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Recent Advances in Structural Systems Reliability Theory

Développements de la théorie de la fiabilité des systèmes structuraux

Fortschritte in der Zuverlässigkeitstheorie für Systeme

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SUMMARY

The paper gives a brief review of recent advances in structural systems reliability with emphasis on the research performed at the University of Aalborg, Denmark. It is shown that stability and fatigue reliability analysis can be integrated into the β-unzipping method. Finally, optimal design with reliability constraints is described.

RÉSUMÉ

L'article rappelle brièvement les développements récents dans la théorie de la fiabilité des systèmes structuraux et mentionne les recherches effectuées à l'Université d'Aalborg, Danemark. Il montre comment les problèmes de stabilité et du comportement à la fatigue peuvent être analysés et introduits dans la théorie de la fiabilité. L'article conclut avec quelques remarques concernant le dimensionnement optimal des systèmes structuraux.

ZUSAMMENFASSUNG

Die an der Universität von Aalborg kürzlich erzielten Fortschritte in der Zuverlässigkeitstheorie für Systeme werden dargestellt. Es wird gezeigt, in welcher Weise auch Stabilitäts- und Ermüdungs-Probleme in eine Zuverlässigkeits-Analyse eingefügt werden können. Zuletzt werden Fragen der optimalen Bemessung von Systemen angeschnitten.

1. INTRODUCTION

In this paper recent advances in structural systems reliability are discussed from an application point of view. It is not attempted to give a concise and comprehensive presentation here, but rather a brief orientation on what can be performed to-day. The design procedure based on a set of partial coefficients is probably the most advanced procedure widely used at present. Clearly, it has many advantages compared with the traditional methods, especially if the partial factors are calibrated by a rational procedure. However, until recently partial coefficients have often been updated on the basis of experience, i.e. if during - say a 10 year period - only few failures are observed for a given type of structure then the corresponding partial coefficients are relaxed and vice versa. A much more satisfactory approach is determination of rational sets of partial coefficients by using modern structural reliability theory. Several different procedures have already been used by code committees. One approach is described in detail by Thoft-Christensen & Baker [1].

It is well known that most structural failures occur for unexpected reasons (gross errors) and therefore, they are not included in the usual structural reliability analysis. In structural reliability theory only recognised failure modes such as buckling, plastic collapse, fatigue failure, etc. and modes of unserviceability are included. Gross errors are usually major mistakes either in planning, design, analysis, construction, use or maintenance of the structure. In general, gross errors are not direct included in the estimate of the safety or reliability of a structure, but they are treated separately. As emphasized by Thoft-Christensen & Baker [1] the most natural way to treat gross errors is to improve the quality assurance.

Estimation of the probability of failure of single structural elements is now considered a rather trivial task although there is still a need for data concerning the probability distributions of material properties (e.g. yield stresses), load parameters (e.g. wind loads), and geometrical quantities (e.g. cross-sectional areas). When probability distributions for these so-called basic variables are known and the failure criterion for a given structural element is given then the probability of failure is

$$P_f = \int_{\omega_f} f_{\overline{X}}(\overline{x})\,d\overline{x} \tag{1}$$

where ω_f is the failure domain (the domain of unsafe states in the space of basic variables) and $f_{\overline{X}}$ is the joint distribution function for the set of basic variables $\overline{X} = (X_1, \ldots, X_n)$. Calculation of P_f on the basis of (1) will require estimation of an n-dimensional integral and such an estimate will in most cases be very time consuming. Further, $f_{\overline{X}}$ is only known in a few simple situations. Therefore, a more simplified measure of the reliability is needed. The so-called reliability index is now accepted by most researchers in this field as a satisfactory measure for the safety (or reliability) of a structural element. This reliability index is described in detail by Thoft-Christensen & Baker [1]. A brief definition of the reliability index can be given in the following way.

Let the relevant basic variables be $\overline{X} = (X_1, \ldots, X_n)$. By a suitable transformation $\overline{X}$ is transformed into a set of independent standard normal variables $\overline{Z} = (Z_1, \ldots, Z_n)$. Further, let the so-called failure function (limit state function) f divide the z-space into a failure region ($f(z) \leq 0$) and a safe region ($f(z) > 0$). The reliability index β is then defined as the smallest distance from the origin to the failure surface ($f(z) = 0$) in the standard normal z-system. It can then be shown that

$$P_f \approx \Phi(-\beta) \tag{2}$$

where Φ is the standard normal distribution function. $M = f(\overline{Z})$ is called the safety margin.

