Limit state of a thin-walled rectangular hollow beam-column

Limit State of a Thin-Walled Rectangular Hollow Beam-Column

Etat ultime des profils creux, de section rectangulaire, à parois minces

Grenzzustand des dünnwandigen Druckstabes

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SUMMARY

A theoretical study of the behaviour of a thin-walled beam-column affected by plate buckling has been conducted. The moment-curvature-thrust relationships for a rectangular hollow cross-section are developed and the elastic response of a column segment subjected to compression combined with a bending moment is studied. The technical analysis known as the «Curvature Method» is used to obtain the solution for a symmetrically loaded beam-column.

RÉSUMÉ

Dans cet article, nous avons étudié le comportement des profils creux à parois minces en voilement local. Nous avons obtenu les relations entre le moment, la courbure de l'élément et l'effort normal pour les profils creux rectangulaires et analysé le comportement élastique d'une poutre-colonne soumise à une combinaison d'actions compression-moment de flexion. La «méthode de courbure» a été utilisée pour résoudre le cas d'une poutre-colonne chargée symétriquement.

ZUSAMMENFASSUNG

Im Aufsatz wurde das Verhalten des dünnwandigen Stabes mit lokal ausgebeulten Wänden analysiert. Die Abhängigkeit zwischen dem Moment (2. Ordnung) und der Krümmung wurden für den geschlossenen, viereckigen Querschnitt dargestellt. Auch das elastische Verhalten eines Stabstückes unter Druck und Biegung wurde analysiert. Für die Lösung des Problems des symmetrisch belasteten Druckstabes wurde die Krümmungsmethode verwendet.

1. INTRODUCTION

Local buckling is usually a governing mode of the behaviour of thin-walled steel structural elements. The basic concept underlying the design of these elements involves the utilization of the post-buckling strength of the compression plate elements which comprise the cross-section. As a steel column is a system of thin plates, its overall and local deformations are interconnected. The cause of collapse is usually an interaction between two different modes of buckling.

In case of compressed thin-walled columns, which are affected by plate buckling, a great deal of research has been carried out recently and a number of different design analysis procedures has been formulated, but they have proved impractical to apply. Most researchers assume ideal geometrical and structural conditions and calculate the bifurcation load of axially compressed thin-walled columns. However, it is known that local and overall buckling cannot occur in a strict theoretical sense. Since the bifurcation concept does not correspond to the behaviour of real columns it results in an overestimation of the critical load.

Only a few researchers have considered a column with unvoidable initial irregularities. Also the action of the combination of bending moments and a direct compression on a beam-column has received relatively little attention.

Hence, the relationships between generalized stress - generalized strain for a thin-walled rectangular hollow cross-section have been presented and the elastic response of a column segment subjected to compression combined with a bending moment has been studied . Next the analysis technique known as the "Curvature Method" has been used to obtain a solution for the beam-column loaded symmetrically.

2. OUTLINE OF THEORETICAL BACKGROUND

The basic equation of beam-column theory is generally expressed in terms of the lateral displacement of the center line $w(x)$. This approach, known as the "Deflection Method", resolves itself into the differential equation of the fourth order for the deflection curve. Analytical exact solutions in elastic range can be obtained in most cases because the basic relationship between the moment and the curvature is linear. When a beam-column is affected by plate buckling, the solutions cannot be obtained in a similar manner because of high nonlinearity which arises from local buckling.

It has been proved that the curvature of a beam-column can be more useful for the analytical solution than the deflection curve.The analysis technique which involves the curvature is known as the "Curvature Method" and has been developed for elastic-plastic beam-columns [1]. Since the "Curvature Method" requires a moment-curvature-thrust relationship , this has been studied in the paper first and foremost.

3.MOMENT-CURVATURE RELATIONS

Fig. 1 shows a column segment of a rectangular hollow section which is subjected to a bending moment, M, in the plane of symmetry and to an axial thrust P. Considering the deflected configuration of the beam-column, Bernoulli's hypothesis concering plane cross-sections is assumed.

