Acceptable Risk Levels for Fatigue Design of Concrete Structures

Niveaux de risque acceptables pour le dimensionnement à la fatigue de structures en béton

Akzeptierbares Risiko-Niveau für die Bemessung von Betonbauten auf Ermüdung

R. F. WARNER
Prof. of Civil Engineering
Univ. of Adelaide
Adelaide, Australia

R. F. Warner was born in 1935. He obtained his doctorate from Lehigh University. With research interests in the behaviour of concrete structures, he has published text books on the design of prestressed and reinforced concrete.

SUMMARY
A method is proposed for determining acceptable risk levels for the fatigue design of concrete structures. The safety level for fatigue failure is found by calibrating back to the safety level for flexure failure. The underlying assumption is that the expected cost of failure should be approximately equal for fatigue and flexure.

RÉSUMÉ
Une méthode est proposée afin de déterminer les niveaux de risque acceptables pour le dimensionnement à la fatigue de structures en béton. Le risque d'une rupture due à la fatigue est comparé au risque d'une rupture par flexion. L'hypothèse retenue est que le coût prévisible de la rupture soit à peu près semblable dans les cas de fatigue et de flexion.

ZUSAMMENFASSUNG
Eine Methode für die Bestimmung akzeptierbarer Versagenswahrscheinlichkeiten für die Bemessung von Stahlbetonbauten gegen Ermüdung wird vorgeschlagen. Die Wahrscheinlichkeit des Ermüdungsbruchs wird an der Wahrscheinlichkeit eines Biegebruchs kalibriert. Dabei wird vorausgesetzt, dass die Versagenskosten für diese beiden Brucharten etwa gleich sein sollten.

1 INTRODUCTION

Before any new, improved structural design procedure can be introduced into a code of practice, an appropriate margin of safety for design has to be chosen, and partial safety coefficients which will provide this safety margin then have to be evaluated. The problems of choosing an appropriate margin of safety and evaluating the corresponding safety coefficients also arise whenever the design of an unusual, one-off structure is undertaken in circumstances to which routine code design procedures are not applicable. With a new generation of limit states codes being introduced progressively in a number of countries, many code writers are currently involved in the tasks of establishing target safety margins for structural design and evaluating the corresponding safety coefficients.

Despite much recent theoretical research into structural reliability theory, the most frequently used method for evaluating design safety coefficients is still by back-calibrating to an existing design procedure which has been shown by extensive previous use to be safe and economic. Back calibration consists essentially of adjusting the safety coefficients in the new design procedure so that, for a number of carefully selected, representative design cases, the resulting designs are very similar to those which are obtained from the existing method. Of course, the back-calibration method can only be used if a well-tried design procedure is already at hand. This is not the situation in the design of concrete structures for fatigue. Indeed, the problem of design for fatigue, as distinct from analysis of fatigue life and fatigue resistance, has received surprisingly scant research attention. Consequently, there is very little guidance presently available to the designer with regard to the safety margins and partial safety coefficients which should be used in fatigue design.

This paper describes a procedure for determining the appropriate risk level for use in the fatigue design of reinforced concrete and prestressed concrete structures. The procedure is called cross-calibration, since it relies on a quantitative comparison of the safety requirements for fatigue with those of another limit state (in this case flexure) which has already been back-calibrated to an existing design method (the ultimate strength method) with a history of successful use. The basis for the quantitative comparison is expected cost of failure.

2. LIMIT STATE FORMAT FOR FATIGUE

In the usual limit states design format [6], a comparison is made between a measure of load intensity S, and a measure of structural resistance R. The margin of safety may be defined as the difference between R and S:

$$Z = R - S \tag{1}$$

All three quantities R, S and Z must of course have the same dimension, which for an ultimate limit state may be force, moment or possibly stress. In fatigue design, R and S may be chosen in a variety of ways: they may refer to the stress intensity at a potential fracture point, to the number of cycles to failure, to accumulated damage in a critical region of the structure, or even to the degradation in static strength of the structure as the result of fatigue damage [6]. Given the limited amount of design data currently available on fatigue of concrete structures, the cycle format appears to be the most promising and the one most readily adaptable to useful design calculation.