The reliability index β can be estimated for any failure mode of a structural element if the corresponding failure function is known. It is much more complicated to estimate the probability of failure for the complete (redundant) structure. However, in the last decade several heuristic techniques have been developed. Two methods - the β-unzipping method and the branch-and-bound method - are presented in detail in a new book by Thoft-Christensen & Murotsu [2]. In this paper only the β-unzipping method developed at the University of Aalborg will be presented.

It has of course for many years been recognised that a fully satisfactory estimate of the reliability of a structure must be based on a systems approach. For statically determinate (non-redundant) structures failure in any member will result in failure of the total system (structure). However, failure in a single element in a structural system will not always result in failure of the total system, because the remaining elements may be able to sustain the external load by redistribution of the internal load effects (statically indeterminate or redundant structures). Further, a structural system will in general have a large number of potential failure modes and the most important modes must be taken into account in an estimate of the reliability of a structure. Identification of the most important (significant) failure modes can be performed by the β-unzipping method.

2. DEFINITION OF FAILURE

It is convenient to divide the different types of failure definitions into three groups:

- failure of a failure element
- failure of a structural element
- failure of a structural system.

To illustrate these different groups of definitions consider the simple frame in figure 1 loaded by two concentrated loads P_1 and P_2. The *structural system* consists of three *structural elements* and each structural element has a number of failure modes called *failure elements.* For the sake of simplicity assume that a failure element is a potential yield hinge. Then the frame has 7 failure elements as indicated by × in figure 1. Let P_1 and P_2 be normally distributed N(55 kN, 5.5 kN) and N(45 kN, 4.5 kN) and let the bending moment capacity R_i of all failure elements be identical and normally distributed N(135 kNm, 13.5 kNm). Further, let R_1, R_2, R_6, and R_7 be fully correlated and likewise R_3, R_4, and R_5 fully correlated. It is then (see Thoft-Christensen & Murotsu [2]) straightforward to calculate the reliability indices β_i, i = 1, 2, . . . , 7 for all failure elements (see table 1).

Failure element, i	1	2	3	4	5	6	7
β_i	4.40	8.48	8.48	4.46	2.55	2.55	1.67

Table 1

A simple definition of failure would be failure in a single failure element. That is, the structure is considered to be in a state of failure when a single failure element fails. The probability of failure P_f is then calculated as the probability of having failure in failure element 1 and/or in failure element 2 . . . and/or in failure element 7, and it can be shown that a good estimate of P_f is

$$P_f \approx 1 - \Phi_7(\bar{\beta}\ ;\ \bar{\bar{\rho}}) = 0.0529 \tag{3}$$

where Φ_7 is the 7-dimensional standardized normal distribution function, $\bar{\beta} = (\beta_1,\ldots,\beta_7)$ and $\bar{\bar{\rho}}$ is the correlation matrix for the safety margins. This definition is called a failure modelling at level 1 and

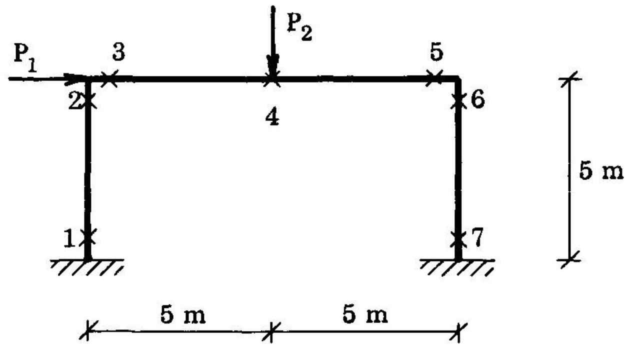

Figure 1. One-storey frame.

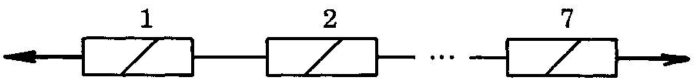

Figure 2. Failure modelling at level 1.

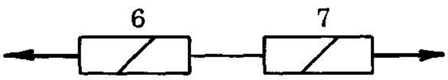

Figure 3. Failure modelling of the right-hand column.

can be illustrated as a series system where the elements are failure elements (see figure 2). A formal reliability index can then be defined as

$$\beta = -\Phi^{-1}(P_f) = 1.62 \tag{4}$$

Note that more failure elements could have been included in the estimate of P_f, e.g. instability failure of the two columns.

