Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte
Band: 51 (1986)

Artikel: Probabilistic appraisal of safety factors in design codes
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DOI: https://doi.org/10.5169/seals-39574

Probabilistic Appraisal of Safety Factors in Design Codes

Apprécation probabiliste des facteurs de sécurité dans les normes de calcul

Wahrscheinlichkeitstheoretische Einschätzung von Sicherheitsfaktoren in Normen

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SUMMARY
This contribution discusses the reliability level for the determination of safety factors in structural codes. The first part is concerned with the target safety level of a component of a structural system, taking into account the variability of loads and uncertainties due to gross errors. The latter part of the paper discusses the balance of safety levels of different components in a structural system using a long-span suspension bridge as an example.

RÉSUMÉ
L'article traite du niveau de fiabilité pour la détermination de facteurs de sécurité dans les normes de construction. La première partie concerne le niveau de sécurité optimal pour un élément du sysstème structural, considérant la variation des charges et l'incertitude possible due à de grossières erreurs. La deuxième partie envisage l'équilibre des niveaux de sécurité de divers éléments dans un système structural, à partir de l'exemple d'un pont suspendu de grande portée.

ZUSAMMENFASSUNG
Der Beitrag befasst sich mit dem Zuverlässigkeits-Niveau für die Festlegung von Sicherheitsfaktoren in Tragwerks-Normen. Zunächst befasst er sich mit dem anzustrebenden Sicherheits-Niveau von Bauteilen, wobei sowohl die Streuungen der Lasten als auch die Unsicherheiten aus groben Fehlern berücksichtigt werden. Sodann wird am Beispiel einer weitgespannten Hängebrücke die Ausgewogenheit des Sicherheits-Niveaus verschiedener Bauteile eines Tragsystems erörtert.

1. INTRODUCTION

One of the most important items in reliability-based code making is to determine the target sefety level. It is often believed in code calibration that the uniform safety level for different loads or their combination is desirable[1]. However, it seems that the situation is not necessarily so in the current design codes. For example, the safety factor or safety level for dead plus live loads is, in most of the codes, taken higher than that for the environmental loads such as wind or earthquake effects[2].

The target safety level is basically determined such that the total cost including the initial and failure costs is minimized. It should be noted that the variability of the loads would considerably affect the initial cost in securing the required safety level, and hence the optimal safety level can be expected to depend on the load variability.

The history of structures indicates that incompleteness of engineer's knowledge or human error has been one of the major causes of the structural failures[3]. It would be true that the traditional safety factor has been expected to cover not only randomness of resistance and load, but also partly to cover the above mentioned uncertainties which are called as 'gross errors' in this paper. Certain gross errors cannot be completely excluded in the present design and construction processes, although their occurrence may be kept below a prescribed level by means of quality control and inspection.

Under these considerations, in the first part of this paper the target safety level of a component of structural system is assessed, taking into account the variability of loads and uncertainties due to gross errors.

Many of civil engineering structures are regarded as a system of several components. The safety levels or safety factors should not be necessarily the same for different load combinations or different structural components[4]; the appropriate safety factors for each component should be chosen depending upon its cost and consequence of its failure. In the second part of this paper, the balanced allocation of safety factors in a structural system is discussed on the basis of the economic optimization. A long-span suspension bridge is used therein as an example.

2. TARGET SAFETY LEVEL

2.1. Target safety level by cost minimization principle

2.1.1 Evaluation of total cost

Total cost C_T of civil engineering structure may be expressed by

$$C_T = C_I + P_F\, C_F \qquad (1)$$

where C_I and C_F represent the construction plus maintenance cost and failure cost of the structure (component), respectively, while P_F is the probability of failure[5]. According to the principle of total cost minimization, the design of structure attaining the minimum total cost is regarded as optimum.