Once the design format has been chosen, simple first-order reliability concepts can be applied in a routine manner. The actual fatigue life N_R of a member subjected to a specified load spectrum shows considerable variability and is best treated as a random variable. For design, a characteristic fatigue life N_{Rk} is chosen to correspond to a specific failure probability P_k:

$$P[N_R < N_{Rk}] = P_k \tag{2}$$

A design value N_{Rd} is defined by means of a safety coefficient γ_{RN}:

$$N_{Rd} = \frac{1}{\gamma_{RN}} N_{Rk} \tag{3}$$

The number of cycles of load which the structure will be designed to resist is denoted by N_S. This may be regarded either as a design constant (such as $2X10^6$) or as a random variable [1]. In the latter case, a characteristic value N_{Sk} has to be chosen:

$$P[N_S < N_{Sk}] = 1 - P_k \tag{4}$$

and hence by means of a partial 'load' factor γ_{SN}, a design value can be defined:

$$N_{Sd} = \gamma_{SN} N_{Sk} \tag{5}$$

Experimental studies of fatigue failure suggest that fatigue life N is best considered on a logarithmic scale. The safety margin Z is therefore defined here in terms of the transformed variables:

$$\bar{N}_S = \log N_S \tag{6}$$

$$\bar{N}_R = \log N_R \tag{7}$$

$$Z = \bar{N}_R - \bar{N}_S \tag{8}$$

The reliability index β then can be defined as follows [4]:

$$\beta = \frac{\mu(Z)}{\sigma(Z)} = \frac{\mu(\bar{N}_R) - \mu(\bar{N}_S)}{\sqrt{\sigma^2(\bar{N}_R) + \sigma^2(\bar{N}_S)}} \tag{9}$$

The nominal probability of failure P_f is related to β by the properties of the normal distribution [4].

Various problems are involved in the evaluation of the safety coefficients γ_{RN} and γ_{SN}, once the reliability index β has been chosen. For example, in dealing with the fatigue life N_R the load spectrum has to be taken into account, usually by some form of cumulative damage calculation. Such matters have been discussed elsewhere [6]. The purpose of the present paper is to deal with the problem of determining an appropriate margin of safety, ie of evaluating β or, equivalently, P_f.

3. CROSS-CALIBRATION AND EXPECTED COSTS

The cross-calibration procedure proposed here is based on a comparison of the safety requirements for fatigue design with those for design for flexure. It is assumed that the latter has already been back-calibrated to the existing ultimate strength design procedures in current use. The quantitative basis for the comparison is expected cost of limit state entry. The probability of occurrence of an event E_j, such as entry into a limit state, will be written as $P[E_j]$. The cost of event E_j, should it occur, is $C[E_j]$, and the expected cost of E_j is defined as:

$$EC[E_j] = P[E_j].C[E_j] \tag{10}$$

A range of consequences, Q_{ij}, follow on from an event E_j, and each consequence has an associated cost $C[Q_{ij}]$. If the consequences all occur simultaneously, then we have:

$$C[E_j] = \sum_i [Q_{ij}] \tag{11}$$

and the expected cost of E_j becomes:

$$EC[E_j] = P[E_j] \sum_i C[Q_{ij}] \tag{12}$$

However, if there exists a range of alternative, mutually exclusive consequences Q_{ij} with costs $C[Q_{ij}]$ and associated probabilities $P[Q_{ij}]$, then we have:

$$C[E_j] = \sum_i P[Q_{ij}].C[Q_{ij}] \tag{13}$$

$$EC[E_j] = P[E_j] \sum_i P[Q_{ij}].C[Q_i] \tag{14}$$

As we shall be concerned only with fatigue and flexure failure, the indices 1 and 2 will be used to refer to these limit states, respectively. Thus, E_1 and E_2 are the events of fatigue failure and flexure flexure.

In order to evaluate the target design probability for fatigue, $P[E_1]$, the criterion of equal expected costs for fatigue and flexure failure is applied:

$$EC[E_1] = EC[E_2] \tag{15}$$

This leads to the following expression for the target design probability for fatigue:

$$P[E_1] = P[E_2] \frac{C[E_2]}{C[E_1]} \tag{16}$$

Nominal probabilities of failure for use in the design of concrete, steel and timber members have recently been evaluated by Leicester [2], by a process of back-calibration to existing Australian design codes. Extreme values obtained by Leicester for the safety index β, together with the corresponding nominal probabilities of failure, $P[E_2]$, for the flexural design of concrete members, are summarised in Table 1.

4. CONSEQUENCES AND COSTS OF LIMIT STATES ENTRY

If estimates can be made of the consequences and hence relative costs of fatigue failure and flexure failure, Eq 16 can be used in conjunction with the information in Table 1 to evaluate $P[E_1]$, the nominal probability of failure for use in fatigue design.