For some structures it is useful to consider the single structural elements separately and define failure of a structural element as failure of a single failure element in the structural element. Consider as an example the right-hand column of the frame in figure 1. This column has two failure elements, namely failure elements 6 and 7. Failure of the structure could be defined as failure of any of the three structural elements (one beam and two columns). For the specific column above the probability of failure is the probability of having failure in failure element 6 or/and in failure element 7. This can be modelled as shown in figure 3. The reliability indices for failure elements 6 and 7 are shown in table 1 and it can be shown that the correlation coefficient between the corresponding safety margins is 0.98. Therefore, the probability of failure of the structural element is

$$P_f \approx 1 - \Phi_2(1.67,\ 2.55\ ;\ 0.97) = 0.04746 \tag{5}$$

The corresponding reliability index is $\beta = 1.67$, which is equal to the reliability index for failure element 6 due to the fact that the correlation coefficient is close to 0.98 and β_6 is much smaller than β_7.

For redundant structures failure in a single failure element or a single structural element will in general not be considered as failure of the complete structure. For some elasto-plastic structures it may be more relevant to define failure of the structure as formation of a mechanism. For other structures is could be more natural to define failure of the structure as failure in two failure elements (level 2 modelling). Independently of the definition chosen it is important to have at disposal a method by which the most significant failure elements, pairs of failure elements or mechanisms can be identified because the total number of failure elements, pairs of failure elements or mechanisms are usually too high to include all in the reliability analysis.

3. IDENTIFICATION OF SIGNIFICANT FAILURE MODES

Return to the simple frame in figure 1. Estimation of the probability of failure when failure of the frame is defined at level 1 is based on the series system shown in figure 2 and the calculation of P_f is given by (3). However, the number of failure elements in the series system can easily without loss of accuracy be reduced to a system with only three elements, namely nos. 5, 6, and 7, because these elements have much smaller β-values than those erased. The calculated probability of systems failure will then be almost unaffected and the calculation procedure simplified. Failure elements 5, 6, and 7 are called significant (or critical) failure elements.

To obtain the significant pairs of failure elements at level 2 the structure is modified by assuming in turn failure in the significant failure elements and adding fictitious loads corresponding to the load-carrying capacities of the elements in failure. As an illustration modify the simple frame in figure 1 by assuming failure (formation of a yield hinge) in failure element 7 and add a set of fictitious bending moments if the element is ductile. If the failed element is brittle no ficitious bending moments are added. The modified structure is then analysed and new β-values are calculated for the remaining elements. In this case failure elements 6, 5, and 1 have the smallest β-values, namely 2.48, 2.64, and 3.39, respectively. By this procedure the significant pairs of failure elements, namely (7, 6), (7, 5), and (7, 1) are identified. By continuing this procedure (assuming failure in failure elements 5 and 6) one more significant pair is identified, namely (6, 7). It turns out that the corresponding reliability index for the frame (at level 2) is 2.45.

When failure is defined as formation of a mechanism the procedure mentioned above can be continued but it is much more efficient to base the identification of significant mechanisms on so-called fundamental mechanisms. This procedure is described in detail by Thoft-Christensen & Murotsu [2] and will not be treated here. It can be mentioned that this definition of systems failure results in a β-value equal to 4.19 for the frame in figure 1.

In a paper by Sørensen, Thoft-Christensen & Sigurdsson [3] a program package is described by which significant mechanisms in plane and space frame and lattice structures are identified automatically.

It is obvious that the method (β-unzipping method) presented above will give an upper bound for the systems reliability index β as some failure modes are omitted. However, it is the experience that the upper bounds obtained are very close to the correct value. Methods exist by which lower values can be obtained, but it seems to be much more difficult to obtain »good» lower bounds.

Combined failure conditions (e.g. between the bending moment and the axial force) for a failure element are a little complicated to include in the systems reliability analysis. However, when failure is defined as formation of a mechanism and when the β-unzipping method is used to identify significant mechanisms, the procedure is quite simple (see Thoft-Christensen, Sigurdsson & Sørensen [4].

4. FATIGUE RELIABILITY ANALYSIS

The β-unzipping method described is quite general in some sense. It can be used in connection with several systems failure definitions, and a large number of failure modes for structural elements and joints can be included by an appropriate choice of failure elements. However, the method has until now only been used for static loading. It is not yet clear how to modify the method so that dynamic effects of dynamically sensitive structures can be included.

