**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte

**Band:** 50 (1986)

**Artikel:** Load combination and load resistance factor design

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**DOI:** https://doi.org/10.5169/seals-39123

# Load Combination and Load Resistance Factor Design

Combinaisons de charges et facteurs de charge et de résistance

Lastkombinationen und Bemessung mit Sicherheitsfaktoren

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## SUMMARY
This paper reviews the current procedure for developing probalility-based load combinations for structural design in a load and resistance factor design (LRFD) format. In this format, the load and resistance factors associated with a particular load combination are determined for use in a limit state design on the basis of a target limit state probability. This paper then provides critical comments on the validity of the methodology by which these load and resistance factors are derived.

## RÉSUMÉ
La contribution traite de la procédure actuelle de détermination probabilistique des combinaisons de charges pour le dimensionnement de structures: elle est basée sur des facteurs de charge et de résistance (LRFD) pour le calcul du projet. Selon cette méthode, les facteurs de charge et de résistance liés à certaines combinaisons de charges sont déterminés pour un calcul à la ruine, sur la base d'une probabilité opérationelle donnée. Une appréciation est portée sur la validité de la méthodologie utilisée pour la détermination des facteurs de charge et de résistance.

## ZUSAMMENFASSUNG
In Bemessungsbedingungen, die mit Last- und Widerstandsfaktoren arbeiten und eine bestimmte operationelle Versagenswahrscheinlichkeit anstreben, sind die Faktoren selbst und die Lastkombinationsregeln festzulegen. Der Beitrag untersucht das hierfür übliche Verfahren und äussert sich kritisch zur Gültigkeit der Methodik.

## 1. INTRODUCTION

In the practice of structural design, both extreme and abnormal loading conditions must be considered. This requirement possibly results in a large number of load combinations in the design criteria. Furthermore, the load and resistance factors specified in the code are usually determined by the code committee primarily on the basis of collective judgment and experience. Hence, a rational procedure is needed to justify the number of load combinations and to determine appropriate load and resistance factors. This paper reviews the procedure of probability-based load combination criteria for structural design and provides critical comments thereon. While the discussion primarily centers on the LRFD methodology, its implementations extend to a re-examination of the basic issues associated with the concept of structural safety and design. In this connection, the notion of the limit state probability curve (surface) is introduced.

## 2. PROCEDURE FOR ESTABLISHING LOAD COMBINATION CRITERIA

While many assert that the current probability-based LRFD criteria are rational, the way in which particular load combinations are chosen for design purposes appears to be rather arbitrary. To be on the safe side, one tends to cover all the possible combinations of all the conceivable loads. Indeed, this appears to be the case for nuclear power plant design where the grave consequences of failure warrants the utmost care in selecting such load combinations. Probabilistically speaking, one can obviously ennumerate all the possible load combinations that are mutually exclusive. For simplicity, consider a structure subjected to a primary load D/L (dead and live load), earthquake load E and wind load W.

Recognizing that highly frequent micro-tremors and constantly present breezes do not really constitute an earthquake and wind loads, respectively, the structure will be subjected to a set of mutually exclusive load combinations consisting of D/L, D/L+E, D/L+W and D/L+E+W:

D/L: dead and live load only; no other load acting;
D/L+E: dead/live load and earthquake load but not wind load;
D/L+W: dead/live load and wind load but not earthquake load;
D/L+E+W: dead/live load, earthquake and wind loads.

However, identifying a certain load combination as part of such a mutually exclusive set does not necessarily warrant that this combination be considered for structural design. Indeed, for the choice of load combinations to be considered for design, limit state probabilities must be taken into consideration.

Structural limit states represent various states of undesirable structural behavior. For example, the allowable stress $\sigma_a$, a state of stress corresponding to the yield stress $\sigma_y$ divided by a (material) safety factor, represents a limit state. Similarly, the yield stress $\sigma_y$ and ultimate stress $\sigma_u$ represent limit states which, however, have more physical significance than the allowable stress. Note the words "allowable," "yield" and "ultimate" stress are used for simplicity of discussion. In the present study, they conceptually represent the following states of structural behavior: allowable stress = threshold of undesirability, possibly with respect to serviceability or to stress history dependent failure such as fatigue, yield stress = threshold of permanent deformation, and ultimate stress = threshold, possibly leading to structural collapse.

The limit state probability associated with $\sigma_a$ is then given by $P_a = P\{\sigma > \sigma_a\}$ where $\{\sigma > \sigma_a\}$ = the event that the state of stress at some location in the structure exceeds $\sigma_a$ at least once in the structure's lifetime. Since the events D/L, D/L+E, D/L+W and D/L+E+W are mutually exclusive, this limit state probability can be written as

$$P_a = P\{\sigma>\sigma_a|D/L\}P\{D/L\} + P\{\sigma>\sigma_a|D/L+E\}P\{D/L+E\} + P\{\sigma>\sigma_a|D/L+W\}P\{D/L+W\} + P\{\sigma>\sigma_a|D/L+E+W\}P\{D/L+E+W\} \quad (1)$$