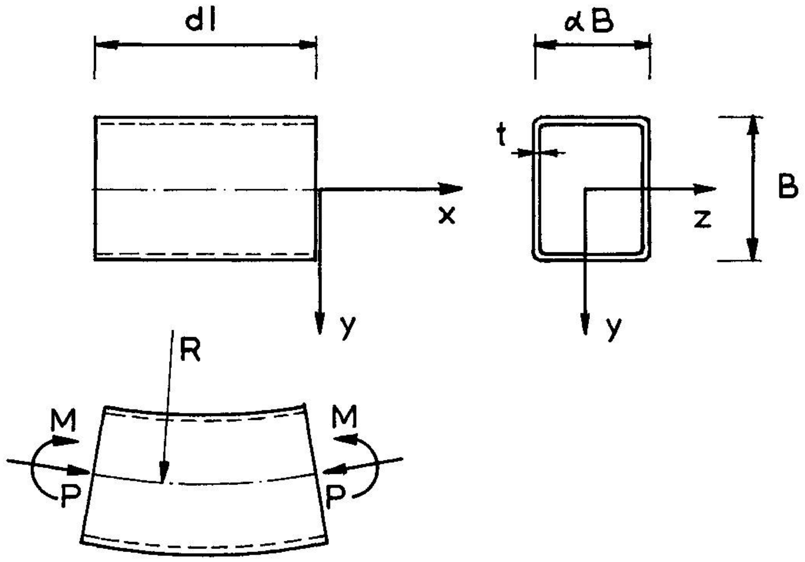

Fig. 1 Compression and bending of a segment

Deflection w of the beam-column is assumed to be of such order that the curvature can be determined by the expression

$$\Phi = - w'' \quad (1)$$

The strain ε in the fiber, which is away from the center line by distance y, can be expressed using the curvature Φ and the mean strain ε_0

$$\varepsilon = \Phi y + \varepsilon_0 \quad (2)$$

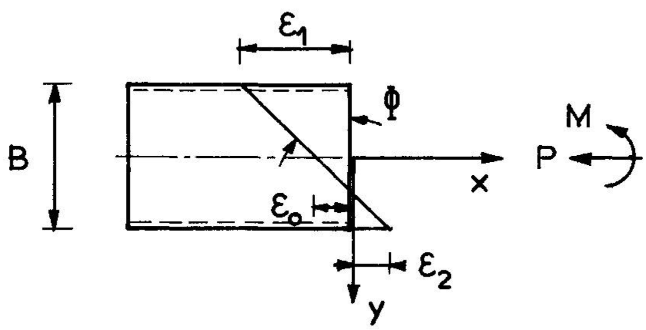

Fig. 2 Strain distribution of a section

The strain distribution is shown in Fig. 2. If the material is ideally elasto -plastic the corresponding stress distribution is obtained directly from Hooke's law

$$\sigma = E (\Phi y + \varepsilon_0) \quad (3)$$

in which E is Young's modulus.

Formulations of the exact moment-curvature-thrust relationship are based on the assumption that von Karman's equation describes the post-buckling behaviour of plate elements. The effective widths B_{eff1} and B_{eff2} are uniquely determined by the edge strains ε_1 and ε_2

$$\frac{B_{eff1}}{B} = \sqrt{\frac{\varepsilon_{cr}}{\varepsilon_1}}, \qquad \frac{B_{eff2}}{B} = \sqrt{\frac{\varepsilon_{cr}}{\varepsilon_2}} \tag{4}$$

in which ε_{cr} is the local buckling strain.

If it is assumed that local buckling occurs only in flanges subjected to a uniform shortening there are three possible modes of the cross-section behaviour as shown in Fig. 3.

