Distortion of Thin-Walled Open Cross Section Members

Distorsion des profils à section ouverte et parois minces

Deformation von dünnwandigen Bauteilen mit offenem Querschnitt

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SUMMARY

In the analysis of thin-walled open-cross-section members, the effect of distorting contours of the cross sections has, up till now, not been taken into account. In this study a «theory of bars with distortion» is developed. This theory includes the conventional theory of warping torsion. An index showing rigidity of distorsion is proposed. In particular, many experiments have been carried out.

RÉSUMÉ

Jusqu'à présent, l'analyse des profils à section ouverte et parois minces ne prend pas en compte l'effet de la distorsion du contour de la section droite. Cette étude présente une théorie qui rend compte de cette distorsion en plus du gauchissement admis en torsion non-uniforme. On propose en particulier un indice de rigidité à la distorsion. Des résultats d'essais permettent de vérifier cette théorie.

ZUSAMMENFASSUNG

Mit den bisherigen Berechnungsmethoden konnte bei dünnwandigen Stäben mit offenem Profil der Verformungseffekt für die Konturlinien nicht berechnet werden. In dieser Abhandlung wird eine «Theorie für Stäbe mit deformierbaren Querschnitten» entwickelt, wobei die bekannte Theorie für Wölbtorsion berücksichtigt worden ist. Im besonderen schlägt man die Einführung eines Steifigkeitsindexes für die Deformation vor. Weiterhin wurde eine grosse Anzahl von Experimenten durchgeführt.

1. PREFACE

Recently, the use of cold-formed thin-walled members for building construction has especially increased. From the viewpoint of the strength and rigidity of thin-walled structural members, closed cross section members are more advantageous than open cross section members, but cases requiring the use of open cross section members often occur. The theoretical analysis of open cross section members has been established in the theory of warping torsion which has been used by structural designers long since. However, this theory of warping torsion is made up on the basis of the assumption that the in-plane distortion of cross sections does not occur. The distortion mentioned here is the deformation of a contour in the plane of a cross section, and is a different deformation from warping which is the deformation in the direction normal to it, that is, in axial direction(Fig.1).

In this paper, a new concept of the theory of bars with distortion is to be discussed, in which the effect of distortion besides warping is taken into account for thin-walled open cross section members(Fig.2).

2. DISTORTION OF CROSS SECTION

The loading condition which causes the deformation of cross sections is typically shown as Figs.3 and 4. The loading condition as shown in Fig.3(a) is often seen in actual structures, but it can be decomposed into two loading conditions as shown in Figs.3(b) and (c). (b) can be regarded nearly as the loading condition of bending, but (c) is a loading condition of a different type, which is neither bending nor torsion, but it is the force causing distortion. Similarly, the load in Fig.4(c) does not cause bending nor torsional deformation. Only distortion arises.
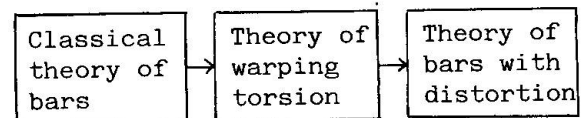
In the loading conditions of Figs.3(c) and 4(c), since the flanges (I) of the upper and lower edges are subjected to in-plane bending, the effect of distortion is transmitted to a long distance. However, if the flanges (I) do not exist, and the members are channels with simple C-form cross section, they will be subjected to the loading by the out-of-plane bending of the flanges (II). In this case, because force is transmitted by out-of-plane bending, the effect of distortion remains in local effect. In this way, it can be decided to distinguish by calling the distortion as subjected to in-plane bending distortion of first kind, and the case as subjected to only out-of-plane bending distortion of second kind(Fig.5). In the first kind,

(a) Warping   (b) Distortion

Fig.1 Deformation of cross section

Classical theory of bars	Theory of warping torsion	Theory of bars with distortion

Fig.2 Theory of bars

Fig.3 Force which causes distortion

Fig.4 Moment which causes distortion

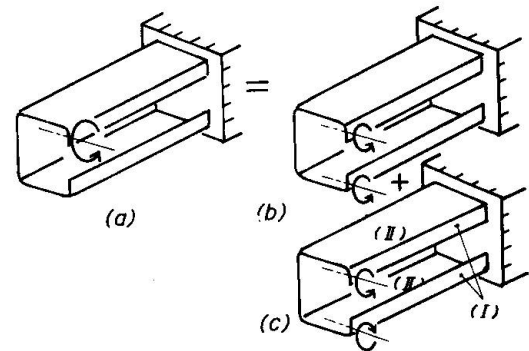the effect is exerted on a whole member, while the second kind remains in local effect. Of course, as shown by the broken line in Fig.5(a), the first kind deformation includes the accompanying deformation of second kind. The cross sectional deformation of second kind can be restrained by local reinforcement, therefore hereinafter, the cross sectional deformation of first kind is discussed in detail.

