Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte
Band: 51 (1986)

Artikel: Monitoring of a historical wall painting
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DOI: https://doi.org/10.5169/seals-39589

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Monitoring of a Historical Wall Painting

Surveillance d'une peinture murale historique

Überwachung eines historischen Freskos

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SUMMARY

A continuous check has been made on the masonry wall on which Leonardo painted the «Last Supper», recording measurements of temperatures, displacements, forces, etc. over about two years. Some of these quantities were chosen to perform a statistical analysis of relevant physical relationships; in particular the time dependence of the displacements has been carefully studied. The present work illustrates the statistical methods and the relationships found and gives an assessment on the safety of the painting on the basis of the examined data.

RÉSUMÉ

Un contrôle systématique a été fait sur la paroi en maçonnerie sur laquelle Leonardo a peint «L'Ultima Cena» à Milan. Des mesures de température, de déplacement, de force, etc. ont été enregistrées pendent deux ans. Certaines de ces valeurs ont été choisies pour effectuer une analyse statistique des relations physiques existant entre elles; l'évolution des déplacements dans le temps a été étudiée soigneusement. Le texte explique les méthodes statistiques employées et les relations physiques trouvées, et formule des jugements sur la sûreté de l'œuvre d'art à partir des données examinées.

ZUSAMMENFASSUNG

Während zweier Jahre wurden am Mauerwerk, welches Leonardo da Vinci's berühmtes Bild «Das letzte Abendmahl» trägt, kontinuierlich Temperatur-, Verschiebungs- und Kräftemessungen durchgeführt. Einige dieser Grössen sind einer statistischen Untersuchung unterzogen worden, um physikalische Beziehungen zwischen ihnen aufzeigen zu können. Insbesondere wurde die Zeitabhängigkeit der Verschiebung aufmerksam beobachtet. Der vorliegende Beitrag erklärt die angewandten statistischen Methoden und die Beziehungen zwischen den verschiedenen physikalischen Grössen und äussert sich auf dieser Basis zur Sicherheit des Kunstwerks.

1. THE PROBLEM

The structural safety of existing structures may be a problem of great interest, even if it does not menace human lifes directly, when the survival of the building is to be assured for his special historical or artistical renown.

These buildings are somehow a really big patrimony of the whole society, and its safeguard is a duty that a civil society will consider of great importance.

In 1979 some alrming crack opening on the wall on which Leonardo da Vinci painted the celebrated "Last Supper" caused the intervention of the "Ministero dei Beni Culturali" by means of "Soprintendenza ai Beni Architettonici" of Milan.

Besides the different works made,some of with concerning the immediate safeguard of the monument, an automatic data acquisition and recording system was designed and set up, in order to collect a continuous checking of 76 instruments measuring temperatures, relative and absolute displacements, forces and settlements in some suitable points of the structure.

The collected data were represented in plots which showd, though in an empirical way, some presumable relationships existing among the observed quantities: in particular a very clear dependence of the displacements and of the forces on the temperature was noted, and an alrming time dependence was also suspected.

Some of these quantities were chosen to represent in the best way the phenomenon and to perform further analysis in order to more accurately describe the functional relationships among them.

2. THE METHOD AND THE STRATEGY

2.1 Basic hypothesis

The quanties selected are:
- 3 series of temperatures (namely T03, T07, T09),
- 4 series of "absolute" displacements of the wall and of the retaining steel frame installed on the backside of the wall (namely D4T, D4M, D7T, D7M);
- 2 series of forces interacting between the wall and the steel frame (namely F05 and f08).

First of all, some hypotheses was made, by engineering judgement, on the relationships existing among the measurements of displacements (d), temperatures (T), forces (f), and time (t), namely:

$$d = d\ (T,t),$$
$$f = f\ (d).$$

The method employed to find the "best" relationships is based on a multiple linear regression by means of least squares technique; the best estimate y of a quantity y on the base of the quantities x_i (i = 1,...,r) is found in the form:

(*) See: Migliacci, A. et al., "Leonardo's Ultima Cena masonry wall", IABSE symposium Venezia 1983.

$$\hat{y} = a_o + \Sigma_i a_i x_i$$

If we know the value of y and $x_1, \ldots, x_r$ in n points, in each of them we define a residual v as:

$$\hat{v}_{yi} = y_{o,i} - \hat{y}_i$$

and impose the condition: $\Sigma_j \hat{v}_j^2$ = minimum, to be realised among the possible choices of the coefficients a_i.
The autocorrelation function of the residuals $\hat{v}_j$ (ordered on a time axis) was then analysed in order to separate the signal (s) from the noise (n) of the process giving a new estimate of y by means of the last square collocation technique:

$$\hat{v}_y = \hat{s}_y + \hat{n}_y$$

$$\hat{\hat{y}} = \hat{y} + \hat{s}_y$$

$$y_o = \hat{\hat{y}} + n_y$$

Similarly a study of the crosscorrelation functions of the residuals of different quantities was performed to control the relationships not explained by the chosen model.