If event E_j represents the entry of a member into a strength limit state, the consequences which need to be considered include the following:

Q_{1j}: damage to, or destruction of, the structural member;
Q_{2j}: damage to, or destruction of, non-structural attachments;
Q_{3j}: damage to the contents of the structure;
Q_{4j}: injury and possible loss of life;

Q_{5j}: inconvenience, including loss of income, during the period that the structure is out of service.

r	β_2 min	β_2 max	$P[E_2]$ ($\times 10^{-6}$) min	$P[E_2]$ ($\times 10^{-6}$) max
0	3.37	3.85	337	60
0.25	3.60	4.25	159	11
0.50	4.08	5.08	21	0.2
0.75	4.00	4.39	32	5
1.00	3.93	3.93	48	48

Table 1 Values of β_2 and $P[E_2]$ for flexure failure (obtained by back calibration [2]).

Note: r is the ratio of wind load to total load, ie wind plus dead plus live loads.

It will be assumed that the cost associated with each of these consequences is a simple multiple of the original cost C_0 of the structural member:

$$C[Q_{ij}] = f_{ij}.C_0 \tag{17}$$

This assumption is probably very reasonable for some of the consequences listed (eg i = 1,2) but rather arbitrary for others (i = 3,4,5). Nevertheless, anything but the simplest approach is unwarranted. Since the consequences being considered can occur simultaneously, we use Eq 12 to express the expected cost of limit state entry as:

$$EC[E_j] = P[E_j].C_0 \sum_i f_{ij} \tag{18}$$

By introducing a factor F into Eq 16 to take account of the multiplying factors f_{ij}, we obtain the following expressions:

$$P[E_1] = P[E_2].F \tag{19}$$

$$F = \frac{\sum_i f_{i2}}{\sum_i f_{i1}} \tag{20}$$

The events leading to failure and the consequences of failure are greatly affected by the structural details of the member, as well as by its use and, where relevant, the occupancy and contents of the overall structure. Of particular structural importance is whether or not the member is statically indeterminate.

Considering first the case of a statically determinate member, we can reasonably assume that flexural failure leads to collapse, with destruction of any attached non-structural members and contents. In contrast, the consequences of fatigue failure in a determinate concrete flexural member are likely to be far less serious. The reason for this is that fatigue failure occurs progressively in a concrete flexural member by successive fracture of the tensile steel elements (prestressing wires or reinforcing bars). In practice there are many individual elements which make up the total tensile steel area, and so the process of fatigue failure is not sudden but gradual, with warning of deterioration given by increasing deflection and widening cracks.

In order to obtain a range of relevant cost values we consider two main structural cases, one with, and the other without, serious consequences of collapse. In Table 2, estimated values for the cost factors f_{ij} are included for the various conditions.

Table 2 Cost factors f_{ij}

i	Consequences of failure: serious		not serious	
	j=1 fatigue	j=2 flexure	j=1 fatigue	j=2 flexure
1	2	2	2	2
2	1	10	2	2
3	1	50	1	10
4	1	150	1	1
5	10	10	10	10
$\sum_i f_{ij}$	15	222	16	25

In the case of flexure failure a factor of 2 is used in Table 2 for the costs associated with repair or replacement (Q_{12}). Although the damage from fatigue failure (fracture of several steel elements in the critical section) is not as severe as for flexure failure, repair is likely to be complicated. Furthermore, fatigue damage is likely to have been initiated in other steel elements and in other critical cross-sections. The cost of repair has therefore been chosen as equal to that for flexure failure. Damage to any non-structural attachments is likely to be severe in the case of flexural failure, and it seems appropriate to choose f_{22} to correspond to the full cost of replacement. These can only be evaluated accurately for a specific example and a rather arbitrary range is given in Table 2. In the case of fatigue failure, collapse does not occur, so that there is likely to be little or no damage to non-structural attachments. The situation regarding damage to contents is similar to that for non-structural attachments. Damage is likely to be zero for fatigue failure and substantial for flexure failure.

Although the likelihood of injury will depend very much on the use and occupancy of the structure, the consequences of fatigue failure remain minimal because collapse is avoided. On the other hand, flexural failure in a determinate member is always potentially serious. The values chosen for f_{42} are intended to reflect this. The situation is rather different with regard to the out-of-service costs, which will be incurred irrespective of the reason for the structure not being able to coninue to function. Out-of-service time is also of importance, but this will depend on the repair process rather than on the cause of failure. Similar values for f_{51} and f_{52} have therefore been introduced into Table 2. From the entries in Table 2, a range of values for the factor F of from 10 to about 250 is obtained for statically determinate members.

