available.

An hypothesis was made: the residuals can be seen as realizations of a broadly stationary random process with mean zero and autocovariance function of the kind

$$c(t_1, t_2) = C(|t_1 - t_2|).$$

Since  $x(t_i)$  are the  $n$  observations at different times  $t_1, \dots, t_i, \dots, t_n$  the estimate of the empirical covariance at the time interval  $\tau_k = |t_1 - t_2|$  is calculated from:

$$\gamma(\tau_k) = 1/n \sum_i (x(t_i) - \bar{x}) 1/n_i \sum_j (x(t_j) - \bar{x})$$

$$\text{where } \tau_{k-1} < |t_i - t_j| \leq \tau_k$$

The "best fit" of the autocovariance function of the signal is then chosen among some available models, namely

E	$\gamma(\tau) = a \exp(-b\tau)$	N	$\gamma(\tau) = a \exp(-b\tau^2)$
EP	$\gamma(\tau) = a \exp(-b\tau) (1-c\tau^2)$	NP	$\gamma(\tau) = a \exp(-b\tau^2) (1-c\tau^2)$
EC	$\gamma(\tau) = a \exp(-b\tau) \cos(c\tau)$	NC	$\gamma(\tau) = a \exp(-b\tau^2) \cos(c\tau)$
ES	$\gamma(\tau) = a \exp(-b\tau) \sin(c\tau)/(c\tau)$	NS	$\gamma(\tau) = a \exp(-b\tau^2) \sin(c\tau)/(c\tau)$

Finally, the noise variance is found as  $\sigma_n^2 = \sigma^2 - \sigma_s^2 = \sigma^2 - a$

The filtering of the process splits the residuals  $v$  in two parts, the signal  $s$  and the noise  $n$ :

$$\begin{aligned}\hat{s} &= C_{ss}^{-1} C_{sv} v \\ \hat{n} &= v - \hat{s}\end{aligned}$$

where  $C_{vv} = C_{ss} + \sigma_n^2 I$

and  $C_{ss}$  is the matrix of the autocovariance of the signal,  $I$  is the unitary matrix of the same dimension as  $C_{ss}$ .

The variance of the error of the estimated signal ( $e = s - \hat{s}$ ) is given by:

$$\sigma_{e_s}^2 = \sigma_s^2 - C_{ss}^{-1} C_{sv} C_{vs}$$

The same relationships are employed for the prediction of the signal  $s$  in points (or at times) where no observations are available:

$$\begin{aligned}\hat{s}_p &= C_{sps}^{-1} C_{sv} v \\ \sigma_{ep}^2 &= \sigma_{sp}^2 - C_{sps}^{-1} C_{sv} C_{vssp}\end{aligned}$$

where  $e_p = s_p - \hat{s}_p$ .

The interest of the prediction is particularly in the study of the future behaviour of the time series.

### 3. THE RESULTS.

Influence of the temperature on the front side of the wall and on the back side was examined, as well as influence of the temperature on the top of the vaults covering the "Cenacolo" (which is nearly the external temperature).

Displacements resulted to be clearly dependent on the difference between external temperature and the temperature on the front side of the wall; this is physically well explained by the variation in the vault thrust due to the differential thermal dilatation.

The quite strong time dependence of the displacements measured on the top of the wall displays a non-stationarity of the phenomenon, and justifies the alarm about wall safety and the works made for its safeguard.

According to physical evidence, forces interacting between the wall and the steel frame results strongly dependent from the difference of the displacements of the wall (DiM) and of the frame (DiT) in corresponding positions, as is shown by the coefficients of the linear regression.

The goodness of fit is very satisfactory: the mean square error (m.s.e.) of the noise is about 6% of the general m.s.e. of the observed data for the forces, and between 7.15 and 16.12% for the displacements.