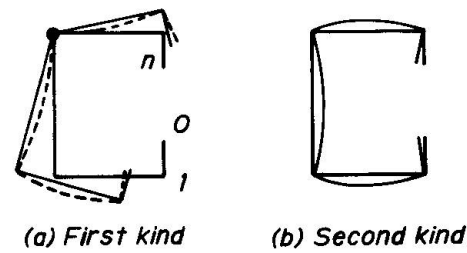(a) First kind   (b) Second kind

Fig.5 Distortion of first and second kind

The cross sectional deformation of first kind can be further classified according to the number of the deformation mode. The number of mode that is, the degree of freedom was enumerated by regarding the case that a cross section deforms making a hinge at only one folded point(mark ●) as one as shown in Fig.5(a) If the end point(point 0 or n) is considered to be a hinge, it becomes ordinary torsion. The degree of freedom m is expressed by m=n-3(n:number of composing plates, n≥3, m≤0→m=0), assuming m=0 as torsion. As mentioned above, analysis can be made generally, including torsion in distortion. The example of classifying cross sections according to this degree of freedom m is shown in Fig.6. As m is larger, the probability that the distortion of first kind occurs under general loading is higher, accordingly, it means that the possibility of causing the effect of distortion to far points is strong.
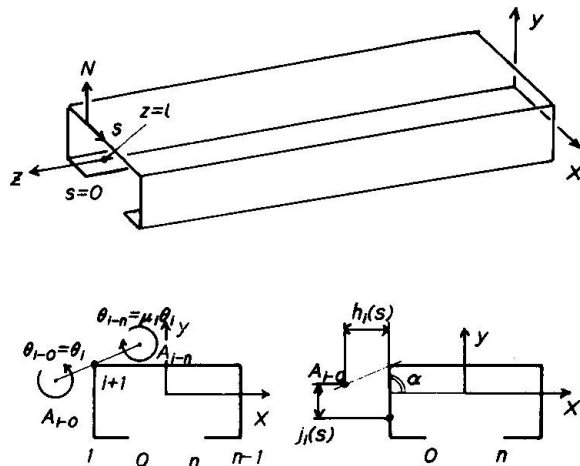

Fig.6 Classification of cross sections

Fig.7 Coordinate system

3. THEORY

As to the distortion of thin-walled open cross section members, research has been carried out already by the author [1], and here, only its outline is shown. The coordinate system is shown in Fig.7. The displacements u,v,w in directions s,N,z of any point of a cross section can be expressed as follows.

$$u(s,N,z)=\xi\cos\alpha+n\sin\alpha+\sum_{i=0}^{m}\theta_i\mu_i(h_i\pm N)+\sum_{i=0}^{m}u_{bi}(s,N,z)\dots\dots(1)$$

$$v(s,N,z)=-\xi\sin\alpha+n\cos\alpha+\sum_{i=0}^{m}\theta_i\mu_i j_i+\sum_{i=0}^{m}v_{bi}(s,N,z)\dots\dots(2)$$

$$w(s,N,z)=\zeta(z)-\xi'x-n'y+\sum_{i=0}^{m}\theta_i'\phi_i+\sum_{i=0}^{m}w_{bi}(s,N,z)\dots\dots(3)$$

Here,ξ,n and ζ are rigid body translations in x, y and z directions. x, y and z are the principal axes of a cross section, and θ_i is the angle of turning around a hinge point corresponding to the i-th degree of freedom. The added displacement (the broken line in Fig.5(a)) due to accompanying out-of-plane bending is expressed by (u_{bi},v_{bi},w_{bi}). $\mu_i(s)$ is a proper rotation ratio. This value of μ_i is determined by orthogonalizing axial stress. Besides, at the stage of orthogonalizing, the necessity to separate two centers of rotation on

the left and right sides of a hinge which were considered to be the centers of rotation arises. These two points (A_{i-0}, and A_{i-n} in Fig.7) obtained by orthogonalizing can be named distortional shear centers. This concept is that extending the ordinary concept of shear center. Besides, $\phi_i(s)$ is a function having generalized a warping function, and forms the system of orthogonal functions regarding i=0,..,m. Differentiating Eqs.(1)-(3), infinitesimal strains are given , and using the constitutive equations for linear elastic bodies, stresses can be determined. It is assumed that the external forces F_q and F_p in s and z directions per unit length of direction s are applied to both end surfaces (z=0,ℓ) of a bar, and the principle of virtual work is used, then the following basis equations are obtained.