It is assumed in this study that both the structural resistance R and the load effect S, treated as random quantities, are log-normally distributed. Then the probability of structural failure P_F is given by

$$P_F = \Phi(-\beta) \qquad (2)$$

where $\Phi(\bullet)$ is the cumulative distribution function of the standard normal distribution and safety index β is

$$\beta = \{\ln(\bar{\nu}\sqrt{1+V_S^2}/\sqrt{1+V_R^2})\}/\sqrt{\ln\{(1+V_R^2)(1+V_S^2)\}} \tag{3}$$

$$\bar{\nu} = \bar{R} / \bar{S} \quad : \text{ central safety factor} \tag{4}$$

$\bar{R}$: mean of R, V_R: coefficient of variation of R,
$\bar{S}$: mean of S, V_S: coefficient of variation of S

In general, initial construction cost C_I increases with the increase of the safety factor ν. The following function proposed in Refs. [6] and [7] is used:

$$C_I(\nu)=C_I(\nu_0)\{1+b(\nu/\nu_0-1)\} \tag{5}$$

where ν_0 is the value adopted in the current code and 'b' is a constant. Then the total cost is obtained as

$$C_T=C_I(\nu_0)\{1+b(\bar{\nu}/\bar{\nu}_0-1)+P_F C_F^*\} \tag{6}$$

Failure cost of structure should be evaluated taking account of the direct loss and the indirect loss associated with social and economical effects caused by failure. Because its evaluation is very difficult at present, the dimensionless failure cost C_F^* is assumed to be constant in this study.

Design resistance R_d and design load S_d are assumed to be the fractile values, corresponding to 10% lower and upper fractile, respectively. Furthermore, the following design format is used:

$$R_d / \nu \geqq S_d \tag{7}$$

Then substituting eq. (2) into eq. (6) and satisfying the condition $dC_T/d\beta = 0$, the optimal safety level βopt can be obtained.

2.1.2 Numerical results and discussions

Fig. 1 shows the calculated optimal safety level βopt as a function of the coefficient of variation, V_S of load effect. It is found that the value of βopt for the small V_S is significantly larger than βopt for the large V_S. This result suggests to assign a relatively high target safety level to the structures subject to the less variable load effects. The reason for being obtained this result is that the safety index β does not effectively increase with the increase of the central safety factor when the value of V_S is large. In other words, the large initial cost is required to achieve high safety level for this case.

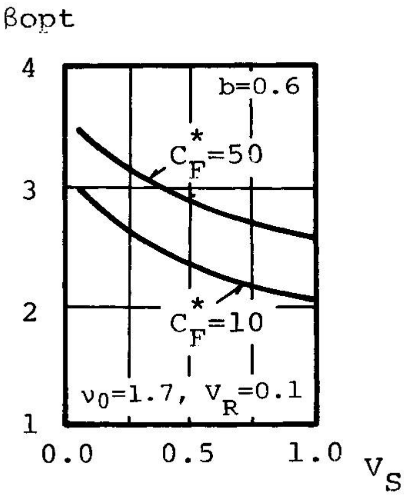

Fig.1 Optimal safety level β_{opt}

2.2 Target safety level in presence of gross errors

2.2.1 Probabilistic model

It is well recognized that structural safety depends not only on statistical uncertainties but also uncertainties due to gross errors such as human errors. Then the effect of the latter uncertainties on structural reliability is investigated by use of the simple probabilistic model defined as below[3].

Assume that the structural resistance R decreases to R_u because of the existence of uncertainties due to gross errors and that R_u is defined as

$$R_U = R \times U \tag{8}$$

where U is the random variable whose probability density function is

$$f_U(x) = P\delta(x-\phi) + (1-p)\delta(x-1) \tag{9}$$

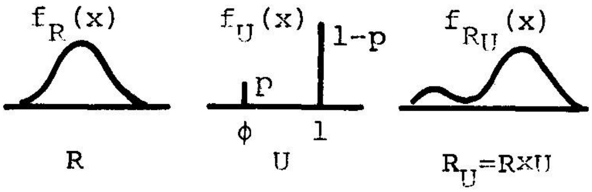

Fig.2 Structural resistance deterioration model

in which $\delta(\cdot)$ is Dirac's delta funcion. In this modeling it is considered that the resistance reduction occurs with probability p and that structural resistance R deteriorates to $(1 - \phi)$ R as shown in Fig. 2. The probability of failure with log-normally distributed R and S is expressed as