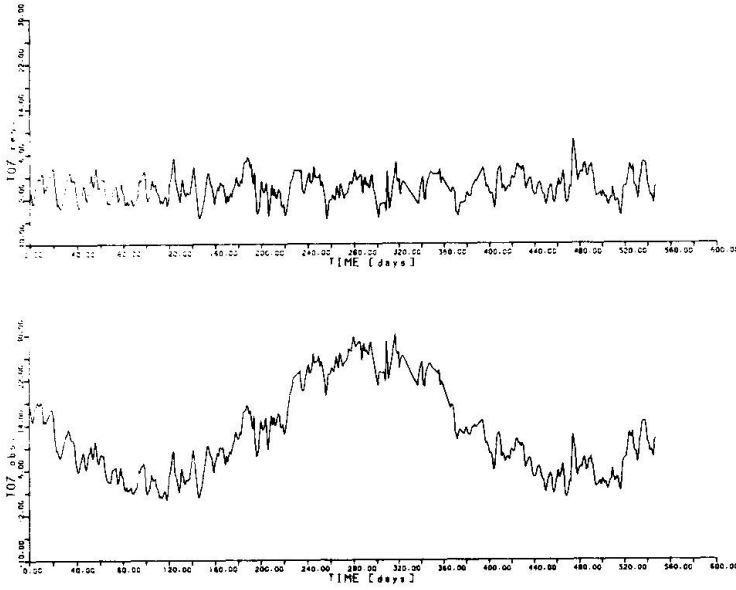


Fig. 1 Plots of temperatures TO7 observed and residuals versus time.

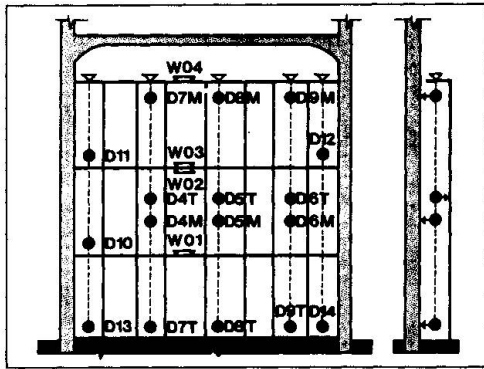


Fig. 2 Plots of displacements D4M observed and residuals versus time.

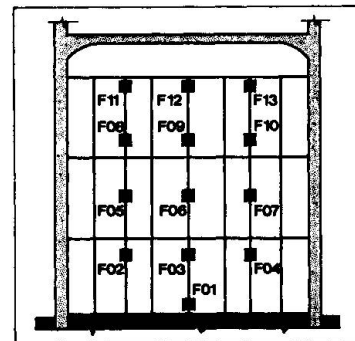
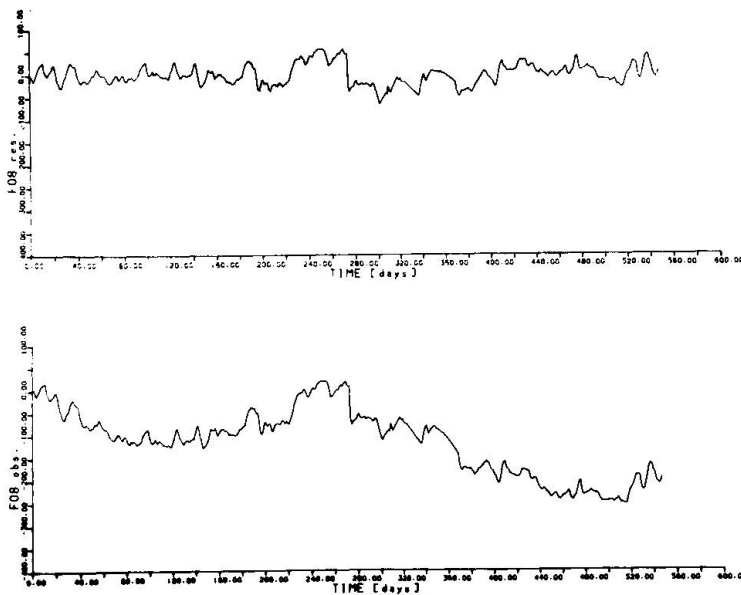
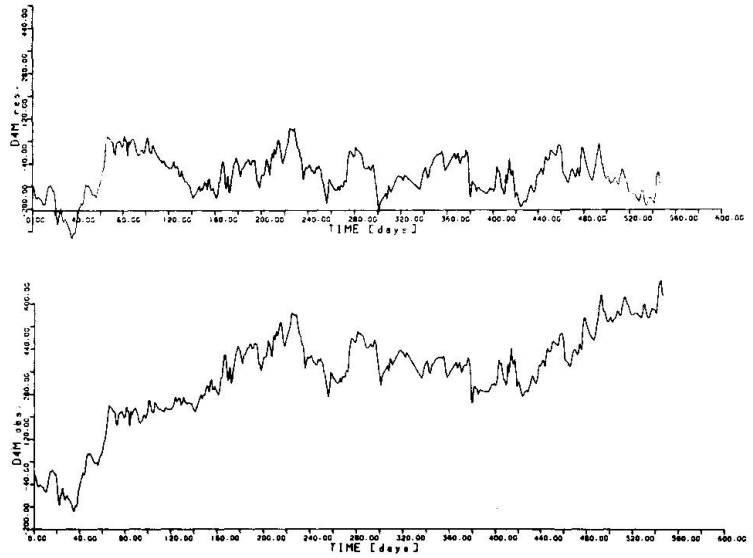


