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Combined Axial Load and Bending in Cold-Formed Steel Members

Profilés formés à froid soumis à l'action combinée
de la compression axiale et de la flexion

Kaltverformte Stahlkonstruktionselemente unter Biegung mit Normalkraft

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Prof. Pekoz obtained his Ph.D. degree in 1967 at Cornell University where he has been involved in research on thin walled members since 1963. He has worked as a visiting Professor in Sweden and the Netherlands. He is active in consulting internationally as well as in the preparation of the American Iron and Steel Institute Specifications and the European Recommendations on Cold-Formed Steel Members.

SUMMARY

A general design approach for combined axial load and biaxial bending of singly and doubly symmetric open and tubular section is developed. The result is a unified design approach for the treatment of several problems. These include the treatment of plate elements, columns, beams and beam columns. The approach covers sections with plate elements that are locally stable as well as those in the post-local buckling range at overall failure. The overall failure modes include flexure and torsional-flexure. The approach developed was checked on the basis of analytical approaches and parametric studies. The results of 260 carefully conducted tests were also used in the verification of the proposed approach.

RÉSUMÉ

On développe une approche générale du dimensionnement de sections ouvertes et fermées à simple et double symétrie soumises à une charge axiale et à une flexion biaxiale. Le résultat en est une méthode unifiée pour le traitement de divers éléments tels que les plaques, les colonnes, les poutres et les poutres-colonnes. Cette approche couvre les sections comportant des éléments plans stables localement aussi bien que des éléments voilés localement lors de la rupture globale, qui se produit par flexion ou par déversement. L'approche développée a été vérifiée analytiquement et par étude paramétrique. Les résultats de 260 essais ont également été utilisés pour cette vérification.

ZUSAMMENFASSUNG

Eine Berechnungsmethode für zweiachsige Biegung mit Normalkraft von einfach- und doppelt-symmetrischen, offenen und rohrförmigen Querschnitten wurde entwickelt. Das Ergebnis ist eine einheitliche Methode für die Behandlung von verschiedenen Problemen (z.B. Platten-elemente, Stützen und Träger), die Querschnitte mit Plattenelementen umfasst. Die Versagens-zustände schliessen einfache Biegung und Biegedrillknicken ein. Die neue Methode wurde anhand von analytischen und parametrischen Studien überprüft. Zusätzlich wurden Mess-ergebnisse von 260 Versuchen für die Überprüfung der Methode ausgewertet.



This study deals with thin-walled doubly and singly symmetric open and tubular sections subjected to biaxial loading. Though singly symmetric open section cold-formed steel members are frequently subjected to biaxial loading, general design provisions do not exist. The design problem is often complicated because the plate elements making up such sections may buckle locally below loads causing overall failure. The subject is relevant to several practical applications including thin walled square or rectangular tubes, end wall columns in metal buildings, many typical industrial storage rack columns and purlins in the end bays of metal buildings.

The author has formulated [1] a unified design approach for beams, columns and beam-columns based on the data and conclusions reached in several research projects reported in [2] through [8] carried out by him and his collaborators. This approach also includes a unified treatment of plate elements making up such members. Due to length limitations, this paper will present only a brief summary of the details and supporting evidence reported in [1]. In each phase of the research, rigorous analytical models were developed and compared with the results of experiments conducted within the project as well as those conducted elsewhere. The results of tests on 51 locally stable columns and beam-columns, 102 locally unstable beams and 107 locally unstable column and beam-columns were used in this process [1].

The proposed unified design approach is being considered for the 1986 Edition of the AISI Specification for the Design of Cold-Formed Steel Structural Members and the ECCS European Recommendations for the Design of Light Gauge Steel Members.

BEHAVIOR OF PLATE ELEMENTS

The post-buckling behavior of plate elements is represented by a generalization of the Winter effective width equation [9]. The Winter equation was derived for plate elements supported adequately along the two longitudinal edges. This concept has been extended in [3] through [6] to unstiffened elements (element supported only on one longitudinal edge) and to elements with an edge stiffener or an intermediate stiffener of any size.

