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Torsional-Flexural Buckling of Partially Closed Thin-Walled Columns

Flambement par flexion et torsion de colonnes à parois minces partiellement caissonnées

Biegedrillknicken von teilweise geschlossenen dünnwandigen Stützen

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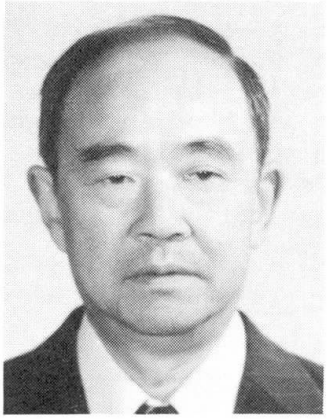

Shi-Ji Wang, born 1924, received his BS at Zhong-Zheng University in 1947. For thirty years he has been engaged in teaching and research on steel structures, and now he is the vice-chairman of the Technical Group of the Chinese National Specification Committee for the Design of Cold-Formed Thin-Walled Steel Structures.

SUMMARY

This paper discusses the buckling behavior of partially closed thin-walled sections, i.e. thin-walled open sections with battens, under concentric or eccentric load. A rational method for determining the torsional-flexural buckling load of such members is presented. The influence of prebuckling displacements due to end couples is included in the analysis of instability for both open and partially closed sections. Theoretical results are compared with experimentally obtained buckling loads.

RÉSUMÉ

Cette contribution concerne l'étude du flambement de colonnes dont la section droite est constituée de sections ouvertes à parois minces reliées par des diaphragmes et soumises à des charges centrées et excentrées. On y présente une méthode rationelle permettant de déterminer la charge de flambement par flexion et torsion de tels éléments. L'influence des déformations dues aux moments d'extrémités est prise en compte dans l'analyse de l'instabilité des sections ouvertes ou partiellement caissonnées. Les résultats de cette analyse sont comparés avec les charges de flambement obtenues expérimentalement.

ZUSAMMENFASSUNG

Dieser Beitrag behandelt das Knickverhalten des teilweise geschlossenen, dünnwandigen Stabes und zwar des offenen dünnwandigen Querschnittes mit Versteifungsblechen unter Axialdruck und exzentrischem Druck. Eine Berechnungsmethode für die Ermittlung der Verzweigungslast bei Biegedrillknicken des Stabes wird vorgestellt. Bei der Analyse der Instabilität wird der Einfluss einer Verschiebung des offenen und des teilweise geschlossenen Querschnitts, die durch Endmomente entsteht, berücksichtigt. Theoretische Ergebnisse werden mit experimentellen Knicklasten verglichen.

INTRODUCTION

It is well known that when torsional-flexural buckling occurs, the critical load of thin-walled open sections will always be less than the in-plane buckling or collapse load. However, if some batten plates are attached along the open side of such members so that there exist several intermittent closed sections, their load-carrying capacity will be increased, and it is possible to transform the mode of failure from torsional-flexural buckling to in-plane instability provided the spacing of battens is short enough to prevent the member from warping and twisting. This problem has been studied by the writer in another paper [1], in which the effect of prebuckling deflection was ignored.

Owing to the fact that the torsional-flexural buckling load of an eccentric column is more or less influenced by its prebuckling deflection, especially for slender member under axial thrust and large terminal moment, the effect of prebuckling deflection has been investigated by Peköz and Winter [2], who assumed a parabolic curve to approximate the in-plane deflection. In this paper, an exact deflection curve is adopted to estimate the influence of prebuckling deflection on torsional-flexural buckling load.