Fig. 3 Plots of forces F08 observed and residuals vs. time.

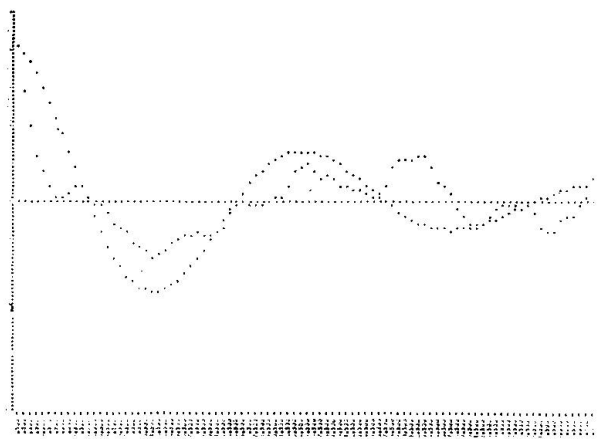


Fig. 4 Autocovariance of T07 residuals

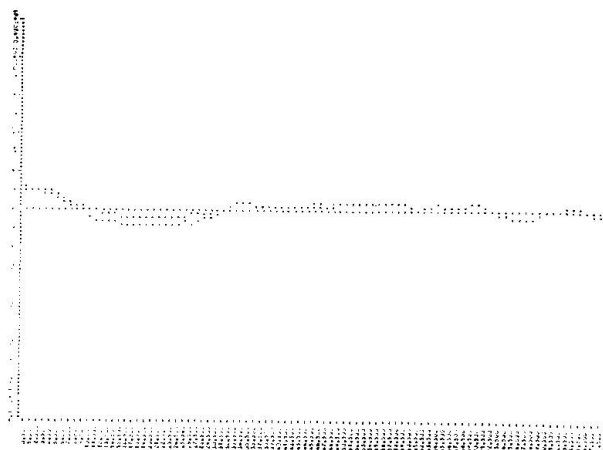


Fig. 5 Crosscovariance of T07 and D4M residuals.

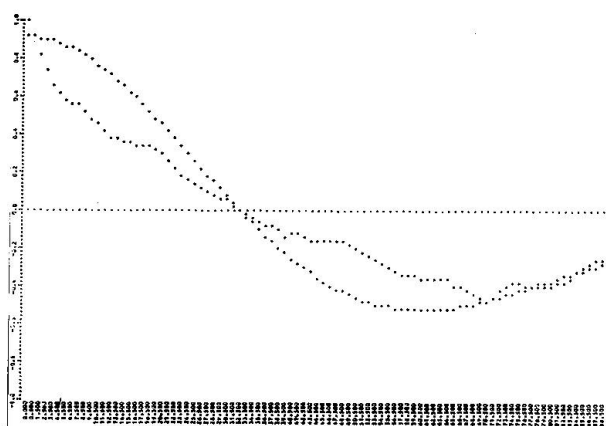


Fig. 6 Autocovariance of D4M residuals

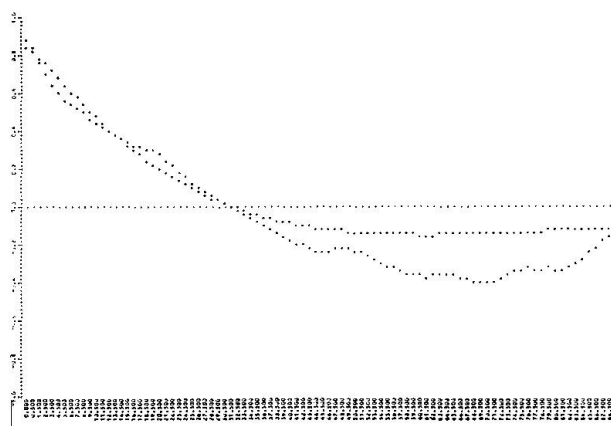


Fig. 7 Crosscovariance of D4M and D4T residuals.