2.2 Modelling the observed quantities

The measured temperatures have shown a typical periodical time dependence, related to seasonal variations: thus they were modelled by means of a Fourier-type series based on harmonics of the foundamental period of one year:

$$T_o = c_T + \Sigma_i (a_{Ti} \sin (i\pi t/365) + b_i \cos(i\pi t/365)) + \hat{v}_T = \hat{T} + \hat{v}_T$$

The second step was to explain the displacements by a linear regression on the explained temperatures $\hat{T}_i$ and the time t:

$$d_o = c_d + \Sigma_i a_{d,i} + b_d t + \hat{v}_d = \hat{d} + \hat{v}_d$$

A particular study was performed to check the possible fitting of a less dangerous time dependence of the kind: $y = c - a e^{-bt}$, which supposes that the phenomenon is tending asymptotically to a constant.
These are the steps:
displacements on temperatures only:

$$d_o = c^*_d + \Sigma_i a^*_{d,i} \hat{T}_i + \hat{v}^*_d$$

time on temperatures:

$$t_o = c_t + \Sigma_i a_{Ti} \hat{T}_i + \hat{v}_t = \tilde{\bar{t}} + (t_o - \bar{t})$$

and so the comparison was made between:

$$\hat{v}^*_d = a^{**}\,\hat{v}_t + v_{\hat{v}_d}$$

and

$$\hat{v}^*_d = c^{**} - a^{**} \exp(-\hat{v}_t/365) + v_{\hat{v}_d}$$

the very little difference in the results between the two extimates of the displacements makes the choise between the two models impossible. So we can state that the phenomenon is possibly exhausting very slowly and a period of time of two years is not enough to predict his real behaviour.

The third step was then to explain the forces by the displacements (measured in correspondence of the force transducers)

$$f = c_f + \Sigma_i\, a_{fi}\, \hat{d}_i + \hat{v}_f = \hat{f} + \hat{v}_f$$

All the residuals v can then be broken in a signal s and a noise n, giving a new estimate of y.

2.3 The least squares collocation method.

The collocation method requires appropriate models to interpolate the empirical autocovariance function of the signal, obtained from the residuals of the linear regression. This interpolated function is then used (in addition to the model found by the linear or non-linear regression) to predict the value of the studied quantities at times where on observations are avaitable.

An hypothesis was made: the residuals can be seen as realizations of a broadly stationary random process with mean zero and autocovariance function of the kind

$$C\,(t_1,t_2) = C(|t_1 - t_2|).$$

Since $x\,(t_i)$ are the n observations at different times $t_1,\ldots,t_i,\ldots,t_n$ the estimate of the empirical covariance at the time interval $\tau_k = |t_1 - t_2|$ is calculated from:

$$\gamma(\tau_k) = 1/n\ \Sigma_i (x(t_i) - \bar{x})\ 1/n_i\ \Sigma_j (x(t_j) - \bar{x})$$

$$\text{where } \tau_{k-1} < |t_i - t_j| \leq \tau_k$$

The "best fit" of the autocovariance function of the signal is then chosed among some available models, namely

E	$\gamma(\tau) = a \exp(-b\tau)$	N	$\gamma(\tau) = a \exp(-b\tau^2)$
EP	$\gamma(\tau) = a \exp(-b\tau)\ (1-c\tau^2)$	NP	$\gamma(\tau) = a \exp(-b\tau^2)\ (1-c\tau^2)$
EC	$\gamma(\tau) = a \exp(-b\tau) \cos(c\tau)$	NC	$\gamma(\tau) = a \exp(-b\tau^2) \cos(c\tau)$
ES	$\gamma(\tau) = a \exp(-b\tau) \sin(c\tau)/(c\tau)$	NS	$\gamma(\tau) = a \exp(-b\tau^2) \sin(c\tau)/(c\tau)$

Finally, the noise variance is found as $\sigma_n^2 = \sigma^2 - \sigma_s^2 = \sigma^2 - a$

The filtering of the process splits the residuals v in two parts, the signal s and the noise n:

$$\hat{s} = C_{ss} C_{vv}^{-1} v$$

$$\hat{n} = v - \hat{s}$$

where $C_{vv} = C_{ss} + \sigma_n^2 I$

and C_{ss} is the matrix of the autocovariance of the signal, I is the unitary matrix of the same dimension as C_{ss}.

