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Single Load on Trapezoidal Steel Sheet

Charge concentrée appliquée sur une tôle profilée en acier de forme trapézoïdale

Einzellast auf einem trapezförmigen Stahlblech

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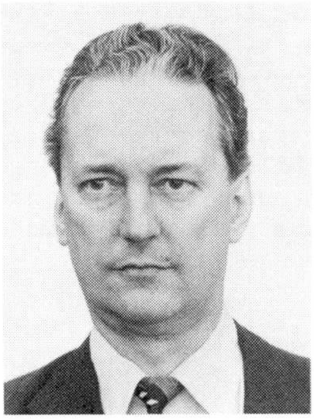

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SUMMARY

A simple model is presented for the calculation of moments and deflections for a trapezoidal steel sheet loaded by a single transversal load at mid-span. The theoretical calculations are compared with laboratory tests on nine different types of steel decks. Results from «in situ» tests are given.

RÉSUMÉ

Un modèle simple de calcul des moments et des flèches d'une plaque profilée de forme trapézoïdale soumise à une charge concentrée transversale à mi-travée est présenté. Les résultats théoriques sont comparés aux résultats d'essais effectués en laboratoire sur neuf types différents de tôles profilées. Des résultats d'essais «in situ» sont également donnés.

ZUSAMMENFASSUNG

Ein einfaches Modell für die Berechnung von Momenten und Durchbiegungen eines trapezförmigen profilierten Stahlbleches, das durch eine Einzellast in der Feldmitte belastet ist, wird beschrieben. Die theoretische Berechnung wird mit Laborversuchen von neun verschiedenen Stahlblechtypen verglichen. Ergebnisse von einigen Feldversuchen werden ebenfalls angegeben.

1. INTRODUCTION

The resistance against a single load on a thin-walled steel deck is very important as most of the steel decks are used as working platforms during the erection period. Two properties of structural behaviour are important - the distribution of deformation over the neighbouring profiles and the capactiy to withstand a single load acting on one of the profile tops. If a load is applied on the steel deck near bonded insulation, it is easy to damage the bonding. What is the load magnitude that can be accepted without permanent local deformation of the flange or the web? Those two questions have been pretty much discussed during the last years.

The concepts of (a) "capacity to carry a single load" and (b) "walkability" are concepts partly overlapping each other. The assessment of "walkability" is mainly subjective. To exceed the load carrying capacity, without deformation restrictions, is very hard in practice. The steel sheet can buckle at a rather low load level but the load can be raised to several times the buckling load due to the "suspension" effect. "Walkability" means among other things that the steel sheet must carry a walking man without permanent indentation. In the Swedish standard it is stated that the residual deformation under the load must not exceed 3 mm.

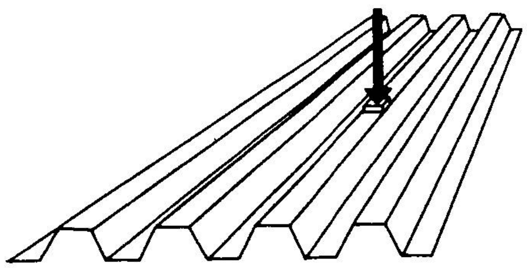
2. THEORETICAL MODEL

Let us start with a simple model, the simply supported trapezoidal steel sheet with a single load applied in the middle. Of all the single profiles ("waves") only three waves are assumed to be active. The loaded wave is supposed to be supported by the two neighbouring waves via springs, c.f. fig.1.

The deflection for the mid-wave and for the side-waves are denoted y_m and y_s respectively. The spring force q acting between these two "beams" is

$$q = c(y_m - y_s) \tag{1}$$

where c is the spring stiffness.