1. ELASTIC TORSIONAL-FLEXURAL BUCKLING OF SINGLY SYMMETRIC OPEN SECTIONS

The total potential energy of a singly symmetric section under axial thrust and equal terminal moments applied in the plane of symmetry as shown in Fig.1 is

$$\Pi = \frac{1}{2}\int_0^L \left[EI_y u''^2 + EI_x v''^2 + EC_w \varphi''^2 + (GJ - Pr_o^2 + M\beta_y)\varphi'^2 - P(u'^2 + v'^2) + 2(Px_o + M)v'\varphi'\right] dz \qquad (1)$$

where x,y= symmetric and unsymmetric axis of the cross-section, respectively; E = modulus of elasticity; G = shear modulus; I_x, I_y = moments of inertia with respect to x- and y-axis; J = St Venant torsion constant C_w = warping constant; x_o = abscissa of the shear centre; u,v = displacements of the shear centre in the x and y direction; φ = angle of rotation of the cross-section about the shear centre; P = axial thrust; M = bending moment in the symmetric plane and

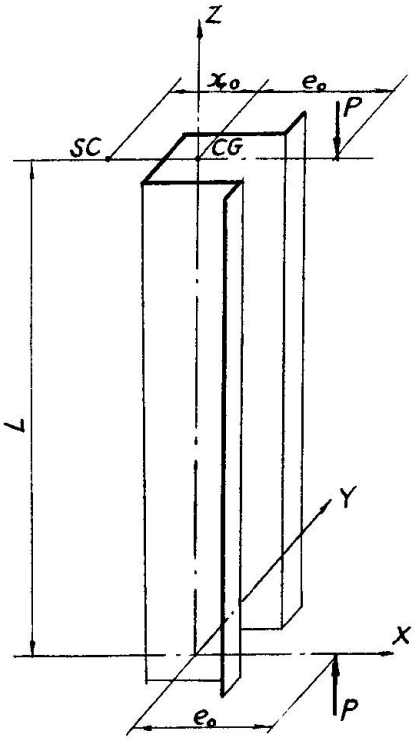

Fig.1 Singly symmetric open section under eccentric load

$$r_o^2 = (I_x + I_y)/A + x_o^2 \qquad (2)$$

$$\beta_y = \frac{1}{I_y}\int_A x(x^2+y^2)dA - 2x_o \qquad (3)$$

For simply supported beam-columns subjected to equal end moments M_o and axial thrust P the deflection curve can be expressed by the following equation [3]

$$u = \frac{M_o}{P\cos(kL/2)}\cos\left(\frac{kL}{2} - kz\right) - \frac{M_o}{P} \qquad (4)$$

from which

$$M = -EI_y u'' = M_o(\cos kz + \tan w \sin kz) \qquad (5)$$

where

$$k = \sqrt{P/EI_y} \qquad (6)$$

$$w = kL/2 \qquad (7)$$

Since we are interested to obtain the torsional-flexural buckling load only, the expression for total potential energy can be

reduced to

$$\Pi = \frac{1}{2}\int_0^L \left[EI_x v''^2 + EC_w \varphi''^2 + (GJ - Pr_o^2 + M\beta_y)\varphi'^2 - Pv'^2 + 2(Px_o + M)v'\varphi'\right] dz \tag{8}$$

1.1 Unbattened Open Sections

The deflection components v and φ under various boundary conditions are assumed as shown in Table 1, in which v_o and φ_o are undetermined parameters.

Boundary Conditions at z=0,L	v	φ
$U=v=\varphi=0,\ u''=v''=\varphi''=0$	$v_o \sin(\pi z/L)$	$\varphi_o \sin(\pi z/L)$
$u=v=\varphi=0,\ u''=v''=\varphi'=0$	$v_o \sin(\pi z/L)$	$\varphi_o [1-\cos(2\pi z/L)]$
$u=v=\varphi=0,\ u''=v'=\varphi'=0$	$v_o [1-\cos(2\pi z/L)]$	$\varphi_o [1-\cos(2\pi z/L)]$

Table 1 Assumed deflection components

Substituting the appropriate deflection components v,φ and the in-plane moment given in eqn 5 into eqn 8, and using the Rayleigh-Ritz method an approximate value of the torsional-flexural buckling load can be obtained. The solution is expressed in a generalized form as follows

$$\begin{vmatrix} P_x - P & K'_{23}P(x_o - C_1 e_o) \\ K'_{32}P(x_o - C_1 e_o) & r_{eq}^2 (P_z - P) \end{vmatrix} = 0 \tag{9}$$

