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Role of Probabilistic Analysis for Quality Assurance

Rôle des calculs probabilistes pour l'assurance de qualité

Die Rolle probabilistischer Berechnungen in der Qualitätssicherung

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SUMMARY

In the present contribution, first a survey of probabilistic analysis methods is given. In two examples probabilistic and deterministic results are compared. The advantages and the limits of probabilistic analyses are discussed. in the conclusions the most important points which have to be considered in order to obtain meaningful probabilistic results are listed.

RÉSUMÉ

La contribution présente un aperçu des méthodes de calcul probabilistes. A l'aide de deux exemples, on compare les résultats des méthodes déterministe et probabiliste. On présente ensuite les avantages et les limites des calculs probabilistes. En conclusion, on relève les principales conditions à respecter pour l'obtention de résultats probabilistes sensés.

ZUSAMMENFASSUNG

Im vorliegenden Beitrag wird zunächst ein Überblick über probabilistische Berechnungsmethoden gegeben. An zwei Beispielen werden probabilistische und deterministische Ergebnisse verglichen. Die Möglichkeiten und Grenzen probabilistischer Berechnungen werden diskutiert. In den Schlussfolgerungen werden die wichtigsten Punkte zusammengestellt, welche für sinnvolle probabilistische Ergebnisse beachtet werden müssen.

1. INTRODUCTION

Many dynamic excitations such as wind forces, waves or strong earthquakes have inherent probabilistic properties. This random character concerns the occurrance as well as the dynamic characteristics of the loads such as frequency content, maximum excitation and length of duration. In order to judge the behaviour of a structure under such loads, their probabilistic nature should be taken into account.

On the side of the structure the resistances are also not sharply defined deterministic quantities but consist of mean values and variances. The variances, however, are usually much smaller than the variances of the loads. Therefore the resistances of the structure are frequently assumed to be deterministic. To assure the integrity of a structure also under extreme loads, failure mechanisms leading to collaps or partial collaps have to be investigated. If for instance the probability of occurrance of certain plastic hinges which lead to a failure mechanism is known from a probabilistic analysis, then the probabilities of corresponding failure mechanisms can be determined. From these the probability of collaps of the structure is obtained.

A probabilistic structural analysis usually involves more complex analysis methods or more time consuming calculations than conventional deterministic methods. Therefore the question of the significance as well as of the cost/benefit-ratio of such investigations has to be asked. In the following some of the characteristic properties of probabilistic analyses are worked out. From these insights their role for the quality assurance of a structure is discussed.

2. METHODS OF PROBABILISTIC ANALYSIS

The methods available for a probabilistic analysis are basically different for linear and for nonlinear structural behaviour. In the simplest case the structure is linear and the loads consist of stationary random processes with normal distributions. In the frequency domain the loads are then completely defined by their power spectral (p.s.d.f.) and cross spectral density functions (c.s.d.f.). For the discretized structural model, the input and output processes are related by the transfer function theorem. In the case of a displacement $q_i(t)$

$$S_{q_i}(\Omega) = \sum_{j=1}^{n} \sum_{k=1}^{n} H_{ij}(\Omega)\overline{H_{ik}(\Omega)}S_{jk}(\Omega) \qquad (1)$$

holds, where $S_{q_i}(\Omega)$ denotes the p.s.d.f. of $q_i(t)$, $H_{ij}(\Omega)$ and $H_{ik}(\Omega)$ are the complex frequency response functions between degrees of freedom i,j and i,k, respectively, and $S_{jk}(\Omega)$ are the p.s.d.f. and c.s.p.f. of the loads. Similar relationships hold for forces, stresses, accelerations etc. Integration of eq.(1) furnishes the variance σ^2. With the expected frequency

$$f = \frac{1}{2\pi}\left[\frac{\int_0^\infty \Omega^2 S_{q_i}(\Omega)d\Omega}{\int_0^\infty S_{q_i}(\Omega)d\Omega}\right]^{1/2} \tag{2}$$

and the duration T of the process the probability distribution p(x) of the extreme values

