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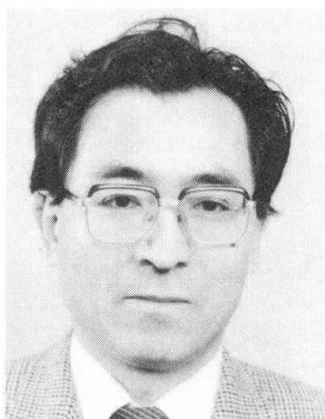
## Rational Analysis of Shear in Composite Columns

Etude systématique du cisaillement dans les colonnes mixtes

Beweisfähige Untersuchung des Schubs in Verbundstützen

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### SUMMARY

An analytic procedure is developed for calculating the ultimate shear capacity of composite (steel-reinforced concrete) columns. The contributions to member strength of various load carrying mechanisms, such as truss actions, arch actions, and those actions which develop in the steel profile and in the cover concrete and core concrete are evaluated independently and then added according to the superposition method. Closed form equations are obtained to define various regions of the failure interaction curve for combined forces. Theoretical predictions show good agreement with available test data.

### RÉSUMÉ

Une procédure analytique est développée pour calculer la résistance ultime à l'effort tranchant d'une colonne mixte (acier et béton armé). Plusieurs modèles sont testés, tels que triangulée de remplacement, effet d'arc, calcul séparé de la résistance des composants (profilé métallique, noyau de béton armé, recouvrement) puis sommation, en vertu du principe de superposition. Des équations précises sont obtenues pour définir les différents domaines des courbes d'interactions donnant les combinaisons d'efforts ultimes.

### ZUSAMMENFASSUNG

Ein analytisches Verfahren zur Berechnung der Schubgrenztragfähigkeit von Verbundstützen wird entwickelt. Die Beiträge von verschiedenen Tragmechanismen, wie Fachwerk- und Bogenwirkung sowie internen Wirkung in den Stahlprofilen und im Beton werden unabhängig voneinander ermittelt und anschliessend nach der Überlagerungsmethode zur Festigkeit des Tragelements addiert. Geschlossene Formeln werden erhalten, um verschiedene Bereiche der Bruchinteraktionskurve für kombinierte Lasten zu definieren. Theoretische Voraussagen zeigen eine gute Übereinstimmung mit den Versuchsergebnissen.



## 1. INTRODUCTION

Most of the design equations provided for estimating the ultimate shear strength of composite structural components were derived empirically, and, therefore, are limited in their applicable range. In fact, there was no formula in which interaction between the axial force, flexural moment and shear force is expressed in a unified manner. These days, we can find studies in which the ultimate shear strength of reinforced concrete and composite structural components is computed based on the concept of the theory of plasticity (1, 2, 3).

Using this concept, the writers developed analytical procedure to estimate the ultimate shear strength of reinforced concrete structural components and proposed relatively simple shear design formulas (4, 5). In this paper, the procedure is extended so that the ultimate shear strength of composite structural components, as shown in Fig. 1, can also be estimated. The basic procedure is explained, and its effectiveness is demonstrated by comparing the results obtained from this analytical procedure with previous test results.

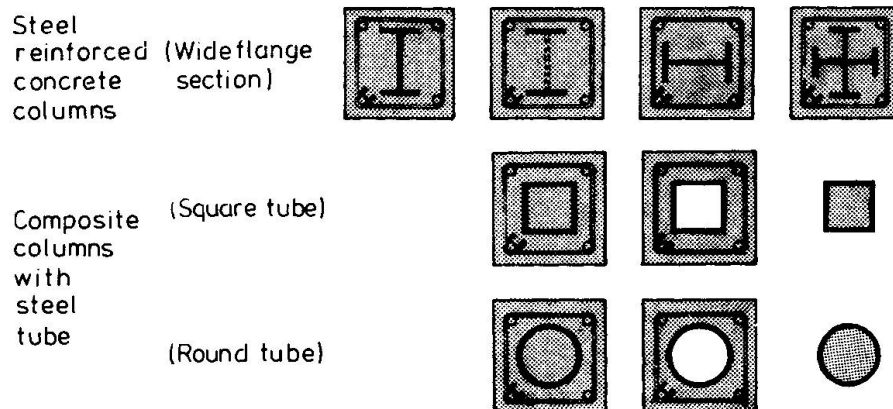


Fig.1 Classification of composite columns

## 2. SHEAR RESISTING MECHANISM

The writers proposed in Ref. 4 that the shear resistance of a reinforced concrete structural component be taken as the sum of the resistances of two shear resisting mechanisms: that is, the beam mechanism and the arch mechanism. For a composite structural component (a reinforced concrete component with embedded H-shaped steel as shown in Fig. 2), two additional mechanisms are assumed also to contribute to the shear resistance. They are the shear resistance of concrete placed both inside and outside of the steel flanges (Fig. 2(c) and (d)) and the resistance of steel (Fig. 2(e)).