Expansion of eqn 9 leads to

$$r_{eq}^2 (P_x - P)(P_z - P) - K_{23}^2 P^2 (x_o - C_1 e_o)^2 = 0$$

Solving this equation yields the torsional-flexural buckling load

$$P_{TF} = \frac{P_x + P_z - \sqrt{(P_x + P_z)^2 - 4DP_x P_z}}{2D} \tag{10}$$

where

$$P_x = K_{22}\pi^2 EI_x / L^2 \tag{11}$$

$$P_z = (K_{33}\pi^2 EC_w / L^2 + GJ)/r_{eq}^2 \tag{12}$$

$$r_{eq}^2 = r_o^2 + C_2 \beta_y e_o \tag{13}$$

$$e_o = M_o / P \tag{14}$$

$$D = 1 - K_{23}^2 (x_o - C_1 e_o)^2 / r_{eq}^2 \tag{15}$$

$$K_{23} = \sqrt{K'_{23} K'_{32}} \tag{16}$$

C_1, C_2 = amplification factors for eccentricity

Values of coefficients K_{ij},K'_{ij} and amplification factors C_1,C_2 are given in Table 2.

Since C_1 and C_2 are functions of P, direct solution of eqn 10 would be either difficult or impossible, and method of successive approximation must be employed. The initial values for both C_1 and C_2 may be taken as unity, i.e., the member is assumed to be undeflected prior to buckling, thus a first approximation of the critical value of P can be found. This value is used to compute C_1 and C_2 and the second approximation of P can be obtained by substituting them into eqn 10. Repeate the procedure until a satisfactory accurate result is reached.

Boundary conditions at $z=0,L$	K_{22}	K_{33}	K'_{23}	K'_{32}	$K_{23}=\sqrt{K'_{23}K'_{32}}$	C_1	C_2
$u=v=\varphi=0$ $u''=v''=\varphi''=0$	1	1	1	1	1	$\frac{\pi^2-2w^2}{\pi^2-w^2}\frac{\tan w}{w}$	$\frac{\pi^2-2w^2}{\pi^2-w^2}\frac{\tan w}{w}$
$u=v=\varphi=0$ $u''=v''=\varphi'=0$	1	4	$\frac{16}{3\pi}$	$\frac{4}{3\pi}$	$\frac{8}{3\pi}$	$\frac{3\pi^2}{4}\left(\frac{3}{9\pi^2-4w^2}+\frac{1}{\pi^2-4w^2}\right)$	$\frac{4\pi^2}{4\pi^2-w^2}\frac{\tan w}{w}$
$u=v=\varphi=0$ $u''=v'=\varphi'=0$	4	4	1	1	1	$\frac{4\pi^2}{4\pi^2-w^2}\frac{\tan w}{w}$	$\frac{4\pi^2}{4\pi^2-w^2}\frac{\tan w}{w}$

Table 2 Values of K_{ij}, K'_{ij} and expressions of C_1, C_2

1.2 Open Sections with Battens

Since batten plates are employed to form several closed sections intermittently along the longitudinal axis of the member, we assume that the rate of change of twisting angle at these sections vanishes provided the shearing rigidity of the batten plates is large enough to prevent the sections from warping. Suppose that the centre lines of battens divide the length of the member into n equal segments, thus the centre-to-centre spacing of battens becomes $a = L/n$ as shown in Fig. 2 .

According to the assumption mentioned above, the deflection component φ must satisfy the following additional boundary conditions:

$$\varphi' = 0 \quad \text{at} \quad z = ja \qquad (j = 1,2,3,\ldots n-1)$$

Two cases are studied for singly symmetric open sections with battens.

1.2.1 The boundary conditions at both ends are: $u = v = \varphi = 0$, $u''=v''=\varphi'= 0$.

For battened members under this end conditions, the assumed deflection function φ given in Table 1 is replaced by

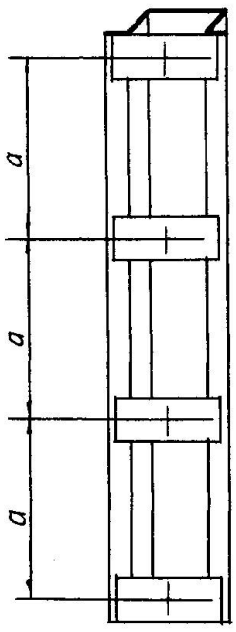

Fig.2 Open section with battens

$$\varphi = \varphi_0\left\{\sin\left(\frac{i-1}{n}\right)\pi+\sin\frac{\pi}{2n}\cos\left(\frac{2i-1}{2n}\right)\pi\left[1+(-1)^i\cos\frac{\pi z}{a}\right]\right\} \tag{17}$$

$$(i = 1,2,3,\ldots,n+1)$$

where $i = j+1$ and n is an even number.