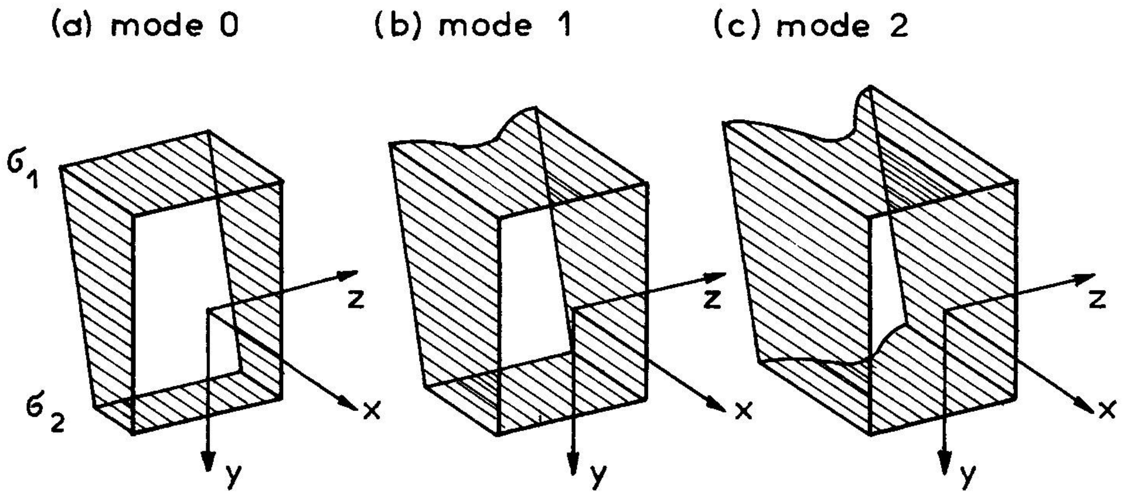

Fig. 3 Modes of the cross-section behaviour

On a cross-section at a distance x, from the end of the beam-column, the generalized stresses or stress resultants are obtained by integrating the axial stress over the entire area of the section, A,

$$\text{Normal Force:} \quad P(\Phi, \varepsilon_o) = \int_A \sigma \, dA \tag{5}$$

$$\text{Bending Moment:} \quad M(\Phi, \varepsilon_o) = \int_A \sigma y \, dA \tag{6}$$

These are the relationships between generalized stresses, P and M, and the generalized strains, Φ and ε_o . Next, the analysis is simplified using nondimensionalized quantities

$$p = \frac{P}{P_{cr}}, \quad m = \frac{M}{M_{cr}}, \quad \varphi = \frac{\Phi}{\Phi_{cr}}, \quad \bar{\varepsilon} = \frac{\varepsilon}{\varepsilon_{cr}} \tag{7}$$

in which the dimensional quantities P_{cr} , M_{cr} and Φ_{cr}, ε_{cr} refer to local buckling of plate elements.

Relationships among the bending moment m, the curvature φ and the axial thrust p are derived in three different regimes 0, 1 and 2 according to three modes of the cross-section behaviour:

Regime 0

$$p = \varepsilon_o \tag{8}$$

$$m = \varphi \tag{9}$$

Regime 1

$$p = \frac{1}{2(1+\alpha)} \left[2\bar{\varepsilon}_o + \alpha\sqrt{\varphi + \bar{\varepsilon}_o} + \alpha(-\varphi + \bar{\varepsilon}_o) \right] \tag{10}$$

$$m = \frac{1}{2(1+3\alpha)} \left[\frac{3}{2}\varphi + \alpha\sqrt{\varphi + \bar{\varepsilon}_o} - \alpha(-\varphi + \bar{\varepsilon}_o) \right] \tag{11}$$

Regime 2

$$p = \frac{1}{2(1+\alpha)} \left(2\bar{\varepsilon}_o + \alpha\sqrt{\varphi + \bar{\varepsilon}_o} + \alpha\sqrt{-\varphi + \bar{\varepsilon}_o} \right) \tag{12}$$

$$m = \frac{1}{2(1+3\alpha)} \left(\frac{3}{2}\varphi + \alpha\sqrt{\varphi + \bar{\varepsilon}_o} - \alpha\sqrt{-\varphi + \bar{\varepsilon}_o} \right) \tag{13}$$

Fig. 4 shows the example of these $m - \varphi - p$ relationships for a beam-column of the square cross-section ($\alpha = 1.0$). The exact nonlinear moment-curvature-thrust relationships cannot be used directly in an analytical solution due to their complicated mathematical form. However the exact $m - \varphi - p$ relationships can be easily approximated by elementary functions.

4. LIMIT STATE, INTERACTION CURVES

The limit state is defined by means of the onset of yielding in the column. The limit state criterion can be expressed in form

$$\sigma_1 = \sigma_y \tag{14}$$

Hence, the yield strength of the material σ_y to the local buckling stress σ_{cr} ratio is indicated as n. The ratio n provides information about the range of the post-buckling