$$EI_y\xi''''=0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(4)$$

$$EI_x\eta''''=0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(5)$$

$$EF\zeta''=0 \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(6)$$

$$EI_{\phi h}\theta_h''''-\sum_{i=0}^{m}GJ_{hi}\theta_i''+\sum_{i=0}^{m}M_{hi}\theta_i=0\dots\dots\dots\dots\dots(7),\text{where } M_{hi}=\int_s\frac{M_hM_i}{EI}ds.$$
$$(h=1,..,m)$$

M_i is the bending moment distributing in a frame having the form of a cross sectional contour, cut into unit breadth in z direction, and I is $t^3/12$. E and G are moduli of elasticity. When the case of m=0 in Eq.(7) is considered, that is, only torsion occurs, it is reduced to the conventional equation for warping torsion. In the above equations, respective constants of cross sections and generalized forces were given in Table 1. J_{hi} is the generalized St. Venant's torsional constant.

Table 1

4. PROPERTIES OF CROSS SECTIONS

In the left column in Table 1, the quantities from the top down to the fourth are those well known so far. That is, F : area of cross section, I_y :moment of inertia around y axis, I_x :moment of inertia around x axis, and $I_{\phi 0}$:constant of warping torsion. All these are expressed as the integral of square of respective generalized coordinates, 1,x,y and ϕ_0 over a whole cross section. Accordingly, the constants regarding squares of $\phi_1,\phi_2,...,\phi_m$ may be grasped similarly as the concept having extended moment of

Constant of cross section	Generalized force	Generalized coordinate
$F=\int\int dsdN$	$F_z=\int F_p ds$	1
$I_y=\int\int x^2 dsdN$	$M_y=-\int F_p x ds$	x
$I_x=\int\int y^2 dsdN$	$M_x=\int F_p y ds$	y
$I_{\phi 0}=\int\int \phi_0^2 dsdN$	$F_{po}=\int F_p\phi_0 ds$	ϕ_0
$I_{\phi 1}=\int\int \phi_1^2 dsdN$ ⋮	$F_{p1}=\int F_p\phi_1 ds$ ⋮	ϕ_1 ⋮
$I_{\phi m}=\int\int \phi_m^2 dsdN$	$F_{pm}=\int F_p\phi_m ds$	ϕ_m

inertia. Now, I_x and I_y express the rigidity resisting bending moments. Similarly, also $I_{\phi 1},I_{\phi 2},...,I_{\phi m}$ can be considered as the rigidity regarding the distortion of the corresponding degree of freedom. Then, these $I_{\phi i}$ (i=1,..,m) can be used as the indices of rigidity peculiar to respective cross sections. It is desirable to take these values large at the time of design.

Next, in the middle column in Table 1, four upper quantities are well known external forces. That is, F_z :axial force, M_y and M_x :bending moments, and F_{p0} :bimoment of torsion. These are all the quantities having integrated F_p multiplied by the orthogonalized coordinates, 1,x,y and ϕ_0, over a whole cross section. Similarly, it may be said that the external forces obtained by integrating F_p multiplied by $\phi_1,\phi_2,...,\phi_m$ are the generalized forces causing corresponding distortion. These are grasped as the concept having extended bimoment in warping torsion, accordingly, these are to be called distortional bimoment. These are to exist as many as the number of the degree of freedom.

The right column in Table 1 shows the orthogonalized generalized coordinates. Furthermore, the longitudinal stress σ_z is to be as follows.

$$\sigma_z = \frac{Fz}{F} - \frac{My}{I_y}x + \frac{Mx}{I_x}y + \frac{F_{p0}}{I_{\phi 0}}\phi_0 + \frac{F_{p1}}{I_{\phi 1}}\phi_1 + \cdots + \frac{F_{pm}}{I_{\phi m}}\phi_m \quad (8)$$

By taking an example of the cross section having the degree of freedom of one, Fig.8 shows the generalized coordinates which are the distribution of σ_z.