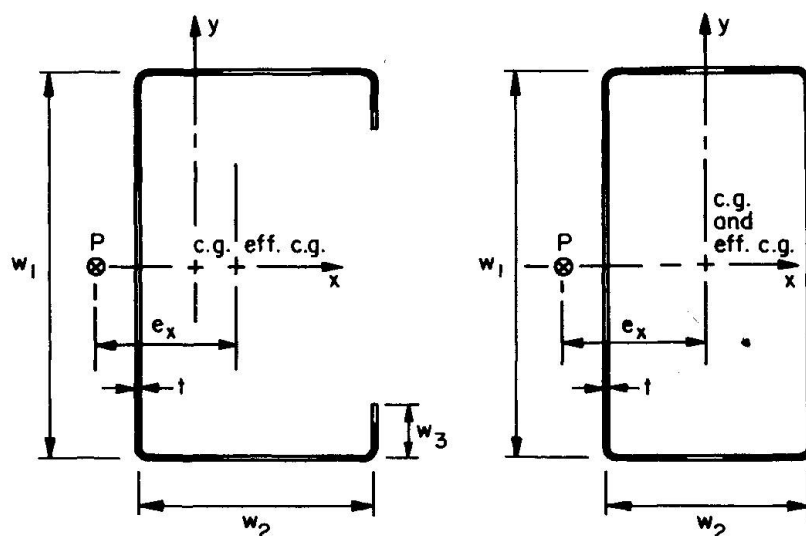


Fig. 1 Section geometry and generalized effective section

The expressions developed in [3] through [6] are the basis of the effective section properties calculated in the proposed approach with the exception of using the actual, rather than the effective moment of inertia in the equations of [5] to assess the stiffener adequacy. This gave improved results. A new approach for webs is also developed [1]. The cross-sectional geometry notation and the generalized effective section for a C and a tubular section is illustrated in Fig. 1.

LOCALLY STABLE BEAM COLUMNS OPEN SECTION

For sections with fully effective plate elements, the studies by the author [10] show that interaction equations can be used. This approach was adopted in the RMI Specification [11]. The validity of the approach was further confirmed in [8] on the basis of extensive analytical and experimental studies. The interaction equation is

$$\frac{P}{P_o} + \frac{M_x C_{mx}}{M_{xo} (1 - P/P_x)} + \frac{M_y C_{my}}{M_{yo} (1 - P/P_y)} = 1 \quad \text{Eq. 1}$$

P , M_x and M_y are the axial force and the moments about the x and y axes, respectively, due to the applied loading. P_o is the failure load in the absence of any moment. M_{xo} is the failure moment for bending about the x axis in the absence of an axial load or bending about the y axis. Similarly, M_{yo} is for the bending about the y axis. P_o , M_{xo} and M_{yo} are determined considering both flexural and torsional flexural buckling. C_{mx} and C_{my} are corrections to reflect the moment gradient in the member. P_x and P_y are the flexural buckling loads about the x and y axes, respectively.

INTERACTION OF LOCAL AND FLEXURAL COLUMN BUCKLING

The effect of local buckling on overall buckling behavior has been studied in several research projects at Cornell and elsewhere. On the basis of tests and analytical studies, [2] and [3] conclude that a satisfactory approach is to calculate the overall buckling load using the effective radius of gyration and the effective area, both calculated at the overall buckling stress. This results in an iterative procedure since the buckling stress depends on the effective section properties which in turn depend on the buckling stress. The iterative approach has been extended in [8] to the treatment of torsional flexural buckling.

The approach of [12] for flexural buckling is tried in [1] for a variety of sections and failure modes. This approach is very similar to the one proposed by the author [10] and adopted in the RMI Specification [11] for the treatment of perforated columns and beam columns subject to torsional flexural buckling. The buckling stress is found for an unperforated column and the allowable load is found by multiplying this stress by the net area.