$$x = |q_{extr.}| \tag{3}$$

can be derived. For example in the case of uncorrelated extreme values

$$p(x) = \frac{2fT}{\sigma}\left(\frac{x}{\sigma}\right) \exp[-2fTe^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2} - \frac{1}{2}\left(\frac{x}{\sigma}\right)^2] \tag{4}$$

is obtained. Again similar expressions hold for the processes of forces, stresses a.s.o. By integrating p(x) the probability of exeedance of a value x is obtained. The inverse relationship furnishes the values of x associated with given confidence levels. The analysis can be performed either in modal coordinates by superposition of the modal contributions or by working with the original degrees of freedom (direct method). In most cases modal superposition is more efficient than direct integration. It should be mentioned that instead of working in the frequency domain the analysis can also be performed in the time domain using auto- and cross-correlation functions.

The resistances of the structure can be taken as deterministic or probabilistic quantities. In the latter case the joint probability density function of the inner forces and of the resistances is the product of both probability density functions:

$$p(r,f) = p_r(r)p_f(f) \tag{5}$$

Here $p_r(r)$ denotes the distribution of the resistance and $p_f(f)$ is the distribution of the force. In this equation the force and the resistance are assumed to be statistically independent. Integration over the appropriate bounds furnishes failure probabilities of specific sections.

In the case of nonstationary excitations, all probabilistic quantities become time-dependent. In some cases, however, the loads can be represented as the product of a stationary process and a time-dependent shape function. If linearity of the structure is maintained, the transfer function approach can still be used in a modified form. Again modal superposition is usually computationally more efficient than direct integration. For a general nonstationary process, Monte Carlo simulations have to be made either in modal coordinates or by direct integration.

If the structural behaviour is nonlinear, the principle of superposition is no longer valid. Therefore the transfer function method and modal superposition cannot be applied. The only applicable method is Monte Carlo simulation using direct integration. In the simulations the structure is analysed many times using samples for the excitations and the resistances. All parameters characterizing the loads and the structure can be changed for each calculation. The structural response quantities are then evaluated by statistics and thus expected maximum values and variances are obtained. In order to get reliable results especially for the extreme values, quite a number of analyses have to be made. Therefore the amount of effort and computing time for such simulations can be substantial.