As, in such a composite structural component, the bond strength acting in interfaces between the concrete and steel is small, and the bond stress cannot be significant particularly after the component receives large deformation, the bond is assumed not to contribute to the shear resistance of the component. In Fig. 2, the portion having the effective width of  $b'$  is assumed to act as an reinforced concrete component. Here,  $b'$  is given as the total width subtracted by the steel flange width,  $b_f$ . Since the concrete of the remaining part is separated by the steel flanges, the concrete is divided into three portions, each assumed to form its own arch mechanism(theory 1). In the study of Ref. 2, the three portions were treated as one element(theory 2). In composite structural components having usual cross sectional configuration, however, the shear resistance of the concrete having the width of  $b_f$  is small as

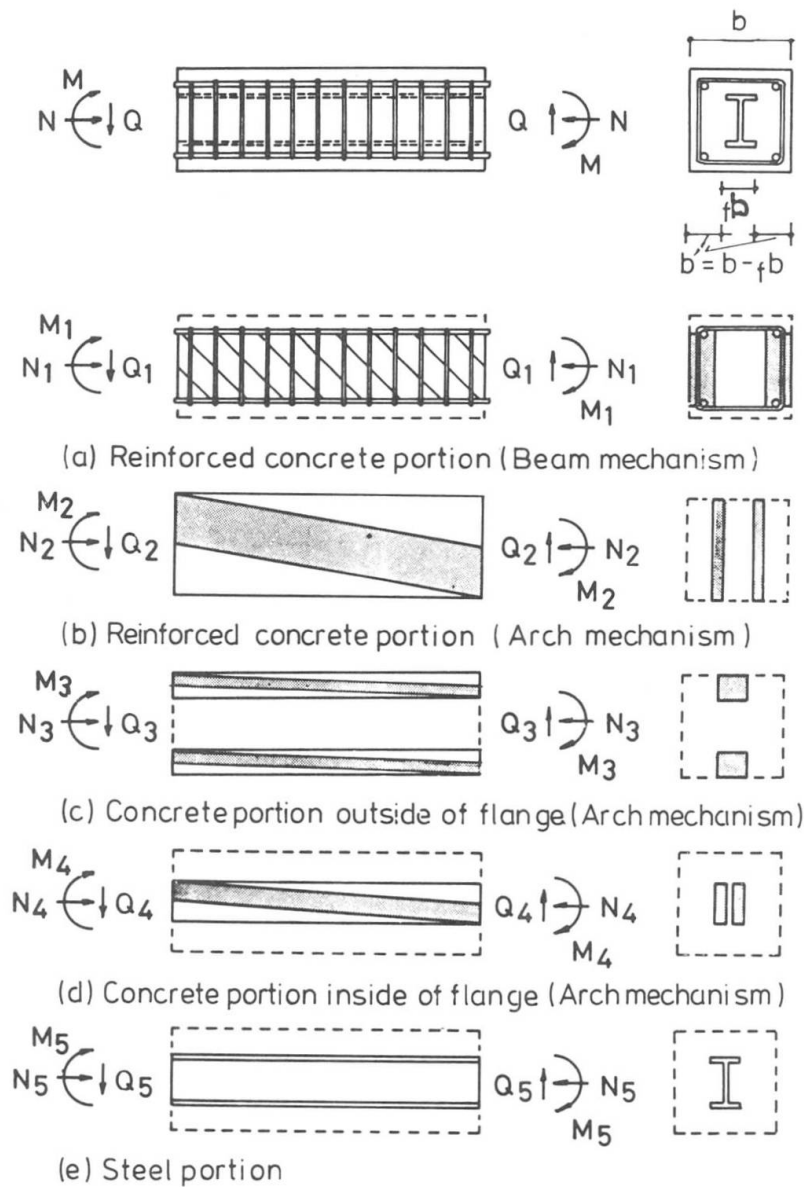


Fig.2 Resistance mechanism of composite columns with wide flange section

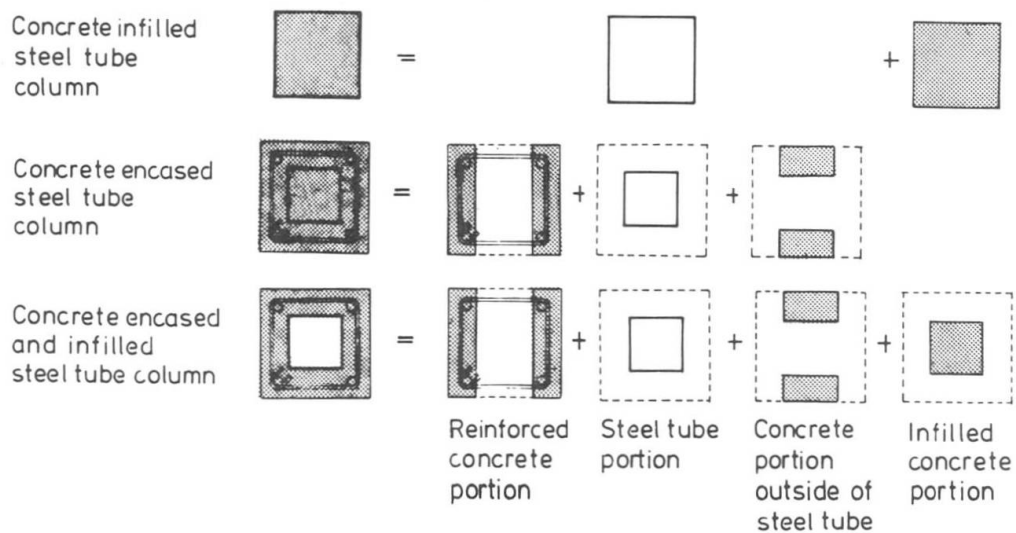


Fig.3 Physical model of stress transfer of composite columns with steel tube

compared to the resistances of other portions. Then, the neglect of this concrete resistance changes the total shear strength estimate very little.

The same procedure can be applied to other types of composite structural components. Figure 3 shows the resisting mechanisms of a concrete infilled steel tube, a concrete encased steel tube, and a concrete infilled and encased steel tube component.

### 3. APPLICATION OF STRENGTH SUPERPOSITION CONCEPT

Suppose that the strength combination ( $M$ ,  $N$ ,  $Q$ ) satisfying the statically admissible stress field is determined for individual resisting mechanisms, the total resistance of the composite structural component is given, using the strength superposition concept, as:

$$\begin{aligned} M &= \sum M_i \\ N &= \sum N_i \\ Q &= \sum Q_i \end{aligned} \quad (1)$$

If the inflection point is assumed to be located in the mid-span of the component:

$$M/Q = M_i/Q_i = l/2 \quad (2)$$

Where  $l$  is the length of the component.