The variance of the error of the estimated signal ($e = s - \hat{s}$) is given by:

$$\sigma_c^2 = \sigma_s^2 - C_{ss} C_{vv}^{-1} C_{ss}$$

The same relationships are employed for the prediction of the signal s in points (or at times) where no observations are available:

$$\hat{s}_p = C_{sps} C_{vv}^{-1} v$$

$$\sigma_{ep}^2 = \sigma_{sp}^2 - C_{sps} C_{vv}^{-1} C_{ssp}$$

where $e_p = s_p - \hat{s}_p$.

The interest of the prediction is particulary in the study of the future behaviour of the time series.

3. THE RESULTS.

Influence of the temperature on the front side of the wall and on the back side was examined, as well as influence of the temperature on the top of the vaults covering the "Cenacolo" (which is nearly the external temperature).
Displacements resulted to be clearly dependent on the difference between external temperature and the temperature on the front side of the wall; this is phisically well explained by the variation in the valut thrust due to the differential thermical dilatation.
The quite strong time dependence of the displacements measured on the top of the wall displays a non-stationnariety of the phenomenon, and justifies the alarm about wall safety and the works made for its safeguard.
According to phisical evidence, forces interacting between the wall and the steel frame results strongly dependent from the difference of the displacements of the wall (DiM) and of the frame (DiT) in corresponding positions, as is shown by the coefficients of the linear regression.

The goodness of fit is very satisfactory: the mean square error (m.s.e.) of the noise is about 6% of the general m.s.e. of the observated data for the forces, and between 7.15 and 16.12% for the dispacements.

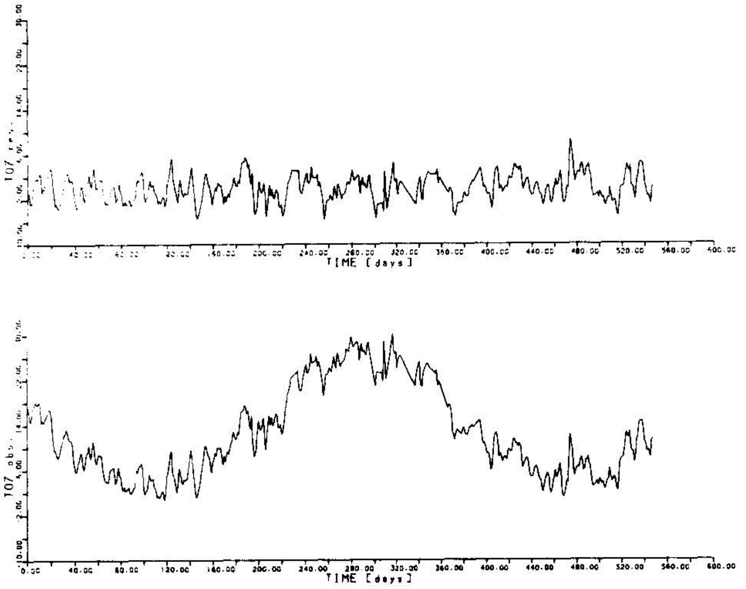
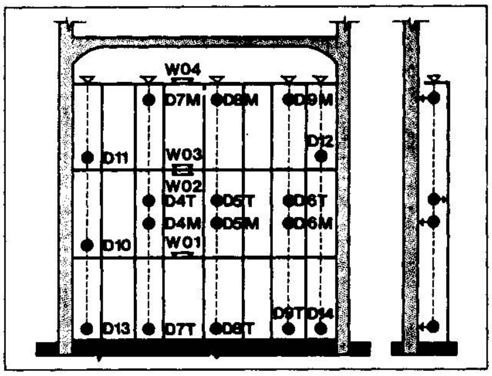


Fig. 1 Plots of temperatures T07 observed and residuals versus time.