Applying the Rayleigh-Ritz method one obtains the same expression for P_{TF} as eqn 10 only if P_z, K'_{23}, K'_{32} and K_{23} are replaced by

$$P_z = (\pi^2 EC_w/a^2 + GJ)/r_{eq}^2 \tag{18}$$

$$K'_{23} = \frac{2n^2}{(n^2-1)\pi}\sin\frac{\pi}{n}\sum_{i=1}^{n}\cos^2\left(\frac{2i-1}{2n}\right)\pi \tag{19}$$

$$K'_{32} = \frac{4n}{(n^2-1)\pi\tan\frac{\pi}{2n}} \tag{20}$$

$$K_{23} = \sqrt{K'_{23}K'_{32}}$$

When $n \geqslant 2$, the values of K_{23} vary from 0.8488 to 0.9003

If n is an odd number, eqn 17 is valid only for $0 \leqslant z \leqslant (n-1)a/2$ and $(n+1)a/2 \leqslant z \leqslant L$. For the mid-segment, i.e., when $(n-1)a/2 \leqslant z \leqslant (n+1)a/2$, the assumed function should be replaced by

$$\varphi = \varphi_0\left[\cos\frac{\pi}{2n} + \frac{1}{2}\left(1-\cos\frac{\pi}{2n}\right)\left(1-\cos\frac{2\pi z}{a}\right)\right] \tag{21}$$

Applying the same procedure as before, the coefficients K'_{23} and K'_{32} can be derived as follows

$$K'_{23} = \frac{4n^2}{\pi}\left[\frac{n}{4(n^2-1)}\sin\frac{\pi}{n} + \frac{2}{4n^2-1}\sin\frac{\pi}{2n}\left(1-\cos\frac{\pi}{2n}\right)\right] \tag{22}$$

$$K'_{32} = K'_{23}\Big/\left[\frac{n^2}{2}\sin^2\left(\frac{\pi}{2n}\right) + n\left(1-\cos\frac{\pi}{2n}\right)^2\right] \tag{23}$$

When $n \geqslant 3$, the values of $K_{23} = \sqrt{K'_{23}K'_{32}}$ vary between 0.8973 and 0.9003.

1.2.2 The boundary conditions at both ends are: $u = v = \varphi = 0$, $u'' = v' = \varphi' = 0$.

In order to satisfy the boundary conditions at both ends and intermediate battened sections, the assumed function for φ should be replaced by

$$\varphi = \varphi_0\left\{\left[1-\cos\frac{2(i-1)\pi}{n}\right] + \sin\frac{\pi}{n}\sin\frac{(2i-1)\pi}{n}\left[1+(-1)^i\cos\frac{\pi z}{a}\right]\right\} \tag{24}$$

$$(n = \text{even numbers},\ i = 1,2,3,\ldots,n+1)$$

from which

$$K'_{23} = K'_{32} = 1 \qquad (n = 2) \tag{25}$$

$$\left.\begin{aligned} K'_{23} &= \frac{n^3}{2(n^2-4)\pi}\sin\frac{2\pi}{n} \\ K'_{32} &= \frac{8n}{(n^2-4)\pi\tan\frac{\pi}{n}} \end{aligned}\right\} \quad (n \geqslant 4) \tag{26}$$

When $n \geqslant 4$, K_{23} vary between 0.8488 and 0.9003.