### 4. INTERACTION EQUATIONS OF RESISTING MECHANISMS

#### 4.1 Interaction Equations of Reinforced Concrete Portion

The interaction of the nondimensionalized axial and shear forces at the reinforced concrete portion is shown in Fig. 4, where the interaction curve comprises seven regions. The interaction equations of those regions is given in Ref. 4, with the reading of  $b$  as  $b'$ .

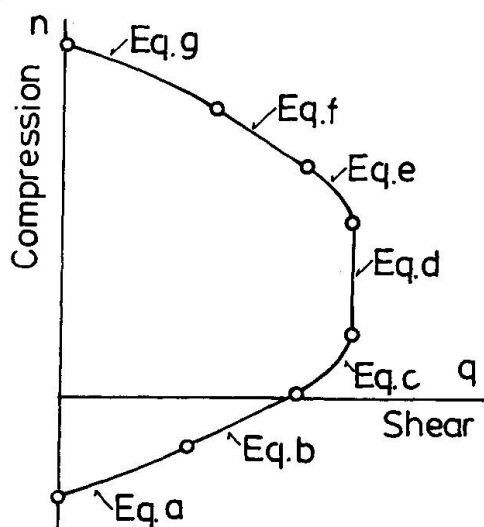


Fig.4 Schematic example of the nondimensionalized axial and shear interaction curves for the reinforced concrete portion

#### 4.2 Interaction Equations of Concrete Portion Having the Steel Flange Width

The interaction equation of the concrete placed inside the steel, the part assumed to form the arch mechanism, can be derived in the same fashion as that of the arch mechanism in the reinforced concrete portion (Fig. 2(b)). On the other hand, the concrete portion outside the steel flanges is assumed to resist only the axial compression since the depth of this portion (measured in the transverse direction) is significantly small relative to its length.

#### 4.3 Interaction Equation of Steel Portion

If the steel stress strain relationship is assumed elasto-plastic, the interaction equation of the steel portion can readily be obtained by employing an appropriate yield criterion.

### 5. INTERACTION CURVES OF COMPOSITE STRUCTURAL COMPONENT

Nondimensionalized axial and shear force interaction equations of the individual resisting mechanisms can be assumed to form interaction equations of the composite structural component. An example of the interaction curve obtained in this manner is shown in Fig. 5. The interaction curve can be expressed with a series of interaction equations (A-K of Fig. 5), each equation representing certain region of the curve. In Fig. 6, interaction curves drawn by using this analytic procedure are compared with some of the previous test results. In this figure, solid lines and chained lines denote the theoretical strength based on theory 1 (concrete is divided into three portions) and the theoretical one based on theory 2 (the three portions are treated as one element). The difference between those curves is found to be minimal. The analysis shows that there is a range in which the shear strength becomes constant regardless of the value of the axial force.