In the case of statically indeterminate members, the consequences of flexure failure become less severe because attainment of the moment capacity in a critical cross-section does not any more imply structural collapse, but rather localised yielding of the reinforcing steel with increased deformations and deflections, possibly with some permanent set. The situation is now more comparable with that of fatigue failure, for which the consequences are rather similar. The cost of repair in each case is probably about equal to the cost of replacement. However, there is little point in considering the indeterminate case in detail because current and proposed design procedures [3] rarely allow for different safety levels according to the degree of statical indeterminacy.

The general figures given in Table 2 are clearly very approximate and even speculative. However, more accurate figures can always be obtained if a specific design case is investigated. It must also be remembered that gross approximations are introduced into the safety treatment for flexural design. As we have just seen, even the influence of statical indeterminacy is at present ignored in the choice of the safety margin for flexural design. For statically determinate members with important and expensive installations and contents, and subject to high occupancy, the cost factors in Table 2 would suggest a value of the ratio F (Eq 20) as follows:

$$F = 20$$

For less important members, the value of F drops markedly:

$$F = 2 \text{ to } 10$$

In the case of indeterminate structures with high levels of built-in indeterminacy, the factor F would also be quite low and similar to the previously quoted range for less important members.

In order to obtain an independent estimate of the factor F, it is possible to use data provided by the Nordic Committee on Building Regulations (NKB). In choosing target failure probabilities, the NKB considers the consequences of failure in three categories, namely not serious, serious, and very serious. Three failure types are considered:
(1) ductile failure with strength reserves due to strain hardening;
(2) ductile failure without reserve capacity; and
(3) brittle failure and instability.

If fatigue failure in a concrete member is identified with first wire or bar fracture, then it is reasonable to treat this as a type 1 failure category, while normal flexure failure for a determinate member could well be regarded as category 2. Assuming the consequences of failure to be serious or very serious, and then taking the corresponding NKB nominal probability levels as quoted in Ref [4], we obtain a range of values for $P[E_1]$ of from 10 to 100 times $P[E_2]$, ie

$$F = 10 \text{ to } 100$$

This range includes the values obtained from the previous considerations.

5. SAFETY COEFFICIENTS FOR FATIGUE DESIGN

With the nominal probability level $P[E_1]$ evaluated, and hence also the corresponding value of β determined, it is possible to estimate values for the partial safety coefficients from the basic limit states requirement:

$$N_{Sd} < N_{Rd} \tag{21}$$

If N_S is treated as a design constant rather than a random variable, γ_{SN} is set equal to unity and the following expression is obtained from the usual second order theory:

$$\gamma_{RN} = \frac{\mu(N_R) - 1.64\,\sigma(N_R)}{N_s} \tag{22}$$

In order to obtain numerical values for γ_{RN}, the design safety coefficient for fatigue, estimates must first be obtained for the coefficient of variation of N_R. It is not the purpose of this paper to discuss the evaluation of γ_{RN}; however this question has been discussed specifically in regard to fatigue in Ref [6].

6 CONCLUDING REMARKS

The proposed back-calibration procedure is open to the criticism that gross and highly approximate estimates of the costs and consequences of entry into the relevant limit states have to be made, in order to obtain numerical values for $P[E_1]$ and hence for the safety index β and the partial safety coefficients. While this is certainly correct, it must be considered in context. There are gross approximations implied in the use of the safety coefficients for flexure, and the error introduced in the present treatment of fatigue reliability is probably not excessive when compared with the errors implied by the gross approximations made in the safety treatment for flexure, which is the best documented of the design limit states.

It should be emphasised that the probability of failure dealt with in simple first order reliability theory is a nominal quantity only, and has nothing directly to do with actual, observed frequencies of failure of real structures. In practice, failure usually occurs in a great majority of cases due to gross error. The important question regarding the relevance of analyses based on nominal probabilities of failure cannot be entered into here. Although it has been argued that design procedures derived from nominal failure probabilities are applicable, even when gross errors govern the rate of occurrence of failure in real situations, the question is far from settled in the opinion of the present author. The main argument which can be raised in defence of the method used here is one of consistency: The proposed cross-calibration procedure will at least give consistency with other limit states design procedures, such as for flexure and shear, for which it has been possible to use back-calibration procedures.

7. REFERENCES

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