In the cases when n are odd numbers eqn 24 is valid only for $0 \leqslant z \leqslant (n-1)a/2$ and $(n+1)a/2 \leqslant z \leqslant L$, in the interval $(n-1)a/2 \leqslant z \leqslant (n+1)a/2$ we use the following function instead:

$$\varphi = \varphi_0\left[\left(1+\cos\frac{\pi}{n}\right) + \frac{1}{2}\left(1-\cos\frac{\pi}{n}\right)\left(1-\cos\frac{2\pi z}{a}\right)\right] \tag{27}$$

from which

$$K'_{23} = \frac{1}{\pi}\left[\frac{n^2}{n^2-4}\sin\frac{2\pi}{n} + \frac{2}{n^2-1}\sin\frac{\pi}{n}\left(1-\cos\frac{\pi}{n}\right)\right] \tag{28}$$

$$K'_{32} = K'_{23}\Big/\left[\frac{1}{2}\sin^2\left(\frac{\pi}{n}\right) + \frac{1}{n}\left(1-\cos\frac{\pi}{n}\right)^2\right] \tag{29}$$

When $n \geqslant 3$, $K_{23} = \sqrt{K'_{23}K'_{32}}$ vary between 0.7839 and 0.9003.

1.2.3 Simplified approach for determining the torsional-flexural buckling load of partially closed members.

It should be noted that if there is only one intermediate batten plate attached at the mid-span of the member, i.e. n=2, its buckling behavior is identical to the unbattened member with the same dimensions. As a result, when n=2, the coefficient K_{23} will automatically coincide with that given in Table 2, and we can conclude that the buckling behavior of thin-walled open sections will not be improved if only one batten plate is used. So that at least two batten plates are required for battened members.

Summarizing the two cases of end conditions in 1.2.1 and 1.2.2, the values of K_{23} vary from 0.7839 to 0.9003 except for n=2. Fortunately, the range of variation is so small that we can use a definite value for K_{23} without regard to the number of battens. For this reason the writter suggests K_{23} = 0.9 for all the cases mentioned above, which yields a slightly conservative value of the theoretical buckling load. Thus, eqns10,11,13 to 16, and coefficients K_{22},C_1,C_2 given in Table 2 are also valid for partially closed sections, but eqn 12 should be replaced by eqn 18 and take K_{23} = 0.9.

2. INELASTIC TORSIONAL-FLEXURAL BUCKLING

For centrally loaded simple columns, the inelastic buckling load can easily be found by using the well known tangent modulus theory. However, when torsional buckling occurs, the problem becomes more complex because both E and G no longer remain constants, so that the elastic section rigidities EI_x, GJ and EC_w should be replaced by the tangent section rigidities $(EI_x)_t$, $(GJ)_t$ and $(EC_w)_t$, respectively. Bleich [4] presented a simplified treatment for inelastic torsional buckling, which was based on the assumption that E and G in the whole section will be reduced to $E\tau$ and $G\tau$ synchronously, where τ is the ratio of tangent modulus E_t to the elastic modulus E. In this paper, Bleich's suggestion is adopted to determine the inelastic torsional-flexural buckling loads. Thus eqn 10 is also valid for inelastic domain if E and G in eqns 11,12 are replaced by $E\tau$ and $G\tau$.

2.1 Equivalent Length for Torsional-Flexural Buckling

For design purposes, a new concept, the equivalent length for torsional-flexural buckling, is introduced herein.

From eqn 9

$$\frac{1}{P} = \frac{P_x+P_z}{2P_xP_z} \pm \sqrt{\left(\frac{P_x+P_z}{2P_xP_z}\right)^2 - \frac{r_{eq}^2 - K_{23}^2(x_o - C_1 e_o)^2}{r_{eq}^2 P_x P_z}} \tag{30}$$

The larger root of $1/P$ corresponds to the smaller root of P, thus

$$P_{TF} = \frac{K_{22}\pi^2 E\, I_x/L^2}{\dfrac{s^2+r_{eq}^2}{2s^2} + \sqrt{\left(\dfrac{s^2+r_{eq}^2}{2s^2}\right)^2 - \dfrac{r_{eq}^2 - K_{23}^2(x_o - C_1 e_o)^2}{s^2}}} \tag{31}$$

where

$$P_x = K_{22}\pi^2 E\tau I_x/L^2 \tag{32}$$

$$P_z = (K_{33}\pi^2 E\tau C_w/L_w^2 + G\tau J)/r_{eq}^2 \tag{33}$$

$$s^2 = \frac{r_{eq}^2 P_z}{P_x} = \frac{K_{33}\pi^2 E\tau C_w/L_w^2 + G\tau J}{K_{22}\pi^2 E\tau I_x/L^2}$$

$$= \frac{L^2}{K_{22}I_x}\left(\frac{K_{33}C_w}{L_w^2} + \frac{GJ}{\pi^2 E}\right) \tag{34}$$

K_{22}, K_{33}, K_{23} and L_w are evaluated as follows:

For unbattened members: $L_w = L$; K_{22}, K_{33} and K_{23} are given in Table 2.