$$P_F^* = \mathrm{Prob}[R_U < S] = pP_{F_U} + (1-p)P_{F_n} \tag{10}$$

in which

$$P_{F_U} = \Phi(-\beta_n - \ln\phi/\sqrt{\ln\{(1+V_R^2)(1+V_S^2)\}}) \tag{11}$$

$$P_{F_n} = \Phi(-\beta_n) \tag{12}$$

βn in eqs. (11) and (12) is the 'apparent' or 'operational' safety index and is the same as that defined by eq.(3). If the values of p and ϕ in eqs. (10) and (11) are given, P_F^* can be calculated by eq. (10).

2.2.2 Values of parameter p and ϕ

We attempt herein to estimate the reasonable range of parameters p and ϕ from the surveys on bridge failures[8]. Suppose that all of structures are designed so as to attain the target safety level βn. Under this condition, the ratio 'a' which leads to the relation between the number of failure caused by uncertainties due to gross errors and that caused by statistical ones is defined from eq. (10) as

$$a = pP_{F_U}/(1-p)P_{F_n} \tag{13}$$

According to the structural failure data in Ref. [8], it can be found that the ratio 'a' takes the values between 0.25 and 2.4. On the other hand, many civil engineers and investigators consider that the actual failure probability of civil engineering structures may be higher than operational one[9], which is generally said to be about $10^{-4} \sim 10^{-6}$. Taking account of these situations, the ratio 'a' is assumed here to take the value between 0.25 and 10.0. Consequently, the fluctuation range of parameter p is $4.1 \times 10^{-3} \sim 1.4 \times 10^{-1}$ for $\phi = 0.7$ under the conditions that $V_R = 0.1$, $V_S = 0.2$ and $\beta_n = 3.0$.

2.2.3 Numerical results and discussions

The apparent β_n to attain the probability of failure $P_F^* = 1.35 \times 10^{-3}$ ($\beta^* = -\Phi^{-1}(1.35 \times 10^{-3}) = 3.0$) is calculated under the conditions that $\phi = 0.7$ and $P = 4.1 \times 10^{-3} \sim 1.4 \times 10^{-1}$ and are presented in Fig. 3. Fig. 3 shows that the case dominated by less variable load effects requirs higher target β_n to attain the same safety level. This means that the probability of failure of the structure subject to less variable loads is strongly increased by the existence of the gross errors. In other words, uniform apparent safety level results in the less safety margin against the uncertainties due to gross errors in the case of the small variance of load effects.

2.3 Target safety level taking account of both total cost minimization and uncertainties due to gross errors.

In this section, target safety level taking account of both total cost minimization and uncertainties due to gross errors is discussed. Substituting eq. (10) instead of eq. (2) into eq. (6) and applying the principle of total cost minimization, βn, opt is calculated. The parameter values used are;

$C_F^*=10$ and 50, $\nu_0=1.7$, $V_R=0.1$, $V_S=0.05\sim1.0$,

$\phi=0.7$, $p=4.1\times10^{-3}\sim1.4\times10^{-1}$, $b=0.6$

Fig. 4 shows the optimum apparent safety level βn, opt calculated as a function of the coefficient of variation of load effect. As expected, βn,opt is larger than that presented in Fig. 1. And it is also found that βn,opt is considerably higher for smaller coefficient of variation V_S of the load effect. This result agrees with that obtained from Fig. 1 and Fig. 3. Accordingly, it can be concluded that relatively higher safety level to the less variable load effects should be assigned and consequently that it is not necessary to provide large difference between load factors according to the variability of load effects.