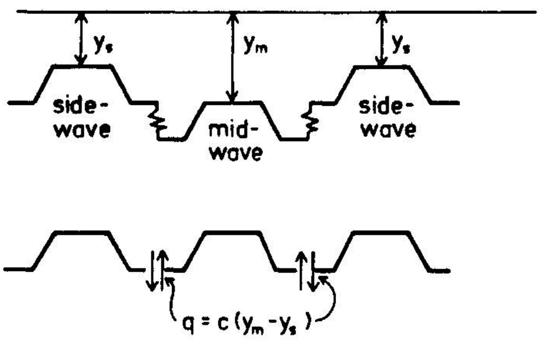


Fig.1 Model

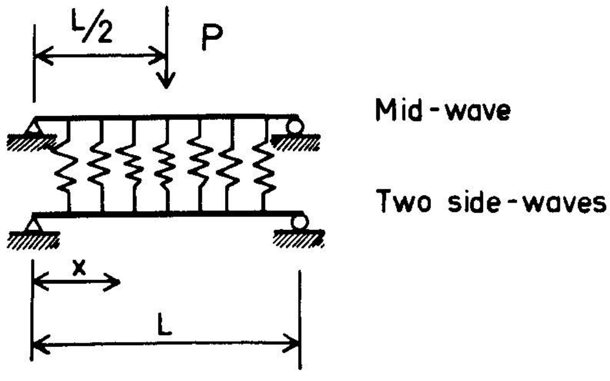


Fig.2 Model with springs

With the assumption that the bending stiffness EI does not vary along the length and that middle- and side-waves have the same stiffness the relationships between forces and deformations are

$$\frac{d^2y_m}{dx^2} = -\frac{M_m}{EI} \quad ; \quad \frac{d^2M_m}{dx^2} = 2q \tag{2}$$

This gives

$$\frac{d^4y_m}{dx^4} = -\frac{2c}{EI}(y_m - y_s) \tag{3}$$

and for the side beam

$$\frac{d^4y_s}{dx^4} = -\frac{c}{EI}(y_s - y_m) \tag{4}$$

Instead of solving these two equations exactly, which leads to cosh·sinh terms - we will use a simple approximation for the form of the deflection curve. We assume that the deflection line can be approximated by the first term of a Fourier series.

$$y = \delta_0 \sin\frac{\pi x}{L} \tag{5}$$

where the boundary conditions y = 0 for x = 0 and x = L are fulfilled. The approximation is rather good because it is known that the form of the deflection curves, both under a single load and a distributed load, is very close to the sinusoidal shape. With the sinusoidal deflection curves also the spring force distribution will be sinusoidal.

$$q = q_0 \sin\frac{\pi x}{L} \tag{6}$$

where q_0 is the intensity at $x = \frac{L}{2}$. At the midspan deflection due to the load q, eq (6), is

$$\delta_0 = \frac{q_0 L^4}{\pi^4 EI} \tag{7}$$

With the assumptions made above the deflection for the mid-wave, the deflection for the side-wave and the relationship between deflection and spring force are

$$\delta_m = \frac{PL^3}{48EI} - \frac{2q_0L^4}{\pi^4 EI} \quad ; \quad \delta_s = \frac{q_0L^4}{\pi^4EI} \quad ; \quad q_0 = c(\delta_m - \delta_s) \tag{8}$$

where

δ_m = midspan deflection, mid-wave
δ_s = midspan deflection, side-wave

3. SPRING STIFFNESS

One key point in this evaluation is the value of the spring stiffness c. The forces between the loaded wave and the supporting waves are transmitted through bending of the steel sheet in the transverse direction. It is difficult to decide exactly which part of the profile will be active to transmit forces. The length of the spring can be adjusted by introducing a correction factor.

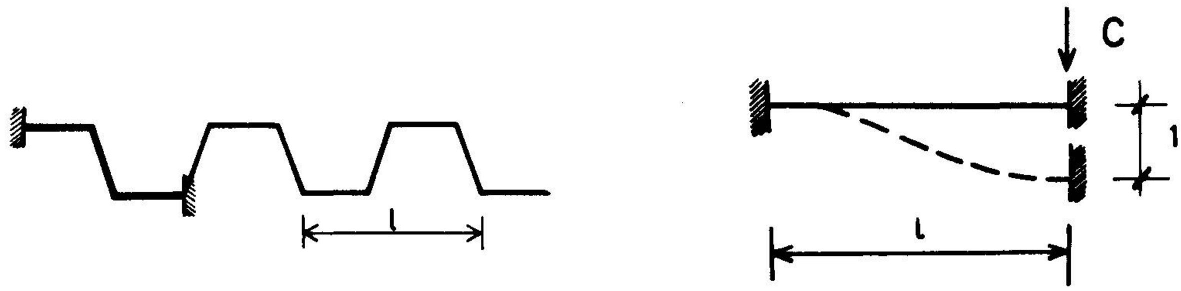


Fig.3 Model for the spring stiffness

Suppose that the spring action will be as shown in fig.3. The spring is assumed to be clamped at both ends. For a perfectly straight "spring beam", fig.3, the spring stiffness c will be

$$c = 12EI_f/\ell^3 \qquad \text{where } I_f = t^3/12 \tag{9}$$
$$t = \text{thickness of the steel sheet}$$