where $P\{\sigma > \sigma_a|A\}$ = limit state probability conditional to A and $P\{A\}$ = probability of A. Similarly,

$$P_y = P\{\sigma>\sigma_y|D/L\}P\{D/L\} + P\{\sigma>\sigma_y|D/L+E\}P\{D/L+E\} + P\{\sigma>\sigma_a|D/L+W\}P\{D/L+W\} + P\{\sigma>\sigma_y|D/L+E+W\}P\{D/L+E+W\} \quad (2)$$

and

$$P_u = P\{\sigma>\sigma_u|D/L\}P\{D/L\} + P\{\sigma>\sigma_u|D/L+E\}P\{D/L+E\} + P\{\sigma>\sigma_u|D/L+W\}P\{D/L+W\} + P\{\sigma>\sigma_u|D/L+E+W\}P\{D/L+E+W\} \quad (3)$$

The target limit state proabilities $P^*_a$, $P^*_y$ and $P^*_u$ are then introduced, being respectively associated with $\sigma_a$, $\sigma_y$ and $\sigma_u$, and the design must satisfy

$$P_a < P^*_a , \qquad P_y < P^*_y , \qquad P_u < P^*_u \quad (4)$$

## 3. LIMIT STATE PROBABILITY DIAGRAM

The notion of a limit state probability diagram is introduced at this point. The diagram plots the common logarithm of the probability $P_f^{(i)}(x) = P\{\sigma_i > x\}$ that the response state $\sigma_i$ will exceed x at least once in the structure's lifetime as a function of x. Curve $B_i$ in Fig. 1 indicates $P_f^{(i)}(x)$ for structure i. Note that such a curve depends on the structure, thus the super- or subscript i. When x assumes specifric limit state values such as $x = \sigma_a$, $\sigma_y$ or $\sigma_u$, $P\{\sigma > x\}$ represents the corresponding limit state probabilities.

The target limit state probabilities $P^*_a$, $P^*_y$ and $P^*_u$ are indicated respectively by points $A_I$, $A_{II}$ and $A_{III}$ in Fig. 1. While it is not a well-recognized notion, the author suggests that conceptually the safety of a class of structures, for which the design code is intended to be used, should be specified by a target limit state probability curve $P^*_f(x)$, as designated by A in Fig. 1. If the state of structural behavior is to be described by more than one variable, say by x and y, the safety should be specified by a target limit state surface $P^*_f(x,y)$.

Since it is impractical to prescribe the entire curve $P^*_f(x)$ as a safety requirement and, even if one could do that, it is impractical to verify if $P_f^{(i)}(x) < P^*_f(x)$ or curve $B_i$ is below curve A for all values of x, one chooses a few values of x to perform such a check. In the present paper, $x = \sigma_a$, $\sigma_y$ and $\sigma_u$ are chosen as an example. Curve $I_i$ in Fig. 1 represents

$$P_{fI}^{(i)}(x) = P\{\sigma_i > x|D/L\}P\{D/L\} \quad (5)$$

Similarly, curves $II_i$, $III_i$ and $IV_i$ represent respectively

$$P_{fII}^{(i)}(x) = P\{\sigma_i > x|D/L+E\}P\{D/L+E\} \quad (6)$$

$$P_{fIII}^{(i)}(x) = P\{\sigma_i > x|D/L+W\}P\{D/L+W\} \quad (7)$$

$$P_{fIV}^{(i)}(x) = P\{\sigma_i > x|D/L+E+W\}P\{D/L+E+W\} \quad (8)$$

Curve $B_i$ in Fig. 1 is the sum of the probability values represented by curves $I_i$,

$II_i$, $III_i$ and $IV_i$.

If the curves $I_i$, $II_i$, $III_i$ and $IV_i$ indeed take the relative positions sketched in Fig. 1, then the D/L combination controls the limit state probability $P_a$, D/L+E the limit state probability $P_y$ and D/L+E+W the limit state probability $P_u$. Note that, in this case, the combination D/L+E+W does not really control any of the limit state probabilities. If all the structures to be designed under the design code exhibit this trend, then the combination D/L+E+W does not have to be considered in the design, and the combination D/L should be considered only for $\sigma_a$, D/L+E for $\sigma_y$ and D/L+W for $\sigma_u$. In fact, this can be interpreted as the conceptual basis for allowing the allowable stress to be increased when combinations of primary and secondary loads are considered in the classical allowable stress design.

Obviously, the dominance of a particular combination of loads for a particular limit state does not necessarily materialize in reality and therefore the above interpretation is most probably too simplistic.

The limit state probability diagram nevertheless clearly indicates the interrelationship among the limit states, limit state probabilities, target limit state probabilities and load combinations. More importantly, the limit state probability diagram as introduced here provides a much more global interpretation of the safety of a structure. Finally, it is pointed out that the state of structural response $\sigma_i$ may take a most undesirable value at different structural locations, depending on the load combinations and therefore the limit state probability diagram may not necessarily be constructed with respect to a specific point in the structure.