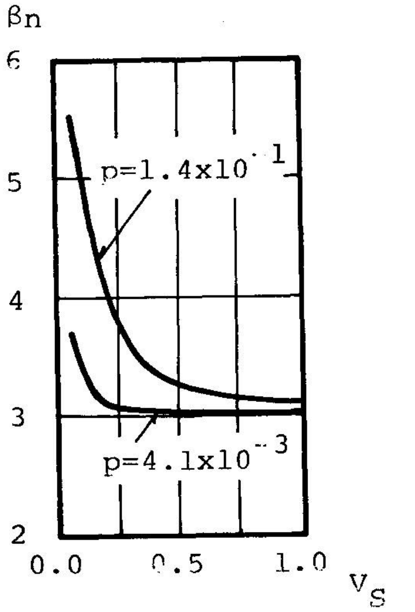

Fig.3 Apparent safety level β_n

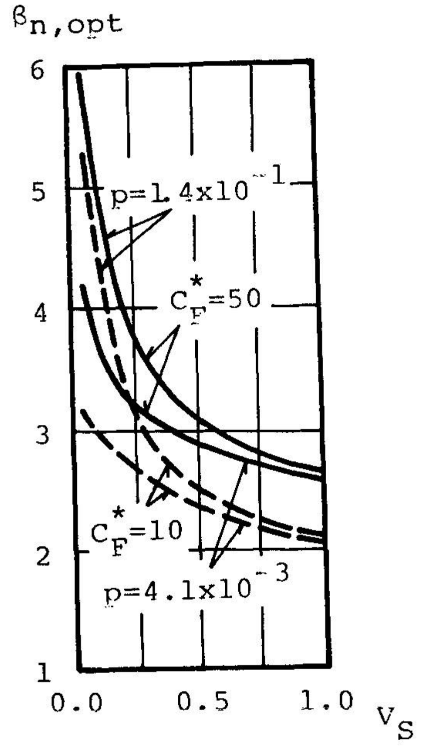

Fig.4 Optimal apparent safety level $\beta_{n,opt}$

3. BALANCE OF SAFETY FACTORS IN A STRUCTURAL SYSTEM

Many of the civil engineering structures are consisted of different components and have various failure modes. Hence they are to be treated as structural systems. Long-span suspension bridge is certainly one of the typical structural systems. Its superstructure is composed of towers, cables and stiffening girder. In towers and cables of a long-span suspension bridge, dead load effect exceeds 90% of the total design load effect. On the other hand, the design of the principal members of stiffening girder is controlled by the wind load, at least in our Japanese practice. This is especially true for the truss type of girders because of the large wind force. The dead load has a very small variation under elaborate quality control, but the wind load and the performance of the suspension bridge under wind action is very much uncertain. This observation leads to smaller safety factor for the towers and cables and to larger safety factor for the stiffening girder. The current design specification for long-span suspension bridges seems opposite; approximately the safety factor of 3.0 for the ultimate strength of the cable and less than 2.0 for the stiffening girder[10].

In this section, balanced allocation of the safety factors in a system is studied again from an economical point of view. Only two components in the suspension bridge, namely cables and stiffening girder are investigated herein.

3.1 Evaluation of total cost

In a similar manner as in section 2, the total cost of each component is expressed as

$$C_{TC}=C_{IC}+P_{FC}C_{FC}\ ,\quad C_{TG}=C_{IG}+P_{FG}C_{FG} \tag{14}$$

in which subscript 'C' and 'G' stand for the cables and girder, respectively.

Then the total cost C_T of the structure is given by

$$C_T=(C_{IC}+P_{FC}C_{FC})+(C_{IG}+P_{FG}C_{FG}) \tag{15}$$

The random variables, i.e., structural resistance, dead and wind loads are assumed log-normally distributed and then the probability of failure P_{FC} and P_{FG} can be obtained from eq. (2). C_{IC} and C_{IG} are assumed to be the form of eq. (5). Break of the cable leads to the collapse of the suspension bridge;failure of the stiffening girder does not necessarily mean the collapse of the whole structure. Considering this, it is reasonable to assume C_{FC} is larger than C_{FG}. The sum of C_{IC} and C_{IG} is assumed to keep constant.

3.2 Numerical results and discussions

Numerical computations are carried out in order to find the optimal set of safety factors ν_C and ν_G.