The proposed approach consists of the following steps. First the elastic flexural buckling stress, F_e , is calculated for the full unreduced section:

$$F_e = \pi^2 E / (KL/r)^2 \quad \text{Eq. 2}$$

Then the failure stress, F_u , is determined:

$$F_u = F_e \quad \text{if } F_u < F_y/2 \quad \text{Eq. 3}$$

$$F_u = F_y (1 - F_y/4F_e) \quad \text{if } F_u > F_y/2 \quad \text{Eq. 4}$$



and the ultimate column load P_u is calculated as

$$P_u = A_e F_u \quad \text{Eq. 5}$$

where A_e is the effective area computed at stress F_u .

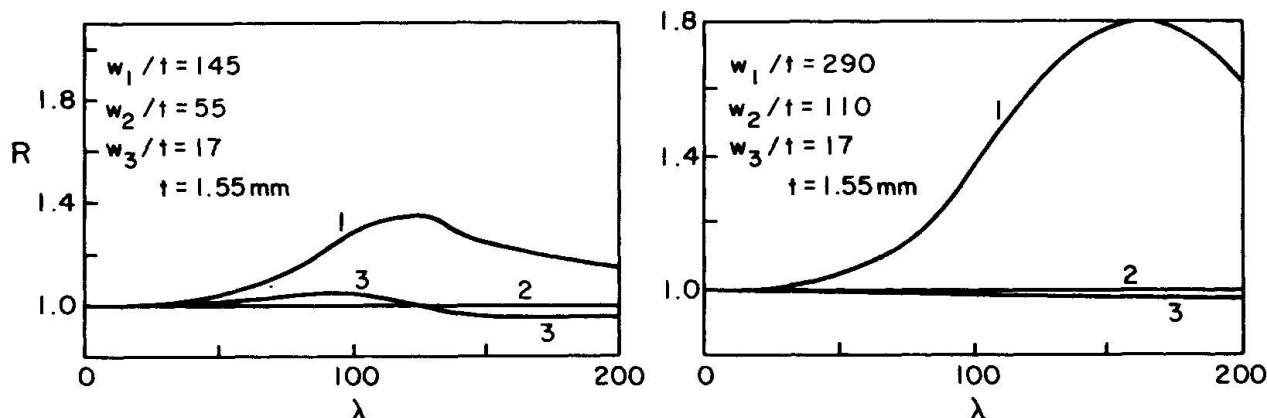


Fig. 2 Interaction of local and overall buckling - C sections ($F_y/E = .0017$)

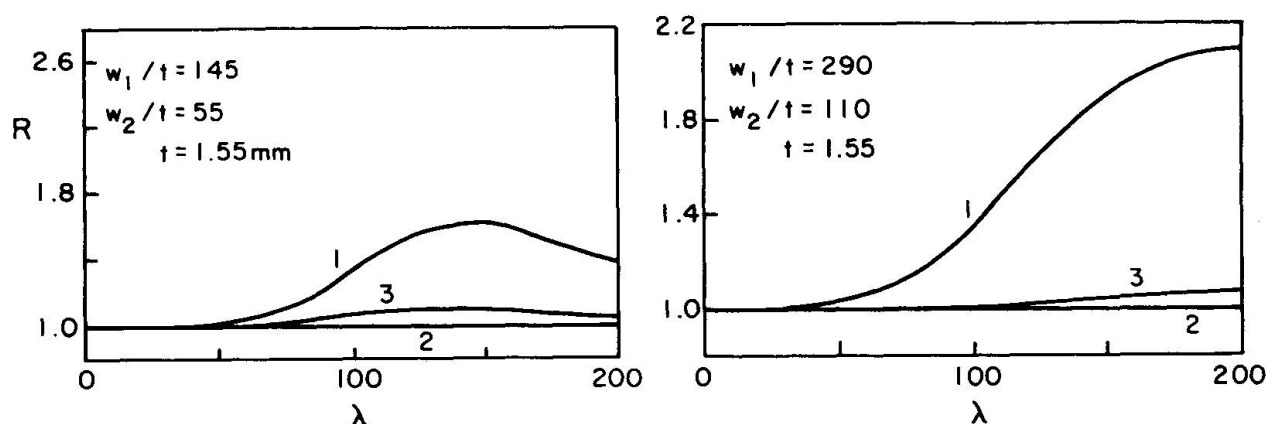


Fig. 3 Interaction of local and overall buckling - tubes ($F_y/E = .0017$)