For battened members: $L_w = a$, $K_{33} = 1$, $K_{23} = 0.9$, K_{22} is given in Table 2.

Let

$$\mu = \sqrt{\frac{s^2+r_{eq}^2}{2s^2} + \sqrt{\left(\frac{s^2+r_{eq}^2}{2s^2}\right)^2 - \frac{r_{eq}^2 - K_{23}^2(x_o - C_1 e_o)^2}{s^2}}} \tag{35}$$

then

$$P_{TF} = K_{22}\pi^2 E\, I_x/(\mu L)^2 \tag{36}$$

in which μL is defined as the equivalent length for torsional-flexural buckling. Since the inelastic factor τ can be eliminated in eqn 34, so that the equivalent-length coefficient μ is valid for both elastic and inelastic buckling, and P_{TF} can be obtained by using the basic column curve.

Specimen	Dimensions (mm)					No. of batten plates	e_o in mm	Axial load (KN)			$\frac{P_{exp}}{P_{yd}}$	$\frac{P_{exp}}{P_{TF}}$
	h	b	c	t	L			P_{yd}	P_{TF}	P_{exp}		
H1-1-0	62.3	77.8	25.5	2.56		0		82.99	78.59	76.10		0.968
H1-1-2	62.0	77.2	25.8	2.52	2710	2	-10	81.08	90.73	86.00	1.061	
H1-1-3	63.1	77.8	25.9	2.42		3		79.84	112.80	95.32	1.194	
H1-2-0	62.9	76.6	26.0	2.53		0		129.04	49.65	50.50		1.017
H1-2-2	62.8	76.8	26.7	2.77		2		141.40	87.98	87.38		0.993
H1-2-3	62.8	77.7	25.7	2.61	2710	3	0	134.39	105.34	98.07		0.931
H1-2-4	63.1	78.1	25.9	2.66		4		137.87	126.90	116.70		0.920
H1-2-5	62.9	76.7	25.6	2.52		5		128.24	129.77	123.56	0.964	
H1-3-0	63.1	76.7	25.7	2.47		0		77.13	37.93	38.83		1.024
H1-3-2	62.5	77.9	25.5	2.55	2710	2	+10	80.63	62.08	62.76		1.011
H1-3-3	62.7	76.0	26.5	2.54		3		78.47	81.43	75.51	1.039	
H1-3-4	62.6	76.5	26.2	2.52		4		78.38	94.36	73.06	1.073	
H1-4-0	62.5	76.8	25.8	2.46		0		59.38	31.30	31.68		1.012
H1-4-2	63.0	77.2	25.8	2.53	2710	2	+20	61.45	52.71	56.00		1.062
H1-4-3	62.3	78.0	25.5	2.46		3		60.61	68.01	63.74	1.052	
H1-4-4	63.2	78.1	26.6	2.58		4		64.46	88.87	67.96	1.054	
H1-5-0	63.0	77.8	25.7	2.45		0		50.14	27.56	27.65		1.003
H1-5-2	63.3	76.8	27.5	2.53	2710	2	+30	52.00	48.49	48.15		0.993
H1-5-3	63.1	77.4	25.6	2.59		3		52.05	63.67	49.43	0.950	
H1-6-0	64.5	78.1	26.2	2.49		0		44.46	25.93	25.99		1.002
H1-6-2	63.2	79.5	25.6	2.49	2710	2	+40	44.98	42.18	39.23		1.075
H1-6-3	62.6	74.8	27.1	2.47		3		41.80	54.41	44.13	1.056	
H1-7-0	62.6	77.5	27.0	2.59	2710	0	+50	40.27	24.07	22.75		0.945
H1-7-2	63.0	76.9	26.8	2.50		2		38.52	37.87	36.28		0.959
H1-8-0	62.5	76.9	25.3	2.51	2710	0	+60	33.52	19.81	18.24		0.921
H1-8-2	62.2	77.5	25.8	2.68		2		36.14	35.75	34.03		0.952
H2-1-0	60.5	77.0	25.9	2.48	1890	0	-10	96.07	122.70	96.60	1.006	
H2-1-2	61.9	76.1	26.1	2.49		2		95.86	134.34	96.11	1.003	
H2-2-0	60.6	76.3	26.9	2.46		0		143.58	86.02	85.81		0.998
H2-2-2	62.8	76.1	25.6	2.46	1890	2	0	142.94	128.24	120.62		0.941
H2-2-3	60.4	77.6	25.2	2.52		3		146.37	191.83	129.94	0.888	
H2-3-0	61.2	78.3	25.9	2.44	1890	0	+10	94.74	67.33	60.80		0.903
H2-3-2	62.0	76.4	26.3	2.49		2		94.24	120.05	86.30	0.916	
H2-4-0	61.7	75.9	26.2	2.41	1890	0	+20	68.13	55.00	54.92		0.999
H2-4-2	61.8	76.3	26.5	2.47		2		70.44	100.84	71.59	1.016	
H2-5-0	61.5	76.6	26.2	2.45	1890	0	+30	56.76	48.67	45.60		0.937
H2-5-2	61.8	76.5	26.6	2.47		2		57.49	88.74	56.88	0.989	
H3-1-0	60.9	77.7	25.7	2.41	1480	0	-10	100.80	133.02	101.01	1.002	
H3-2-0	61.9	77.2	25.4	2.47	1480	0	0	150.75	124.01	115.72		0.933
H3-2-2	62.4	76.4	26.4	2.47		2		151.53	249.86	141.71	0.935	
H3-3-0	61.2	76.4	25.5	2.41	1480	0	+10	96.26	99.41	91.20	0.948	
H3-4-0	60.4	76.6	25.0	2.43	1480	0	+30	58.07	70.22	56.88	0.980	