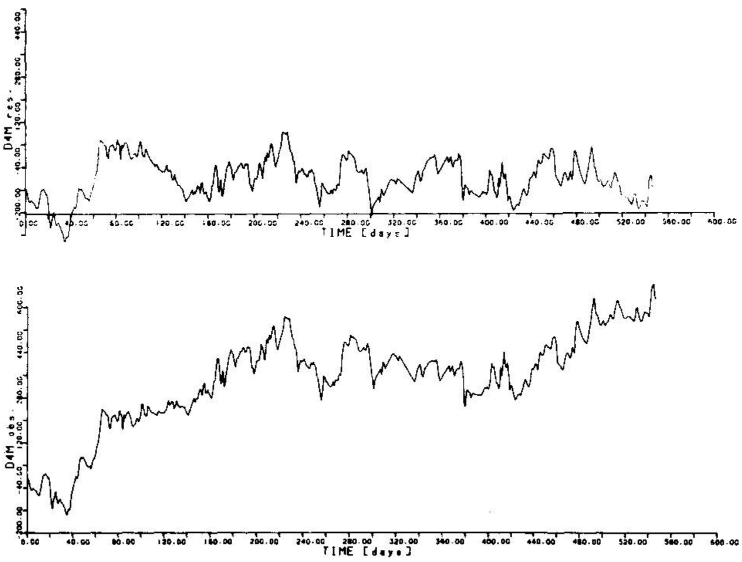
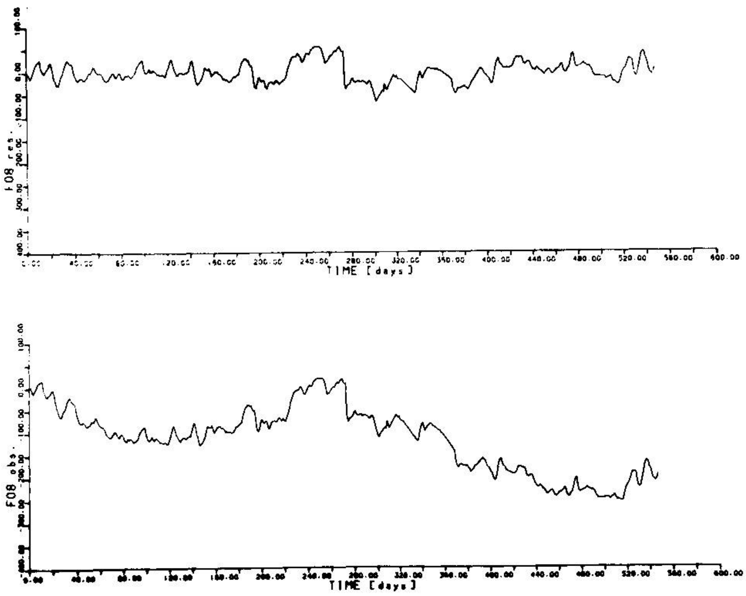


Fig. 2 Plots of displacements D4M observed and residuals versus time.



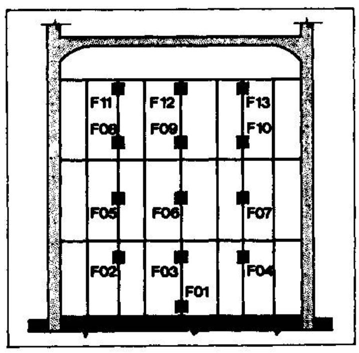


Fig. 3 Plots of forces F08 observed and residuals vs. time.