The real spring will have the point of inflexion in the web and a different deformation length. However, usually the web deformation will not influence the total deformation more than a few percent. This justifies that we use the expression (9), which gives

$$c = E\ t^3/\ell^3 \tag{10}$$

4. FINAL EXPRESSIONS

4.1 Deflection

The expressions (8) together with (10) give

$$\delta_m = \frac{PL^3}{48EI} \cdot \frac{1}{1 + 2/(1+\alpha)}$$

$$\delta_s = \delta_m/(1+\alpha) \tag{11}$$

where

$$\alpha = \pi^4 EI/cL^4 = \pi^4\ \ell^3\ I/t^3L^4.$$

For $\alpha = 0$, which corresponds to an infinitely large stiffness of the spring, we get $\delta_s = \delta_m = 1/3 \cdot PL^3/48EI$ and for $\alpha \rightarrow \infty$ we get $\delta_m = PL^3/48EI$ and $\delta_s = 0$. The deflections as a function of α are shown in fig.4.

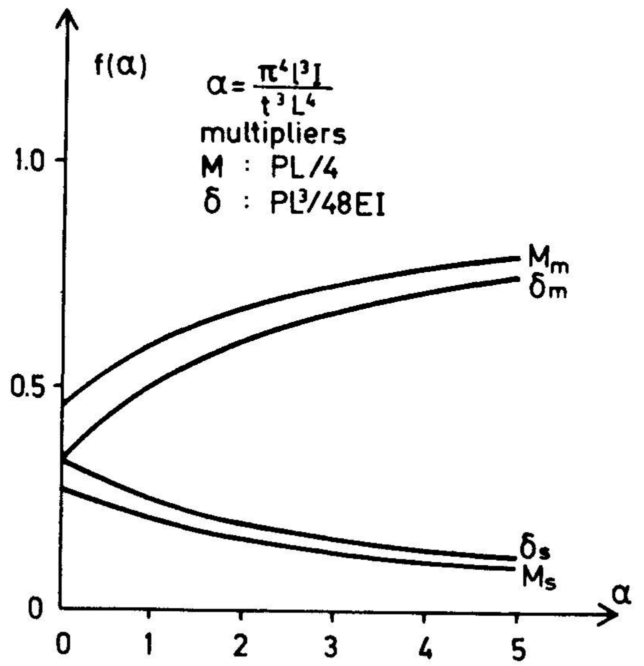


Fig.4 Deflections and moments as a function of α

4.2 Moment

The midspan moment for the loaded profile is

$$M_m = \frac{PL}{4} - 2\,M_q \qquad (12)$$

where

M_q = the moment in the mid-wave caused by the spring forces and at the same time the moment in the side-wave.

An expression for M_q is derived from (2), (6), and (11):

$$M_q = M_s = \frac{PL}{4}\,\frac{\pi^2}{12}\,\frac{1}{3+\alpha} \qquad (13)$$

Then we get

$$M_m = \frac{PL}{4}\left(1 - \frac{\pi^3}{6}\,\frac{1}{3+\alpha}\right) \qquad (14)$$

The variations of the moments are also shown in fig.4.

5. DIFFERENT MODELS - COMPARISONS

In the above model a section with three "waves" was studied. For a section with five waves we instead get the following expressions for the deformation.

$$\delta_m = \frac{PL}{48EI}\,\frac{1}{1+\dfrac{2(2+\alpha)}{\alpha^2+3\alpha+1}}\ ;\quad \delta_s = \delta_m\,\frac{1+\alpha}{\alpha^2+3\alpha+1}\ ;\quad \delta_e = \delta_m\,\frac{1}{\alpha^2+3\alpha+1} \qquad (15)$$

where δ_e is the mid-span deformation of the exterior wave.

A comparison between the two models shows that for the maximum moment and the mid-span deflection the results just differ slightly.

As an example we show the results for $\alpha = 1$:

	3 profile model	5 profile model	3 profile "exact"	mult
Deflection under load	0.50	0.46	0.51	$* \boxed{\frac{PL^3}{48EI}}$
Moment under load	0.59	0.55	0.59	$* \boxed{\frac{PL}{4}}$

In the table above results from an exact solution for the differential equation system (3) + (4) are also shown for comparison.

It may also be interesting to have an idea of the size of the parameter α for some commonly used steel sheets. The range of variation is approximately between $\alpha = 0.5$ and $\alpha = 4$.