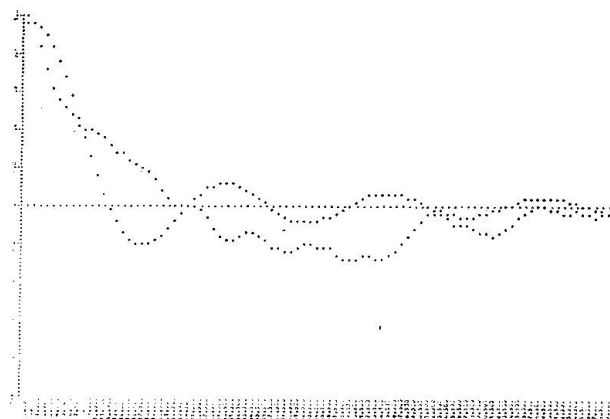


Fig. 8 Autocovariance of F08 residuals

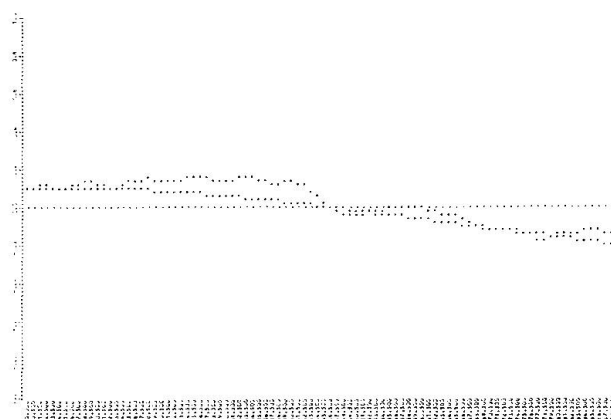


Fig. 9 Crosscovariance of F08 and D4M residuals.



## REFERENCES

1. DAVIES O.L., Statistical methods in research and production , Oliver and Boyed, Londra, 1961.
2. DRAPER N.R., SMITH H., Applied regression analysis, Wiley and Sons, New York, 1961.
3. HEISKANEN W.A., MORITZ M., Physical geodesy, W.H. Freeman and Company, San Francisco, 1967.
4. KENDAL M.G., STUART A. , The advanced theory of statistics, vol. 1, 2 e 3, Griffin, Londra, 1968-'73.
5. MIDDLEBROKS E.J. , Statistical calculation, How to solve statistical problems, Ann Arbor Science, Ann Arbor (Michigan), 1976.
6. MORITZ M., Advanced physical geodesy, Herbert Wichmann Verlag, Karlsruhe, 1980.
7. MORITZ M., Covariance function in least squares collocation, Reports of the Department of Geodetic Science, n. 240 Ohio State University, Columbus, giugno 1976.
8. MORITZ M., Least squares collocation, DGK Reihe A, Heft n. 75, 1973.
9. MORITZ M., Statistical foundations of collocation , Bolletino di Geodesia e Scienze Affini, anno 39, n. 2, aprile-maggio-giugno 1980.
10. MORITZ M., The geometry of least squares , Publication of the Finnish Geodetic Institute, n. 89, Helsinki, 1979.
11. MIKHAIL E.M., con contributi di Ackermann F.. Observations and least squares, Thomas J. Crowell Company Inc., New York, 1976.
12. PAPOULIS A., Probability, random variables, and stochastic processes, Mc Graw-Hill Book Company, New York, 1965.
13. TEWARSON R.P., Sparse matrices, Academic Press, New York, 1973.
14. WATSON G.N., Theory of Bessel functions, Mc Millan Company, New York, 1948.

Leere Seite  
Blank page  
Page vide