On the other hand, when load is applied in the direction orthogonal to the longitudinal axis of a member, the position of shear centers is related. The practical examples of shear centers are shown in Fig.9. A_0 is the shear center of torsion. A_{1-0} and A_{1-n} are the centers of turning of the left and right parts of a cross section regarding the distortion of the first degree of freedom, which can be called shear centers of distortion. A_{2-0} and A_{2-n} are the shear centers regarding the second degree of freedom. When load P in the figure is applied through these points, distortion of its degree of freedom is not to occur. This fact may serve as a reference concerning the position to fix a bracket for transmitting load.

5. EXPERIMENTAL RESULTS

In order to verify the above theory, the following experiments were carried out. The specimens for the experiment were made by bending structural aluminum sheets. Figure 10 shows the case of applying the self-equilibrated force to the cross section having the degree of freedom of one. One end of the specimen was fixed, and load was applied to the other end. (a) and (b) in Fig.10 are the strains measured along the cross section. Where, ε_z is the longitudinal strain, and ε_s is the strain in direction s. Fig.10(c),(d) show the longitudinal distribution of strain ε_z and of quantity $\theta_1(z)$. $\theta_1(z)$ is defined in Chapter 3, and it indicates the magnitude of distortion.

Fig.8 Generalized coordinates: distribution of σ_z

Fig.9 Shear centers of distortion

(a) ε_z at z=600(P=68.6N)   (b) ε_s at z=200(P=68.6N)

(c) ε_z at s=7(P=29.4N)
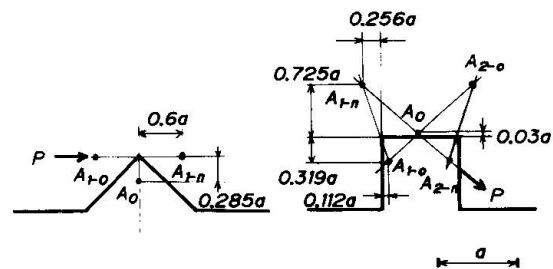
(d) Distortion θ_1 (P=68.6N)
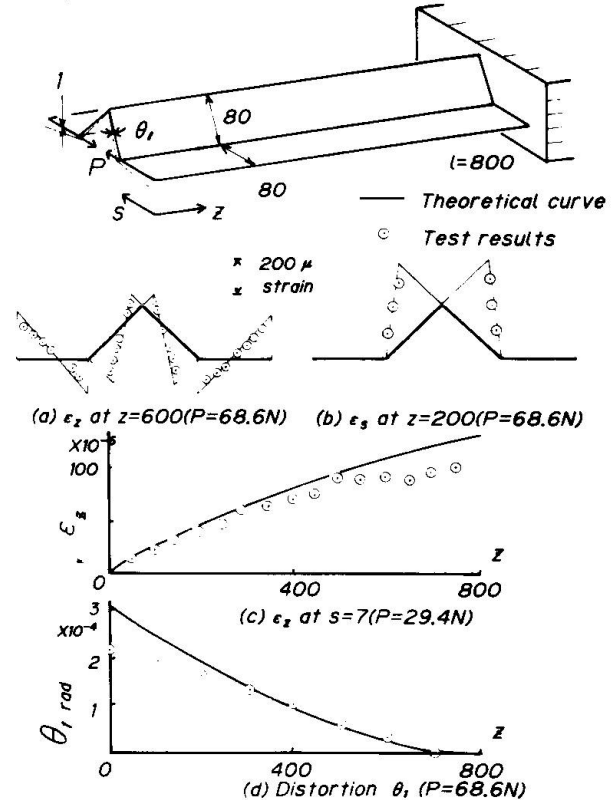
Fig.10 Test results(cross section of one degree of freedom

Figure 11 shows the similar measured results for the cross section having the degree of freedom of two.