6. CONTINUOUS CASE

In the continuous case, e.g. for a two span "beam", fig.5, we get

$$\delta_m = \frac{PL}{48EI} \cdot \frac{23}{32} \frac{1}{1 + \frac{2}{1+\alpha_k}}$$

$$M_{AB,m} = \frac{13\ PL}{64} \left(1 - \frac{\pi^2 \cdot 23}{13 \cdot 12} \left(1 - \frac{3}{4\pi}\right) \frac{1}{\alpha + 3\left(1 - \frac{3\pi}{32}\right)}\right) \qquad (16)$$

$$M_{B,m} = \frac{3PL}{32} \left(1 - \frac{23\pi}{48} \frac{1}{\alpha + 3\left(1 - \frac{3\pi}{32}\right)}\right)$$

where

$$\alpha_k = \alpha / \left(1 - \frac{3\pi}{32}\right)$$

$M_{AB,m}$ = mid-span moment for the loaded profile
$M_{B,m}$ = support moment for the loaded profile

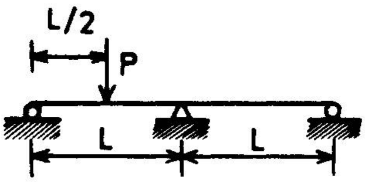


Fig.5 Continuous beam

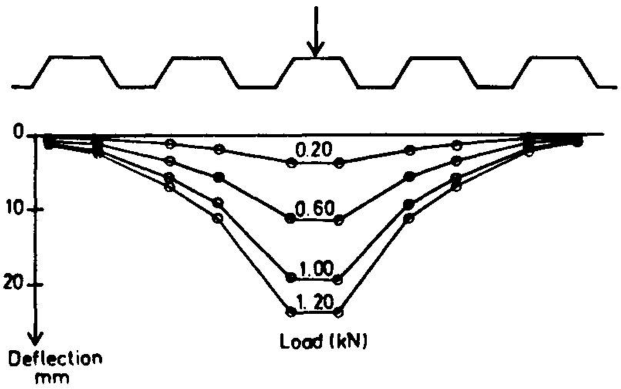


Fig.6 Measured deflections at mid-span for a 3 m span simply supported steel sheet

7. TESTS

7.1 In situ

Different test series have been conducted. In one interesting series roofers, insultation manufacturers and steel sheet manufacturers had to grade different types of steel sheets just by walking on small "test roofs".

In the test series 8 different types of steel sheets were used. The test roofs consisted of 2 steel sheets side by side (popped together) and continuous over three supports. The depth of the profiles used varied from 45 up to 110 mm. The results showed a very big scatter in the judgements - some roofs got both the marks 1 and 5, on the scale 1-5. Three very important points were found at this occasion:

1. The judgements were heavily influenced by the type of industry to which the judging persons belong.

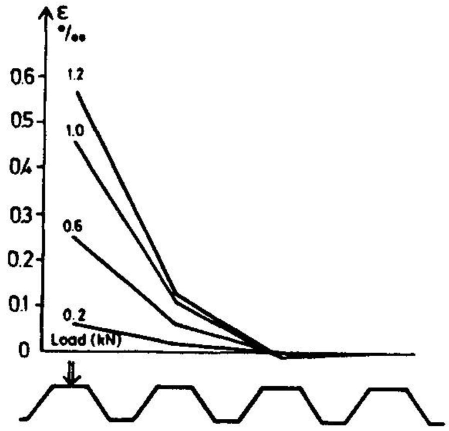
2. Two of the steel sheets were judged "not acceptable" and both showed large deflections in a subsequent test with a load of 1.1 kN (37 and 42 mm respectively). The next roof on the scale had only 27 mm deflection.

3. The judgements also show that a small difference between the deflections of adjacing profiles often gave a good walkability mark.

7.2 Laboratory tests

In the laboratory some tests have been conducted, both simply supported and continuously supported sheets have been tested. Each specimen consisted of two adjacent sheets fixed with pop rivets. For each specimen two different spans were tested. Different load locations were also tested. Both deflections and strains were measured. Just as an example of deflection measurements the results of a simply supported profile having 50 mm depth, 0.6 mm thickness and a span length of 3 m are shown in fig.6. The result of strain measurements are shown in fig.7 both for a midspan section and for a support section.

The tests have confirmed that it is nearly only the loaded profile and the two adjacent profiles that are active in carrying the load.