3. COMPARATIVE EXAMPLES

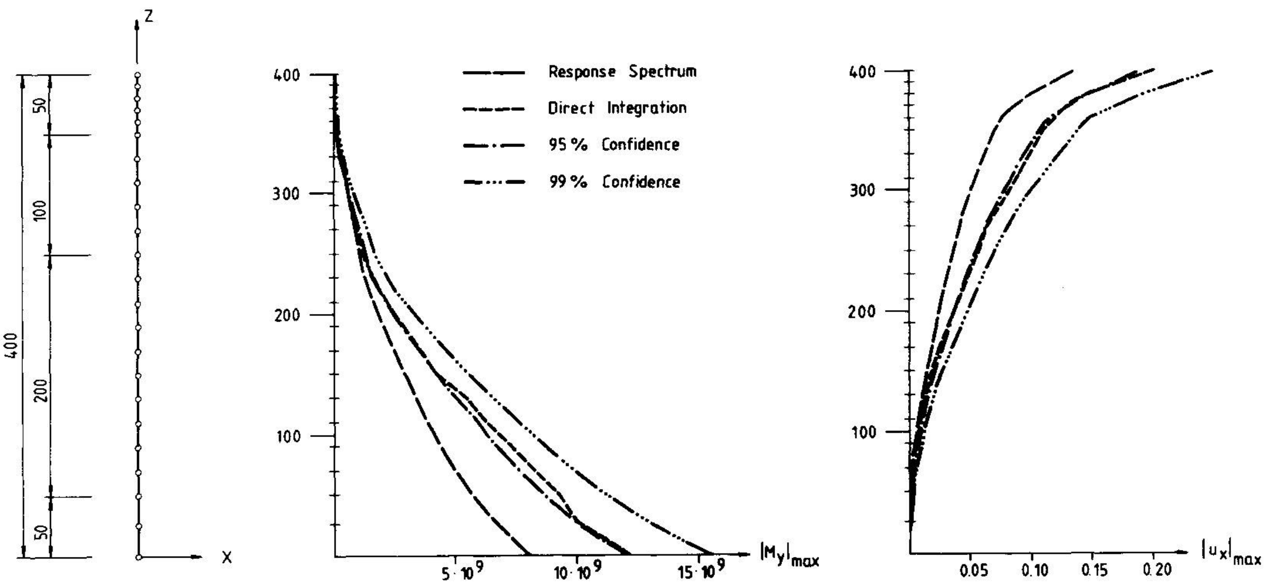

Fig. 1 Probabilistic Analysis of a Skyscraper

In order to get some insight into the results obtained from a probabilistic dynamic analysis as compared to conventional methods, two examples of seismic analysis are shown. Fig.1 shows the FE-model of a high-rise building which is uniformly excited in the x-direction at node 1. The structure has a structural damping coefficient of 0.10. From a probabilistic analysis by the transfer function method the 95 per cent and 99 per cent confidence ranges for the horizontal displacement $|u_x|$ and the bending moment $|M_y|$ were determined. The duration of the output processes was 5 sec. For 5 time functions determined from the p.s.d.f. also Monte Carlo simulations were made by direct time integration to obtain the extreme values. In addition an equivalent response spectrum was determined and a response spectrum analysis was performed. It is seen, that the extreme values from the Monte Carlo simulation are in the vicinity of the 95 per cent confidence range. If the number of samples is increased, these maximum values will further go up. The amount of computing time for the probabilistic analysis was much smaller than for the simulations and was comparable to the computing time for the response spectrum analysis. The results from the latter analysis are much smaller than the results from the simulations and from the probabilistic analysis. This is primarily due to the fact, that response spectra are obtained from average maximum values. In cases of high damping, however, the peak values are considerably reduced. In addition, modal coupling is neglected in the response spectrum method but is important for higher damping ratios and/or close frequencies. In the probabilistic analysis, on the other hand, modal coupling is always taken into account. This example demonstrates that a probabilistic reanalysis of the structure provides an inexpensive way to check the results from other analyses.