Under these observations as well as the design caluculation used in the proposed Akashi Straits Bridge, the longest bridge of the Honshu-Shikoku project, the following values are subjectively chosen and used in the example calculation:

$\nu_0=1.7$, $C_{IC}:C_{IG}=2:1$, $C_{FC}:C_{FG}=2:1$, $C_{IC}:C_{FC}=1:100$, $V_{RC}=V_{RG}=0.1$,

$V_D=0.1$ (dead load effect), $V_W=0.3$ and 0.5 (wind load effect)

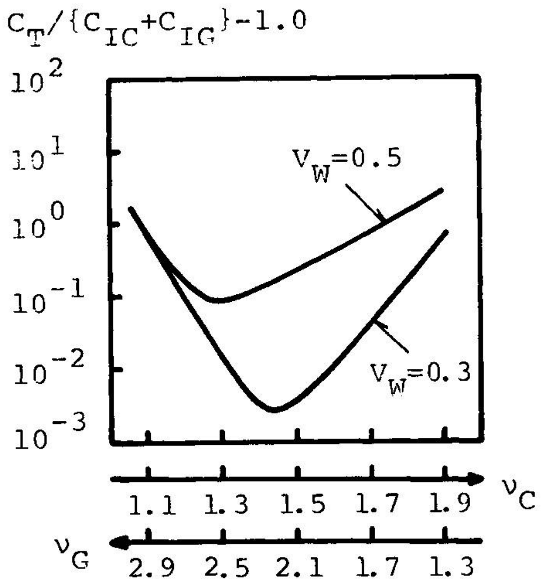

Fig.5 Total cost vs. Safety factors ν_C and ν_G

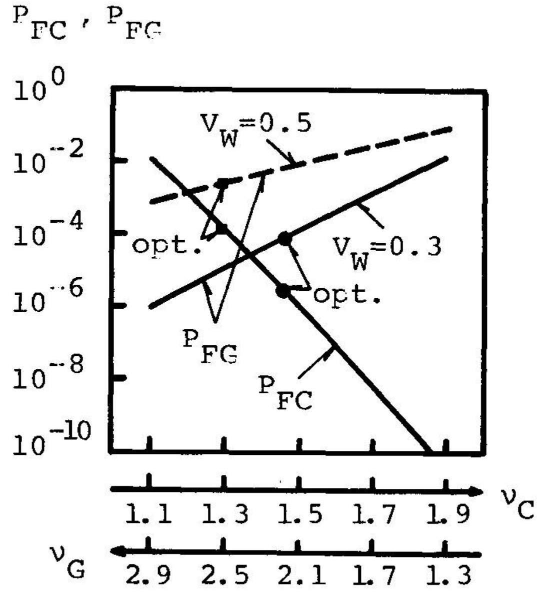

Fig.6 P_{FC} vs. ν_C and P_{FG} vs. ν_G

The dimensionless total cost $C_T(\nu_C, \nu_G)/\{C_{IC}(\nu_0) + C_{IG}(\nu_0)\} - 1.0$ and probability of failure P_{FC} and P_{FG} were calculated as the function of ν_C and ν_G as shown in Fig. 5 and Fig. 6, respectively. It can be found that the optimal safety factor $\nu_{C,opt}$ is noticeabley smaller than $\nu_{G,opt}$, while the optimal value of P_{FC} corresponding to $\nu_{C,opt}$ is smaller than optimal P_{FG} corresponding to $\nu_{G,opt}$. Although P_{FC} decreases remarkably as the increase of ν_C, further increase of ν_C is not profitable refering to Fig. 5. The combination of ν_C and ν_G in the current design specificaitons for long-span suspension bridges contradicts with the above findings. Although the parameter values need to be more carefully chosen, reconsideration of the selection of safety factors might be desirable.

4. CONCLUDING REMARKS

The target safety level of a structural component as well as structural system has been discussed. Introducing a gross error model, the optimal safety level of a structural component was calculated on the basis of cost optimization.

The resluts show that it is reasonable to assign relatively higher safety level to the less variable load effects. This implicitly supports the situation of safety level in the current design code.

Employing a long-span suspension bridge as a structural system example, the optimal allocation of the safety levels for different components was studied. The numerically higher safety level for the components subject to less variable load effects is found optimal in this example as well; namely, the higher safety level for the cable and the lower for the stiffening girder. The reconsideration of the safety factors in the current design practice of long-span suspension bridges is suggested on the basis of these findings.

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