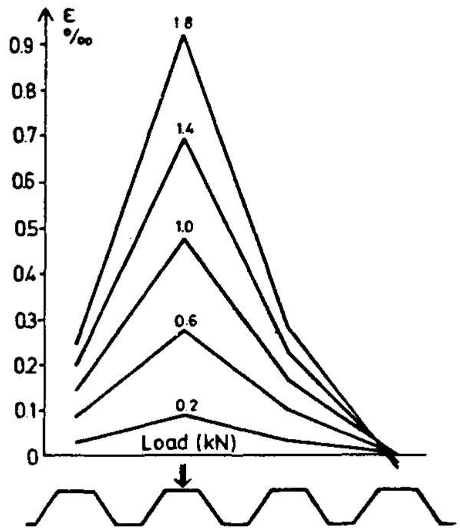


Fig.7 Measured strain distribution, midspan (left) and support section (right) for two different sheets

It is also interesting to see if the test results support the simple model. For this reason the quotients $\delta_m/\Sigma\delta$ and $\varepsilon_m/\Sigma\varepsilon$ are compared with the theoretical results. By comparing this kind of "relative" properties you "get rid" of variations in modulus of inertia, modulus of elasticity and the exact load level. The results from this comparison are shown in fig.8.

7.3 Deformations

Tests and theory agree very well for $\alpha < 1.5$ but for $\alpha > 1.5$ the model gives too big deflection share for the mid-profile. However, the parameter α is very sensitive to the size of the included parameters. If, for instance, we use 0.8 ℓ instead of ℓ as a length of the spring there will be better agreement between tests and theory for $\alpha > 1.5$.

The absolute value of the midspan deflection is more influenced by the variations in span length than by a variation in α. A rather good estimate of the deformations under the load is obtained using the assumption that the deflection of the loaded profile is 50% of the total, or in other words, the loaded profile carries 50% of the load.

7.4 Strain

The strain measurements give an idea how the moment is shared between the waves. These measurements are shown in two different diagrams for the continuous sheets, Fig. 8b,c. As you can see there is a fairly good agreement between theory and tests for the mid-span moment but a big scatter for the support moment, up to 50% of the estimated moment. However, this does not matter, because the support moment due to a single load is just half the mid-span moment.

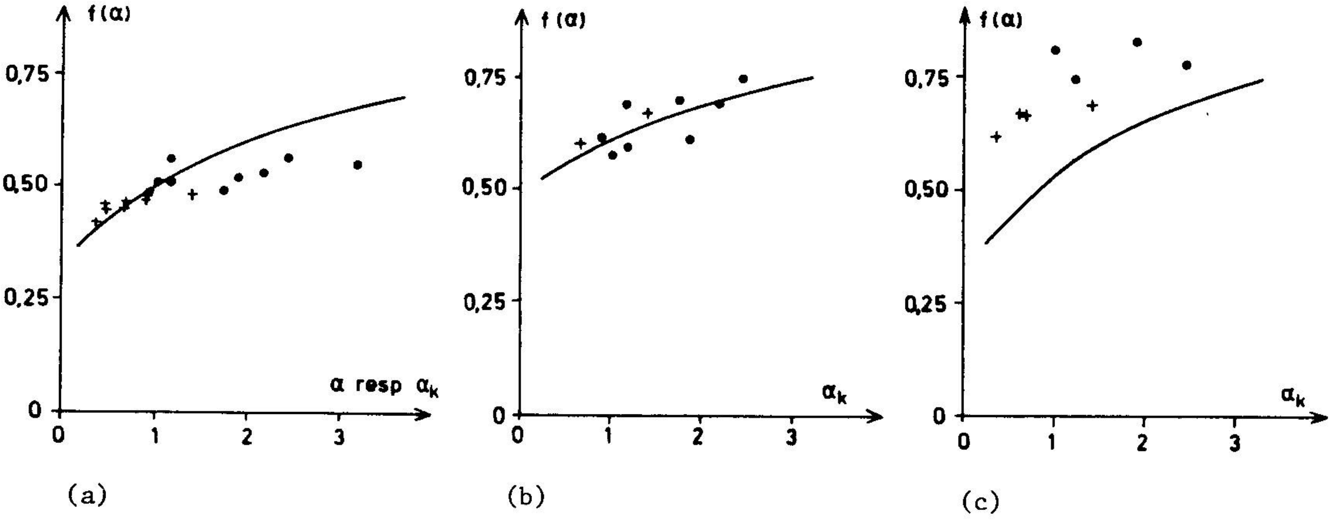


Fig.8 Measured $\delta_m/\Sigma\delta$ (left), $\varepsilon_m/\Sigma\varepsilon$ in midspan (middle) and $\varepsilon_m/\Sigma\varepsilon$ for support moment. The curves are theoretically calculated.

8. ACKNOWLEDGMENT

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