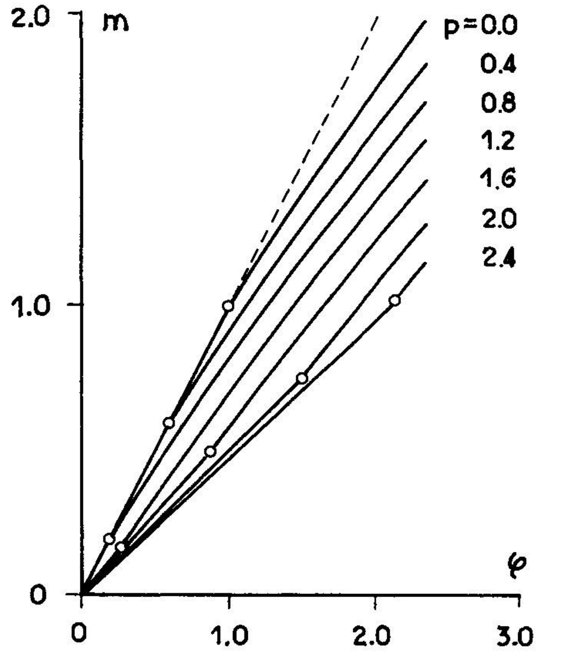

Fig. 4 Moment-curvature-thrust relationships

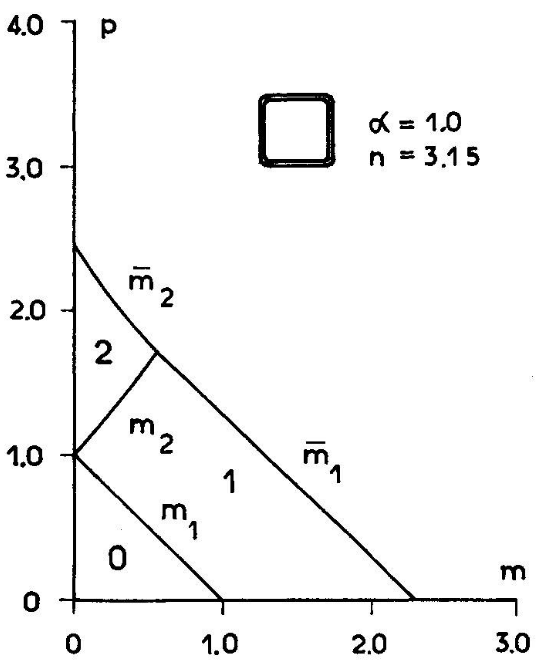

Fig. 5 Interaction curves

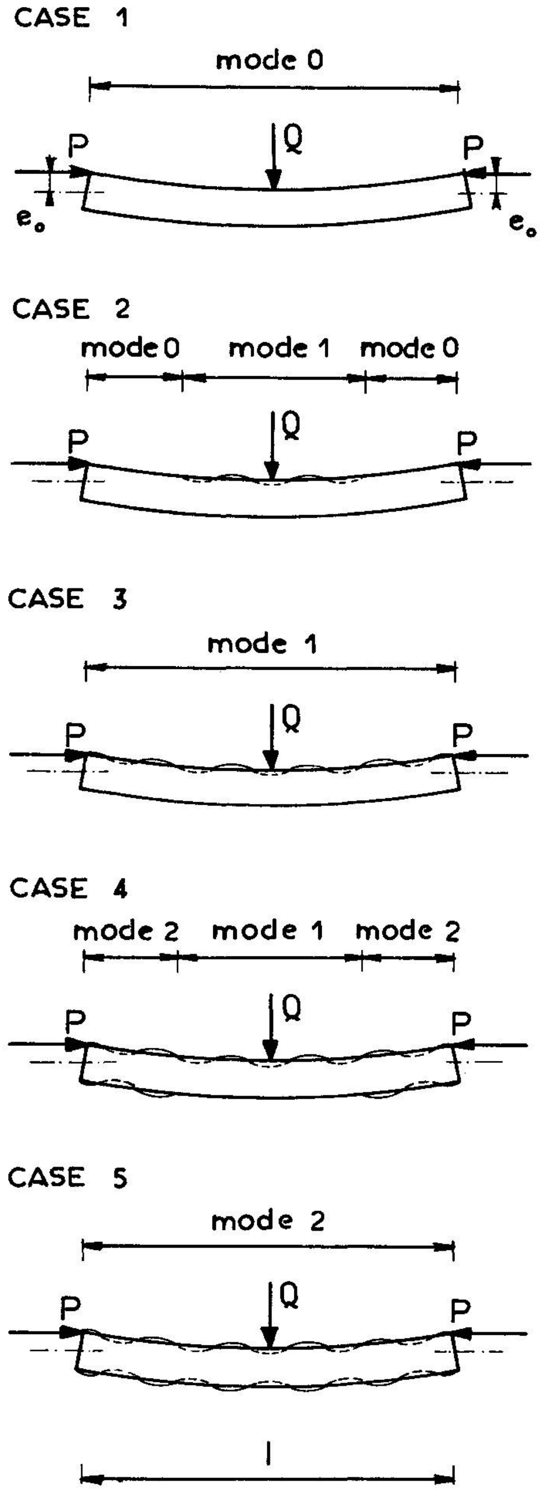

Fig. 6 An eccentrically loaded beam-column (possible cases)

behaviour.