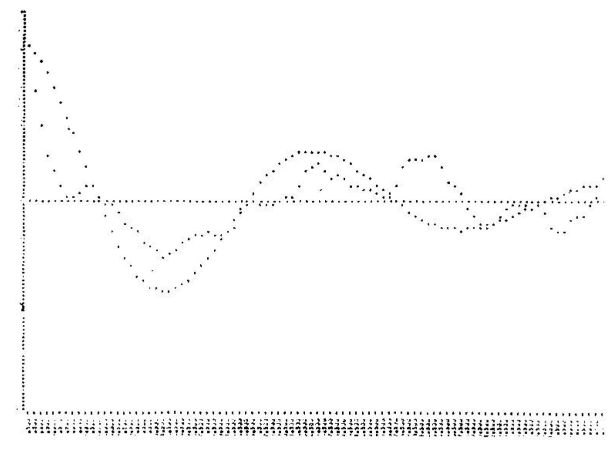


Fig. 4 Autocovariance of TO7 residuals

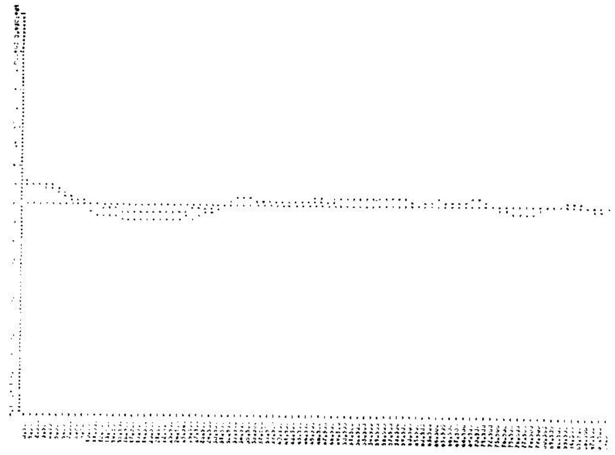


Fig. 5 Crosscovariance of TO7 and D4M residuals.

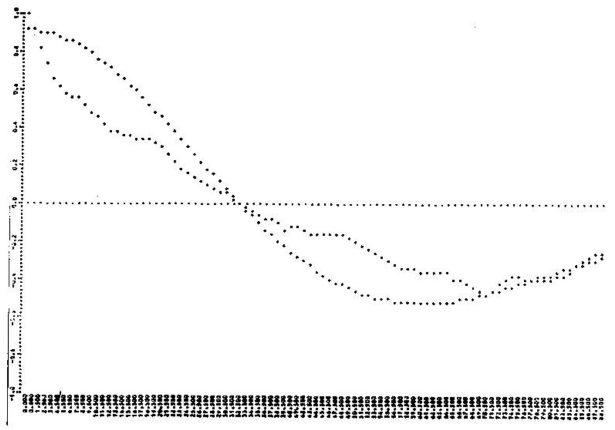


Fig. 6 Autocovariance of D4M residuals

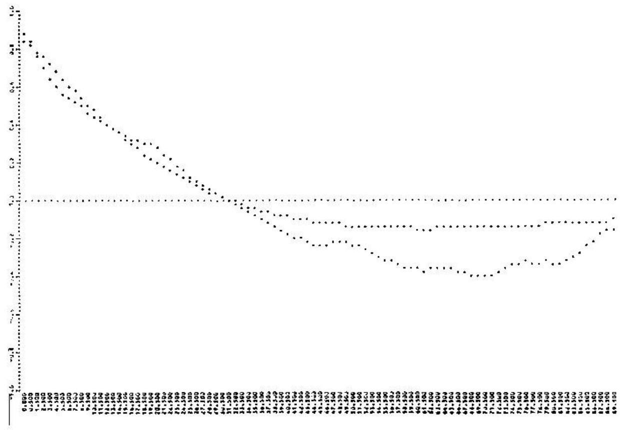


Fig. 7 Crosscovariance of D4M and D4T residuals.