For some structures one of the most important forms of failure is due to fatigue. This is e.g. the case for offshore structures of the jacket type on deep water where estimates of the fatigue life of the welded tubular joints are associated with great uncertainty. Two different approaches in fatigue life assessment, namely

- the S-N approach
- the fracture mechanics approach

can be used. The traditional applications of these approaches are non-probalistic since all parameters affecting the fatigue life are assumed deterministic. Conservative values for the relevant parameters are chosen to ensure a satisfactory safety factor. In the last decade probabilistic models for fatigue damage and assessment of fatigue reliability of structures have been developed (see e.g. Madsen, Skjong & Maghtaderi-Zadeh [5], Wirsching [6]).

Both approaches can easily be integrated with the systems reliability analysis at level 1 presented above by simply considering fatigue of a joint as a failure element. The corresponding reliability index is calculated on the basis of a safety margin which can be formulated in at least two equivalent forms when the S-N approach is used, namely

$$M_1 = D - D_0 \tag{6}$$

or

$$M_2 = L - L_0 \tag{7}$$

where D is the actual accumulated fatigue damage and D_0 the accepted accumulated fatigue damage in the required lifetime L_0 of the structure. L is the actual lifetime of the structure. When the fracture mechanics approach is used the following type of safety margin can be applied

$$M_3 = C_0 - C \tag{8}$$

where C_0 is the critical crack size and C the actual crack size in the required lifetime of the structure.

As an example consider fatigue reliability analysis of a tubular joint of an offshore structure (see Wirsching [6]). Such an analysis is not trivial due to the many sources of uncertainty involved. All the way from the statistical description of the wave loading to the calculation of the stress process considerable uncertainty must be taken into account. A number of parameters will affect e.g. the accumulated fatigue damage D used in the S-N approach. First of all, model uncertainty must be modelled by a random variable B. The load effect on D is modelled by two variables, namely the number of zero upcrossings ν of the assumed narrow-banded stress response process and the variance σ_S^2 of the stress range process. The S-N curve (S is the stress range and N the number of cycles to failure) characterizing the fatigue performance under constant amplitude loading is given by two empirical constants m and K. Correction for non-narrow banded response is introduced by a parameter η. The effect of the material thickness t is modelled by t and a power parameter M_1. Finally, D will of course depend on the required lifetime ℓ_0. Therefore,

$$D = f(\ell_0, \nu, \sigma_S, B, m, K, \eta, t, M_1) \tag{9}$$

where f is a function of 9 variables. B, K, and M_1 are modelled by random variables and the remaining parameters are assumed deterministic. If f is known and data for all parameters (including the random variable D_0) are given then the reliability index β for the fatigue reliability of the joint can be calculated.

From a systems point of view fatigue failure of a joint is considered a failure element and is therefore included in the series system used by level 1 modelling. The correlation between the safety margin used for the fatigue failure element and the bending moment failure elements need to be estimated e.g. by simply assuming independence. When systems failure is defined at level two it is more difficult to include fatigue failure elements because the reserve strength of e.g. a joint after fatigue has taken place must be estimated. For a tubular joint fatigue failure will often occur as a brittle failure between the brace and the chord. When this is the case the brace is simply completely removed and the structure modified in this way is then reanalysed.

5. STABILITY RELIABILITY ANALYSIS

Stability reliability analysis can be performed at two levels, namely analysis of the local stability of the single structural elements and global analysis of the complete structure. The critical load P_{cr} of a single structural element in compression is usually written as

$$P_{cr} = \left(\frac{\pi}{\ell_e}\right)^2 EI \tag{10}$$

where E is the modulus of elasticity, I the moment of inertia, and ℓ_e the so-called effective length of the structural element. ℓ_e is readily determined as a function of the rotational restraints at the end of the bar. Within the limits of validity of (10) it is straightforward to integrate the stability reliability analysis with the β-unzipping method simply by considering the stability failure mode as a failure element with the safety margin

$$M = P_{cr} - AP \tag{11}$$

where A is a model uncertainty variable and P the axial load in compression in the structural element.

When a beam is subjected to a combination of axial load P and a bending moment B the stability reliability can in some cases be estimated on the basis of an interaction formula as the following:

$$\frac{P}{P_{cr}} + \frac{B_0}{B_{cr}}\left(\frac{1}{1-(P/P_E)}\right) = 1 \tag{12}$$

where P_{cr} is a function of the effective slenderness of the beam, B_0 is the maximum moment, B_{cr} the critical moment, and P_E the critical Euler load for elastic buckling in the plane of applied moments. P_{cr}, B_{cr}, P_E, and the interaction curve will in general be associated with some uncertainty. This can be taken into account, e.g. by introducing four model uncertainty variables X_1, X_2, X_3, and X_4 in the safety margin in the following way

$$M = X_1 - \frac{P}{X_2 P_{cr}} - \frac{B_0}{X_3 B_{cr}}\left(\frac{1}{1-(P/X_4 P_E)}\right) \tag{13}$$

The mean values and standard deviations of X_i, $i = 1, \ldots, 4$ must be estimated from experimental data.