The studies summarized in [1] show that proposed approach approximates the iterative approach very closely for flexural, torsional flexural and lateral buckling as discussed below. Figures 2 and 3 give some typical examples of the many comparisons presented in [1]. In these figures Curve 1 is for an approach using Eqs. 2, 3 and 4 taking the section to be fully effective and multiplying the yield stress by the ratio A_{eu}/A where A_{eu} is the effective area for yield stress and A is the full area. P_u is taken as $A F_u$. Curve 2 is for the iterative approach which is the best fit to the test data. Curve 3 is obtained with the proposed unified approach. R is the ratio of the P_u obtained by a particular approach to that obtained by the iterative approach. λ is the slenderness ratio KL/r_y .

The remarkable accuracy of the proposed approach can be explained as follows. The reduction in the value of the radius of gyration resulting from local buckling is rather small. For small slenderness ratios where the column buckling stresses are high compared to the yield stress, the buckling stress is quite insensitive to the changes in the radius of gyration. For small stresses, namely large slenderness ratios, the local buckling is not significant. However, the effective area gets influenced directly and significantly by local buckling. Therefore the behavior is well represented by ignoring the change in the radius of gyration and accounting for the reduction in the effective area in finding the ultimate load of the column.

For locally buckled C and other singly symmetric sections, concentric axial loading with respect to the centroid of the effective section is not typical in structures. The centroid of the effective section depends on the magnitude of loading. The location of the centroid moves as the load is increased. The allowable concentric loading is important as a parameter in the interaction equation.

INTERACTION OF LOCAL AND TORSIONAL-FLEXURAL COLUMN BUCKLING

An analytical model for the behavior of locally unstable open sections is developed in [8] on the basis of the torsional flexural theory for the effective section. The theory was confirmed by correlation with test results. As in the case of flexural buckling, the approach involves iterations. Again concentric buckling load is important for a locally buckled section only as a parameter in the interaction equation.

The proposed approach for the torsional flexural buckling of locally buckled columns is exactly the same as that for columns subject to flexural buckling. In the equations 2 through 5 above, only the determination of F_e changes. F_e is determined according to the torsional flexural buckling theory for the full unreduced section.

EVALUATION OF STUB COLUMN TEST RESULTS

The proposed approach necessitates an expression for the effective area A_e as a function of the stress on the effective area f . The stress f is taken as F_u in calculating column strength. When A_e cannot be calculated, such as when the column has dimensions or geometry outside the range of applicability of the generalized effective width equations, a functional relation between f and A_e can be obtained by stub column tests. A stub column is a short column that is long enough to reflect the local buckling behavior but preferably short enough so that the behavior is not affected by the overall buckling. The effective area, A_{eu} , at ultimate load, P_u is:

$$A_{eu} = P_u / F_y \quad \text{Eq. 6}$$

The effective area at any stress on the effective area f can be calculated as follows [1]:

$$A_e = A - (A - A_{eu}) (f / F_y)^{(A_{eu} / A)} \quad \text{Eq. 7}$$

where A is the full unreduced area of the section.