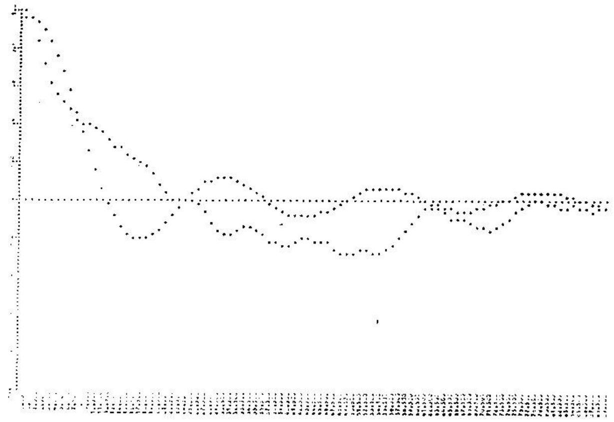


Fig. 8 Autocovariance of FO8 residuals

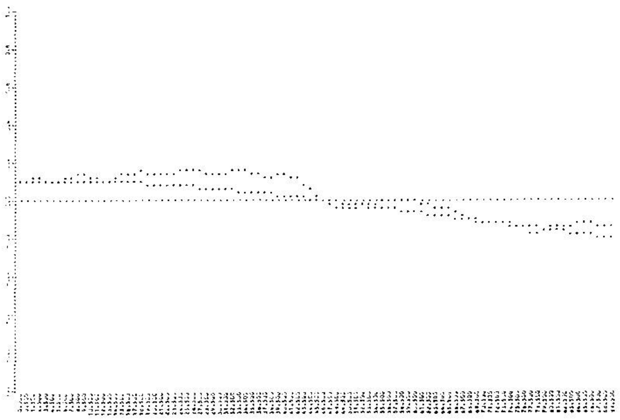


Fig. 9 Crosscovariance of FO8 and D4M residuals.

TAB. I - TEMPERATURES

	TO3	TO7	TO9
x_m	17.37	12.02	18.77
σ_g	4.856	7.692	3.596
σ_e	4.688	7.338	3.381
σ_r	1.266	2.308	1.225
regression coefficients:			
k.t.	19.24	14.97	20.18
sin $2\pi t$	-6.788	-10.81	-4.912
cos $2\pi t$	2.808	3.394	1.877
sin $4\pi t$	-1.206	-0.738	-1.256
cos $4\pi t$	-1.099	-0.354	-1.081
sin $6\pi t$	0.262		0.331
cos $6\pi t$	-0.234		-0.407
sin $8\pi t$	-0.734	-0.640	-0.644
cos $8\pi t$	0.285	-0.241	0.173
autocovariance function:			
kind	N.S.	E.C.	E.C.
$\tau(\gamma=0)$	19.0	8.0	12.4
$\tau(\gamma=.5)$	5.3	2.0	4.4
σ_s	1.233	2.077	1.176
σ_n	0.219	0.968	0.293

TAB. II - DISPLACEMENTS

	D4T	D4M	D7T	D7M
x_m	117.7	339.2	396.6	458.9
σ_g	108.4	175.2	197.7	229.6
σ_e	87.5	158.6	182.6	214.6
σ_r	63.9	74.4	75.8	81.6
regression coefficients:				
k.t.	-271.3	-361.2	66.5	248.8
t	--	.7714	1.009	1.138
TO3	-247.5	-258.1	-194.7	-156.3
TO7	70.39	67.27	53.53	35.42
TO9	205.5	221.8	148.9	116.7
autocovariance function:				
kind	N.P.	E.P.	N.P.	N.P.
$\tau(\gamma=0)$	33.5	31.6	29.0	25.6
$\tau(\gamma=.5)$	20.4	11.5	17.5	14.9
σ_s	63.52	73.98	75.33	81.18
σ_n	17.47	16.25	15.15	16.42

TAB. III - FORCES

	FO5	FO8
x_m	-51.05	-116.6
σ_g	34.77	76.83
σ_e	33.43	73.27
σ_r	9.54	21.92
regression coefficients:		
k.t.	-6.766	-10.12
D4T	0.237	0.373
D4M	-0.260	-0.447
D7T	0.340	0.814
D7M	-0.259	-0.701
autocovariance function:		
kind	N.C.	N.S.
$\tau(\gamma=0)$	20.83	12.3
$\tau(\gamma=.5)$	8.8	7.17
σ_s	9.502	21.83
σ_n	2.091	4.77

TAB. IV - CROSSCORRELATION FUNCTIONS : ρ_0, ρ_1, τ $(\gamma \to 0)$.

	TO3	TO7	TO9	D4M	D4T	D7M	D7T	FO5	FO8
TO3	1.0 .969 110.	0.642 .615 88.5	.928 .898 120.	-.084 -.081 61.5	.056 .055 68.5	-.307 -.296 78.5	-.168 -.163 100.		
TO7		1.00 .821 100.	.681 .631 110.	-.031 -.033 110.	.053 .037 110.	-.218 -.205 47.5	-.090 -.090 140.		
TO9			1.00 .942 120.						
D4M				1.00 .952 160.	.890 .832 220.			-.115 -.111 540.	-.111 -.107 220.
D4T					1.00 .912 180.			.105 .110 50.	.104 .109 210.
D7M						1.00 .936 180.	.963 .925 100.	-.418 -.399 540.	-.459 -.440 160.
D7T							1.00 .938 150.	-.241 -.231 120.	-.291 -.281 160.
FO5								1.00 .951 47.5	.956 .911 78.5
FO8									1.00 .952 120.

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