## 4. LOAD AND RESISTANCE FACTOR DETERMINATION

The currently practiced procedure for determining load and resistance factors can then be extended to deal with a more general interpretation of the safety as introduced above. For example, consider the following LRFD format:

$$\phi_I R_n = \gamma_{DI} D_n + \gamma_{LI} L_n \tag{9}$$

$$\phi_{II} R_n = \gamma_{DII} D_n + \gamma_{LII} L_n + \gamma_{EII} E_n \tag{10}$$

$$\phi_{III} R_n = \gamma_{DIII} D_n + \gamma_{LIII} L_n + \gamma_{WIII} W_n \tag{11}$$

where $R_n$, $D_n$, $L_n$, $E_n$ and $W_n$ are the nominal values of resistance, dead load, live load, earthquake load and wind load, and the $\gamma$'s and $\phi$'s represent the load and resistance factors, respectively.

Consider, then, a set of N representative structures (i=1,2,...,N) and assign initial values to all the load and resistance factors, design each representative structure, develop an objective function which measures the difference between the target limit state probabilities and the computed limit state probabilities, determine a new set of load and resistance factors in the direction of maximum descent with respect to the objective function, and repeat these steps until a set of load and resistance factors that miniimizes the objective functions is found.

A possible form of the objective function $\Omega = \Omega(\phi_I, \phi_{II}, \phi_{II}, \gamma_{DI}, \gamma_{LI}, \gamma_{DII}, \gamma_{LII}, \gamma_{EII}, \gamma_{DIII}, \gamma_{LIII}, \gamma_{WII})$ is

$$\Omega = w_I \sum_{i=1}^{N} \left(\frac{\log P_a^i - \log P_a^*}{\log P_a^*}\right)^2 + w_{II} \sum_{i=1}^{N} \left(\frac{\log P_y^i - \log P_y^*}{\log P_y^*}\right)^2 +$$

$$+ w_{III} \sum_{i=1}^{N} \left(\frac{\log P_u^i - \log P_u^*}{\log P_u^*}\right)^2 \qquad (12)$$

where $P_a^i = P_f^{(i)}(\sigma_a)$, $P_y^i = P_f^{(i)}(\sigma_y)$ and $P_u^i = P_f^{(i)}(\sigma_u)$, and $w_I$, $w_{II}$ and $w_{III}$ are the weights that are assigned to the limit states $\sigma_a$, $\sigma_y$ and $\sigma_u$, respectively. In principle, the optimum values of the load and resistance factors can be obtained from

$$\frac{\partial\Omega}{\partial\delta} = 0 \qquad (\delta = \phi_I, \phi_{II}, \cdots, \gamma_{WIII}) \qquad (13)$$

## 5. CONCLUSION

The LRFD format is considered from a more global point of view than that currently prevailing. In this connection, the notion of the limit state probability diagram is introduced to conceptually clarify the interrelationships among the limit state probability, target limit state probability and load combinations. A method consistent with the limit state probability diagram concept introduced here is suggested to determine the load and resistance factors. How to specify the target limit state probabilities or target limit state curve still remains rather elusive, however.

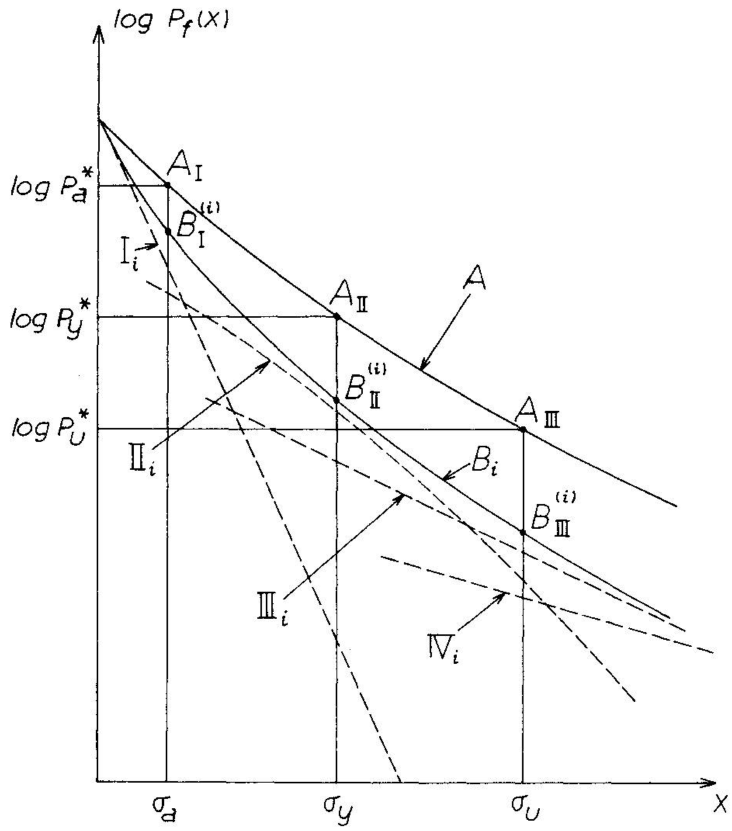


Fig. 1 Limit state probability diagram

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