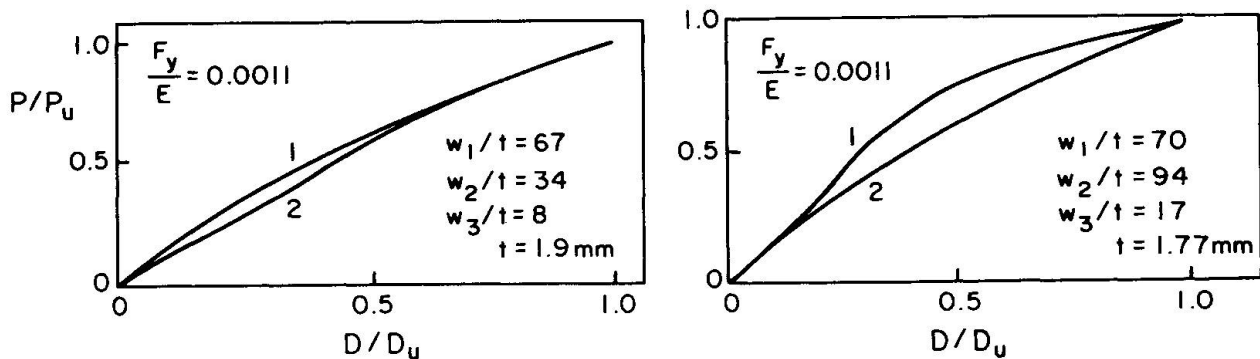


Fig. 4 Evaluation of stub column test results



The validity of the above equations is verified in [1] by a rather extensive parametric study. Many plots as those given in Fig. 4 are presented in [1]. In these figures D is the axial shortening of the stub column at an axial load P . D_u and P_u are the ultimate values of D and P . Curves 1 are based on actual tests. Curves 2 are calculated on the basis of Eqs. 6 and 7. It is seen that the equations are satisfactory and give conservative (low) values of the axial stiffness and consequently the value of A_e .

Approaches for determining an expression for A_e versus f from the measured axial shortening and for the treatment of the case when the stub column is not short enough are formulated in [1].

INTERACTION OF LOCAL AND LATERAL BEAM BUCKLING

The approach proposed in [1] for this case is consistent with the one proposed for columns. First, the elastic lateral buckling stress, F_e , is calculated on the basis of the torsional flexural buckling theory for the full unreduced section using the equations of [13]. Then the failure stress F_u is determined using Eqs. 3 and 4. The lateral buckling moment is determined by multiplying F_u by the effective section modulus calculated for an outer fiber stress of F_u .

The proposed design approach gives results virtually identical with those of the analytical approach developed in [8] on the basis of torsional-flexural buckling theory. There is no direct test data on the lateral buckling of cold-formed steel beams. However some data exists on the behavior of sections with eccentric axial loading. These test results show that the proposed approach is satisfactory [1].

BIAXIALLY LOADED LOCALLY UNSTABLE BEAM COLUMNS

The interaction equation given above was studied extensively and extended to locally unstable sections in [8] and [1]. The approach of [1] involves the use of Eq. 1 for singly or doubly symmetric open sections and closed tubes with some of the terms redefined to account for locally buckled plate elements. P_0 is determined as described above for locally unstable columns. It may be governed by flexural or torsional flexural buckling. M_{x0} and M_{y0} are determined by the approach described above for lateral buckling. All eccentricities (for example e_x in Fig. 1) are taken with respect to the centroid of the effective section for the axial load alone. The parameters P_x and P_y are the elastic buckling loads for the full unreduced section.

The proposed formulation is confirmed in [1] by theory and 107 tests on simply supported, locally unstable C, channel and hat section beam columns. Correlation for angle and lipped angle sections is needed. The extension of the use of the interaction equations for frames is discussed in [1].

An example of the correlation with the test results is illustrated in Fig. 5. This figure presents the results of all the tests with loads with uniaxial or biaxial eccentricities. The figure on the left illustrates the presentation of the results. In this figure R_p , R_x and R_y represent the first, second and the third terms of Eq. 1. Eq. 1 defines the plane ABC. For a given test, the observed values of P , M_x and M_y are substituted into the equation and a point with the resulting R_p , M_x and M_y values is plotted. The results that fall outside the volume OABC indicate that the proposed interaction equation is conservative for those cases. This three-dimensional situation is represented in the figure on the right in two dimensions by plotting the projections of the test points on the R_p - R_0 plane. Thus from geometry R_0 is equal to $.707 (R_x + R_y)$. The points that fall outside the area OAD in the figure on the right show that the Eq. 1 is conservative. The few points that fall within this area have been mostly explained in [1] and the approach is judged satisfactory.