Table 3 Theoretical and experimental failure load comparisons

3. EXPERIMENTS

In order to confirm the theory, a series of test were performed. The tests consisted of cold-formed thin-walled hat sections with various slenderness ratios under concentric or eccentric load. Two or more intermediate batten plates with equal spacing were welded on the open side of the member except those unbattened ones. The shape of the cross section is shown in Fig.3, and their dimensions are given in Table 3. A thick plate was welded to each end of specimens, and both ends were seated on the cruciform-knife supports, thus the boundary conditions of the test columns approach to $u = v = \varphi = 0$, $u'' = v'' = \varphi' = 0$ at both ends and $\varphi' = 0$ at battened sections. The experimental ultimate loads P_{exp} and theoretical values of P_{TF} and the initial yield loads P_{yd} are also given in Table 3.

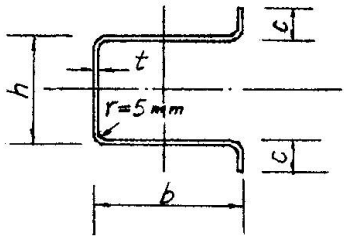

Fig.3 Cross section of specimens

4. CONCLUSIONS

A rational method for determining the torsional-flexural buckling load for thin-walled columns of open cross sections is presented. This method is valid for battened and unbattened members under concentric or eccentric load. The influence of prebuckling deflection is taken into consideration. For the convenience of design, the equivalent-length coefficient is introduced. This coefficient is applicable for both elastic and inelastic buckling. Comparing the theoretical values with experimental results the following conclusions may be drawn:

The torsional-flexural buckling load of an open thin-walled column can be increased by attaching batten plates on its open side.

It is possible to transform the mode of failure from torsional-flexural buckling into in-plane instability if sufficient number of battens are used.

When $P_{TF} \geqslant P_{yd}$ torsional buckling does not occur, more batten plate is unnecessary because the ultimate load of the member can not be further increased.

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