Formulas like (10) and (12) are based on the assumption that the beams are perfectly straight. However, in practice this assumption is seldom satisfied. The reduction effect of imperfections in the load-carrying capacity of structural members with instability failure modes is a research area which has been subject to growing interest in the last decades (see e.g. Elishakoff [7]). It is a fact that imperfections ranging from residual stresses, from manufacturing to eccentricities in joints and loading are unavoidable in real structures, and they are very seldom predictable. Imperfections are best introduced in the stability analysis through a probabilistic approach where the imperfections are modelled as random variables (or stochastic processes).

Faber & Thoft-Christensen [8] have investigated the reliability of a plane lattice structure (see figure 4) with local imperfections. In the paper it is shown that the reduced critical load-carrying capacity P_i due to imperfections can be written $P_i = \lambda_i 2P_{CL}$, where P_{CL} is the local critical load of the individual simply supported columns. λ_i is the reduced critical load-carrying intensity given by

$$\lambda_i = \lambda_1 \left(1 + \frac{1}{2} b(1-\lambda_i)^{-3}\right)^{-1} \tag{14}$$

where $\lambda_1 = P_{CG}/2P_{CL}$ (P_{CG} is the global critical load of the structure) and where $b = b_0/i_m$ (b_0 is the amplitude of the imperfections and i_m the radius of gyration of the horizontal members). The load intensity is λ (see figure 1).

The load intensity λ and the imperfection amplitude b are modelled by random variables Δ and B. Then λ_i will be the outcome of a random variable Δ_i and a natural safety margin is $M = \Delta_i - \Delta$. The reliability index β can then be calculated. The conclusion from this special example was that β is insensitive to variations in the imperfection uncertainty.

The global buckling capacity of a complete structure is in general very time-consuming to obtain. Research in this field is being performed at the University of Aalborg, but the results are not yet published. The approach can be briefly described in the following way. By non-linear analysis (proportionally increased loading) of the structure a number of points of the unknown limit curve are identified. The safety margin corresponding to the limit curve is then used in estimating the stability reliability of the structure.

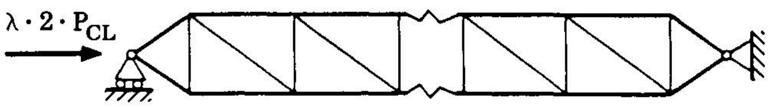

Figure 4. Plane lattice structure.

6. OPTIMAL DESIGN AND RELIABILITY

In the classical deterministic structural optimization for truss and frame structures all variables (dimensions, loads, strengths, etc.) are assumed to be deterministic and the design variables are usually the cross-sectional areas A_i, $i = 1, \ldots, k$, where k is the number of structural elements. The objective function is naturally the total cost of the structure. However, it is often assumed that the cost of the structure is proportional to the weight of the structure. In structures where only one material is used the weight is proportional to $W(\overline{A}) = \overline{\ell}^T \overline{A}$, where $\overline{A} = (A_1, \ldots, A_k)$ and $\ell = (\ell_1, \ldots, \ell_k)$ is the deterministic lengths of the structural elements. The constraints signify that the stresses and the displacements should everywhere be smaller than some prescribed design values.

In modern design theory based on a probabilistic point of view loads and strengths are modelled by random variables. The objective function is unchanged but the constraints are now replaced by only one constraint, namely $\beta_S(\overline{A}) - \beta_S^0 \geqslant 0$, where β_S is a measure of the systems reliability and β_S^0 the corresponding target value.

Optimal design with reliability constraints is an area where significant progress has taken place in the last few years (see e.g. Thoft-Christensen & Murotsu [2]). Research, where the β-unzipping method is used in estimating the systems reliability index β_S has been conducted at the University of Aalborg (see Thoft-Christensen & Sørensen [9] and Sørensen & Thoft-Christensen [10, 11]). A number of different effective optimization procedures are discussed in these papers and applied to a number of examples, e.g. jacket structures with tubular joints.

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