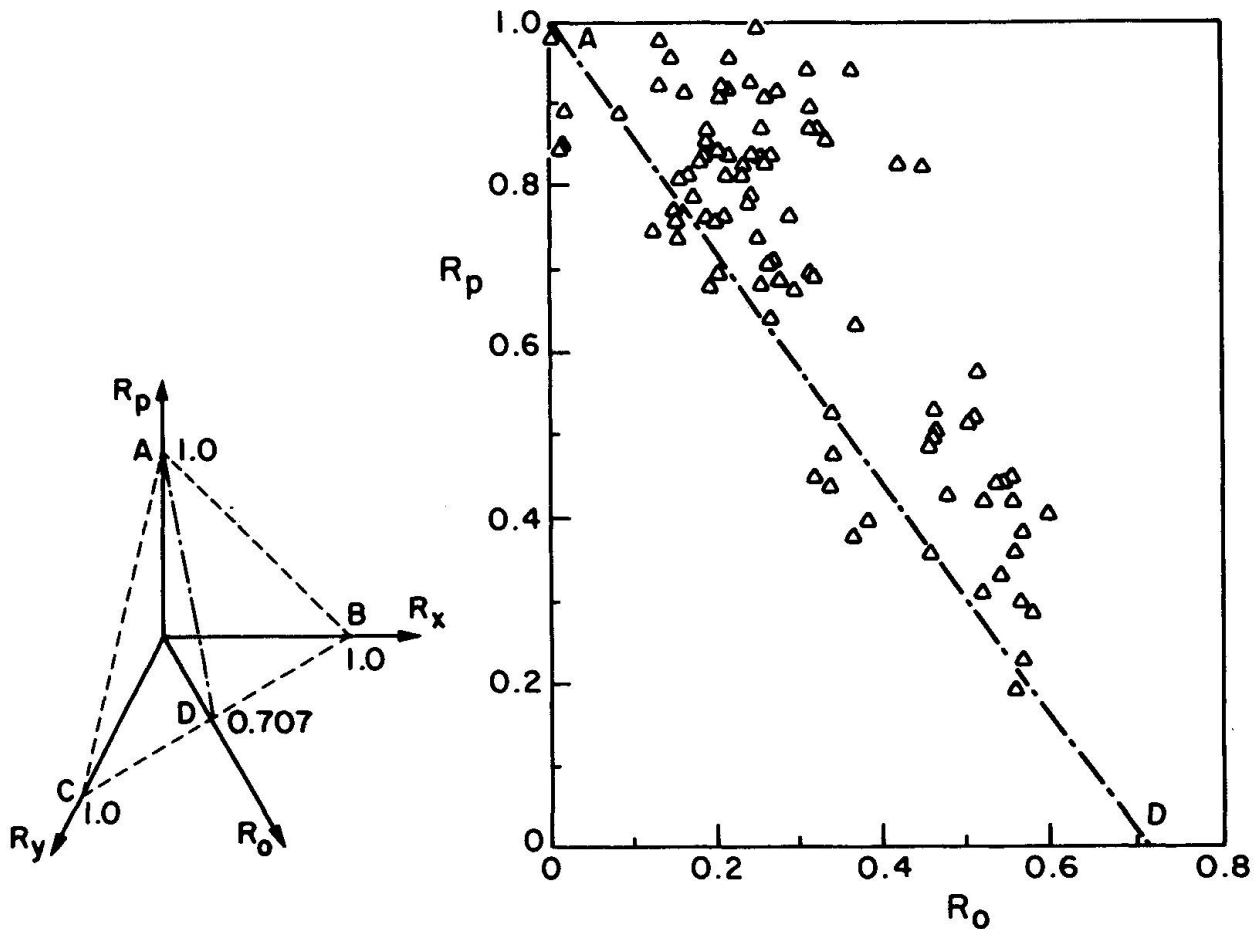


Fig. 5 Correlation of beam column test results with Eq. 1

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