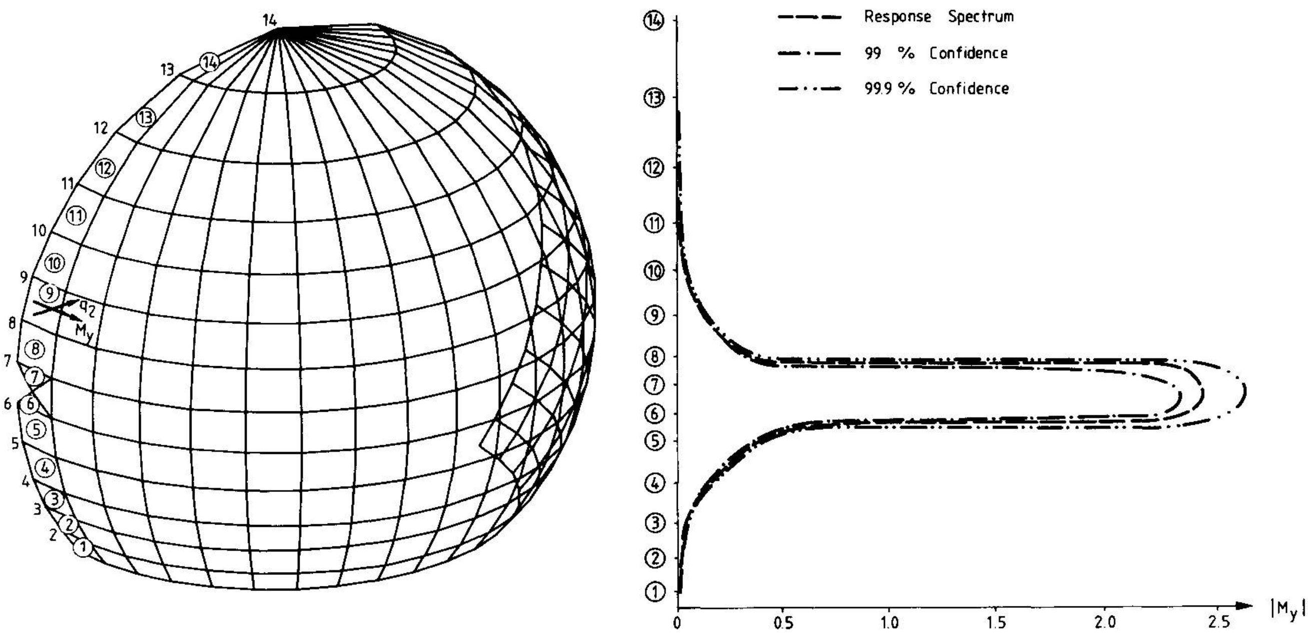

Fig. 2 Probabilistic Analysis of a Nuclear Containment

Fig.2 shows the model of a nuclear containment. The load again consists of uniform seismic excitation in x-direction. The structure has 2 per cent viscous damping. The results show the bending moment $|M_y|$ along a section as calculated by probabilistic analysis with a duration of 2.5 sec. and by response spectra. Here the response spectrum method furnishes results between the 99 per cent and 99.9 per cent confidence interval of the expected maximum values. The sharp increase of the bending moment in the vicinity of nodes 6 and 7 is due to a concentrated mass in that area. If the duration of the excitation increases, the probabilistic results are directly influenced. In the response spectrum analysis, however, the results will not change unless a new set of spectra is used. In this example modal coupling is not important.

In both examples the probability distribution of the forces can be combined with the distribution of the corresponding resistances leading to failure probabilities. Furthermore, also the occurrance of the excitation can be modelled by a probability distribution. Thus for instance the probability of failure during the lifetime of the structure can be determined.

4. ADVANTAGES AND RESTRICTIONS OF PROBABILISTIC ANALYSES

A probabilistic analysis permits the quantification of uncertainties on the side of the loads and of the structure. By introducing confidence levels as in the above examples, different risks are made comparable in a rational way. Probabilistic investigations also allow to take into account risks stemming from different sources and to identify their influence. Considering the computational effort, a probabilistic analysis of a linear structure under stationary excitation is comparable with the effort for a traditional response spectrum analysis and thus much cheaper than time integration. The results are, however, more realistic than response spectrum results because the duration of the excitation as well as modal coupling effects are taken into account.

There are, on the other hand, a number of restrictions to a probabilistic analysis. As all probabilistic formulations are based on random deviations, they will not take into account non-random effects such as design and construction errors. This is, however, also true for conventional calculations. The difficulties to obtain reliable analysis data for the structure and the excitations are usually greater for a probabilistic analysis than for conventional methods. If simulation techniques have to be used such as in the case of nonlinear structural behaviour, they are usually very expensive with respect of problem preparation, computation and evaluation of the results. Thus in order to reduce this effort there is a danger of using too few samples or an oversimplified structure.

5. CONCLUSIONS AND RECOMMENDATIONS

A probabilistic analysis can be a valuable source of information if the following points are kept in mind:

1. All results are only as good as the input data. Therefore great care has to be taken to derive a realistic characterisation of the loads and of the structural properties.

2. For linear structures the transfer function method is usually most efficient in modal coordinates.

3. As for conventional analyses, a probabilistic analysis should first of all always be performed for the linear case and stationary excitation.

4. The duration of the processes directly influences the results. It therefore has to be carefully estimated.

5. Monte Carlo simulations have be done with a sufficient number of samples. The demand of computing time and man hours should be estimated before the beginning of the task.

6. The results of a probabilistic analysis should be considered as one additional source of information on the behaviour of the structure. Together with the results from conventional analyses it provides an improved basis to judge the quality of the design of a structure.

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