By using $m - \varphi - p$ relationships and the limit state criterion the interaction equations are obtained. Fig. 5 shows the interaction curves.

5. CURVATURE METHOD

The deflection curve $w(x)$ of an elastic beam-column is governed by the following differental equation if there is no lateral load, $q = 0$,

$$M'' - P\,w'' = 0 \tag{15}$$

Substitution of Φ from Eq. (1) into Eq. (15) results in the basic differential equation as follows

$$[M\,(\Phi,\,P)]'' + P\,\Phi = 0 \tag{16}$$

Combining the equilibrium equation with the $m - \varphi - p$ relationships the basic differential equation for the curvature Φ is obtained.

The concave or both flanges may be buckled locally. Depending on the extent of the local buckling zones, the state of the beam-column is categorized into five possible cases as shown in Fig. 6.

The boundary conditions at the ends of the column and continuity conditions at the boundaries between two adjacent regimes are needed. Since the location of the boundaries are not known, the formulation of these conditions cannot be stated in a routine way. The jump conditions for the derivate of the curvature must be obtained for the determination of the integration constants.

If the $m - \varphi - p$ relationships are approximated by linear functions the solution has a closed form.

5. EXAMPLE OF ANALYSIS

An eccentrically and symmetrically compressed beam-column is studied herein as the particular case of the general solution. Since the analytical $m - \varphi - p$ relationships are linearized, the closed-form solution has a simple form. The results obtained from the analysis are valid for a wide range of cross-section proportions due to the nondimension form.

Fig. 7 shows the relationships between the slenderness ratio and the limit load for the beam-columns of the square hollow section ($\alpha = 1$). The ratio n defines properties of the material and proportions of the cross-section.

The eccentricity ratio e/r = 0.05 is assumed to obtain the limit loads for various values of the slenderness ratio l/r. In Fig. 7 the curve with the eccentricity ratio e/r = 0.05 is compared with the exact curve determined for beam-colums of the same parameters but not affected by local buckling (n = 1.0). The computed results show a very significant influence of local buckling on the limit state of thin-walled steel columns of small and medium slenderness ratios.

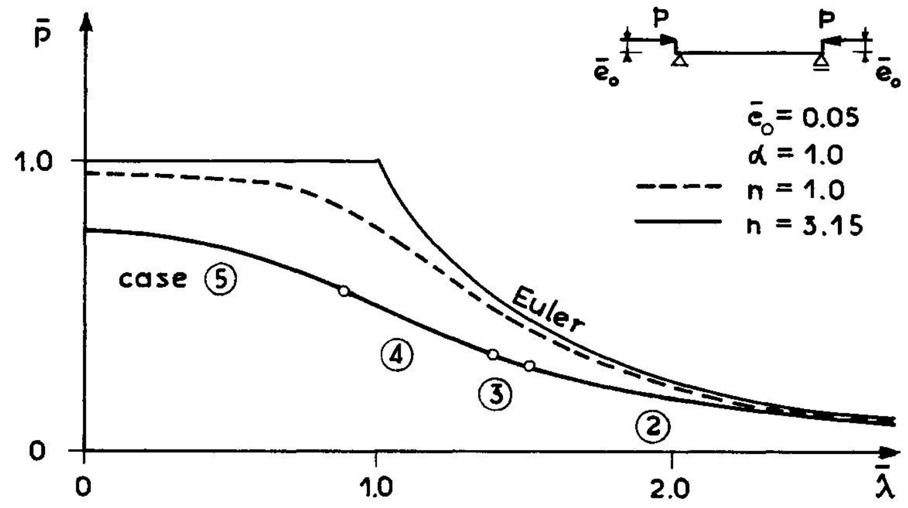

Fig. 7 The effect of local buckling, illustrated by a comparison of the two curves n=3.15 and n=1.0

6. SUMMARY AND CONCLUSIONS

The Curvature Method, applied for the analysis of thin-walled beam-columns affected by local buckling, permits to solve one of the most difficult problems in the field of structural stability. This method enables designers to assess the limit load of these colums in a simple manner. It is quite general in its applicability to any cross-sectional shape though only the rectangular hollow cross-sections have been considered in this paper.

The moment-curvature-thrust relationships, which play the key role in the study, have been based on the von Karman's expression. The modification to the expression for the effective width and the consideration of local buckling of webs under stress gradient will make for greater accuracy.

REFERENCES

1. CHEN W. F. and ATSUTA T., Theory of Beam-Columns. Vol 1 - In-Plane Behaviour and Design, McGraw-Hill, New York, N.Y., 1976.