Figure 12 shows the measured results when four bulkheads were used.
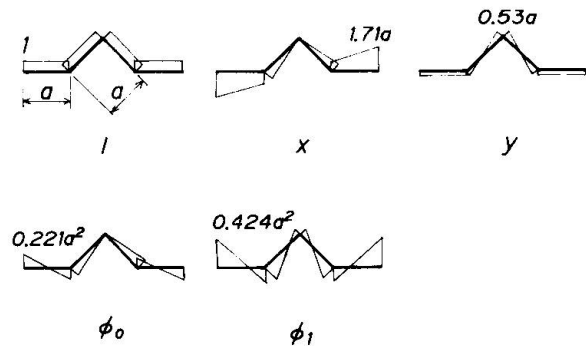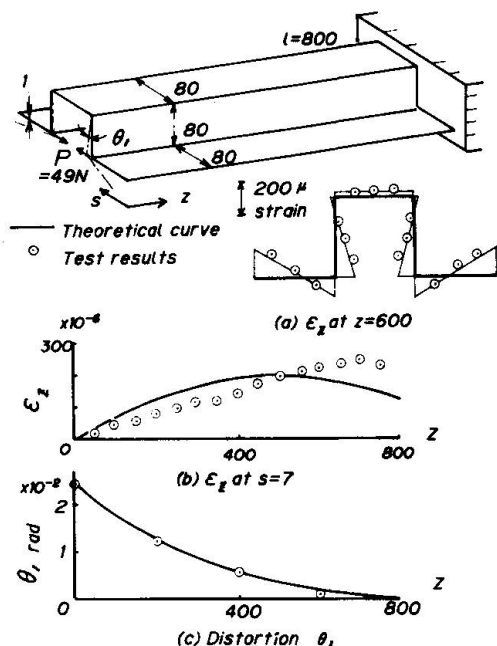
Fig.11 Test results(cross section of two degree of freedom)

Fig.12 Distortion of a bar with bulkheads

The calculated values at this time can be obtained by solving a statically indeterminate problem based on the equations in Chapter 3.

Finally, the author would like to discuss a buckling problem which was not mentioned in Chapter 3. Flexural buckling and torsional-flexural buckling have been well known, but in addition, the new buckling mode by distortion as shown in Fig.13 exists. The distortional buckling load for the cross section as Fig.13 is expressed by the following equation. The details of this equation is discussed in the paper which is to be published.

$$P_{cr}=(EI_{\phi 1}(\frac{n\pi}{\ell})^{2}+M_{11}(\frac{\ell}{n\pi})^{2})/r_{d}^{2}\ldots\ldots(9)$$

Fig.13 Distortional buckling

Here, r_{d} is the value peculiar to the cross section. In this example, the calculated value of P_{cr}=5.98 KN was obtained, and it is lower than conventional elastic or plastic buckling loads. As the result of the experiment using a structural aluminum column having the dimensions shown in Fig.13, the column buckled in distortional mode and critical load was about 7 KN.

6. CONCLUSION

In the conventional theory of warping torsion, the strain and deformation of Figs. 10,11,12 due to distortion have become zero. Then,it is important to estimate the effects of the distortion in the design stage of the thin-walled metal structures in buildings.

REFERENCE

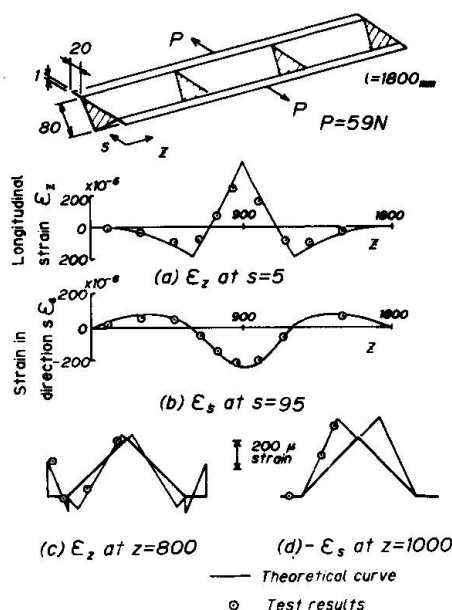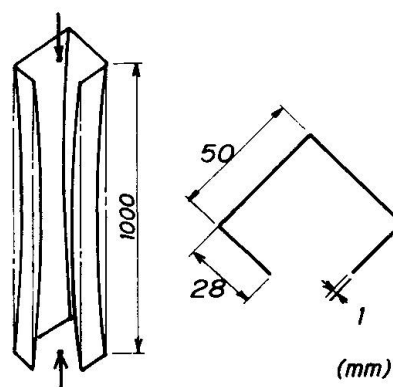1. TAKAHASHI K.,M. MIZUNO, Distortion of Thin-Walled Open-Cross-Section Members. Bull.JSME,23-182,1980,1290-1296.