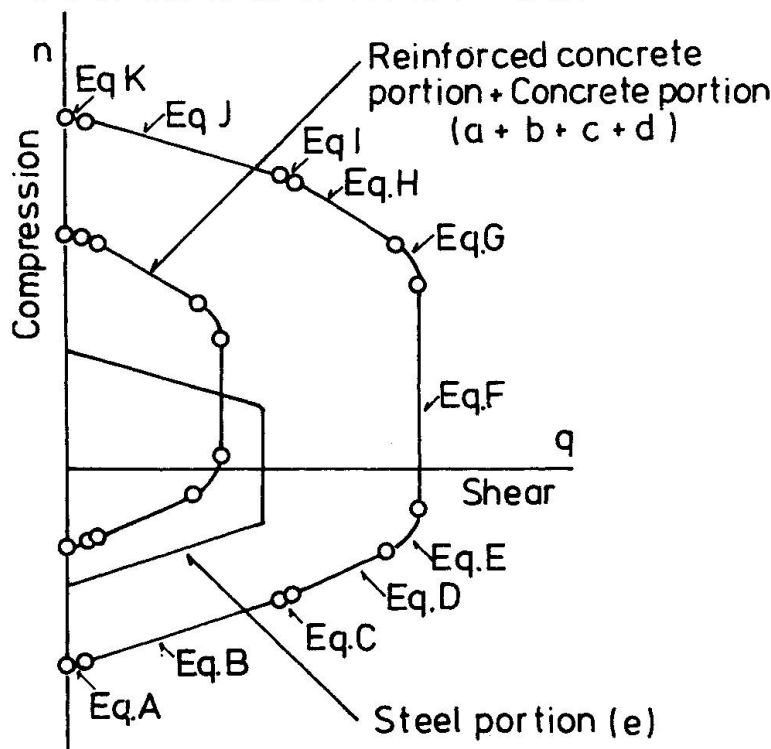
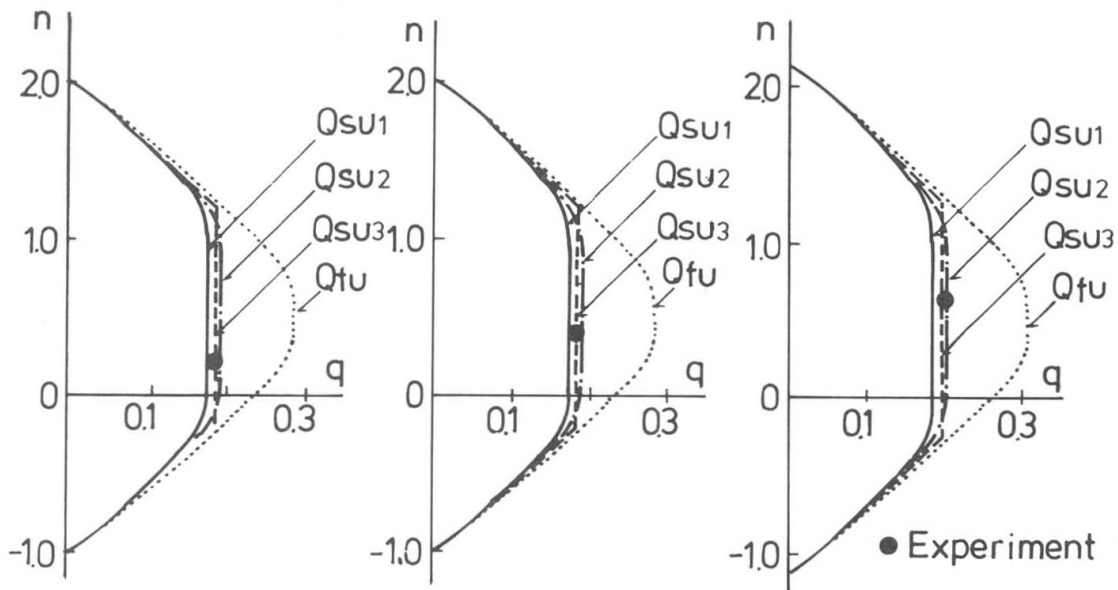


Fig.5 Schematic example of nondimensionalized axial and shear interaction curves



$q_{fu}$  : Flexural strength

$Q_{su1}$  : Shear strength based on theory1 (proposed theory)

$Q_{su2}$  : Shear strength based on theory2

$Q_{su3}$  : Shear strength obtained by AIJ standard (Ref. 9)

Fig.6 Comparison of theoretical prediction with test results for selected example of composite columns with wide flange section (Ref. 6)

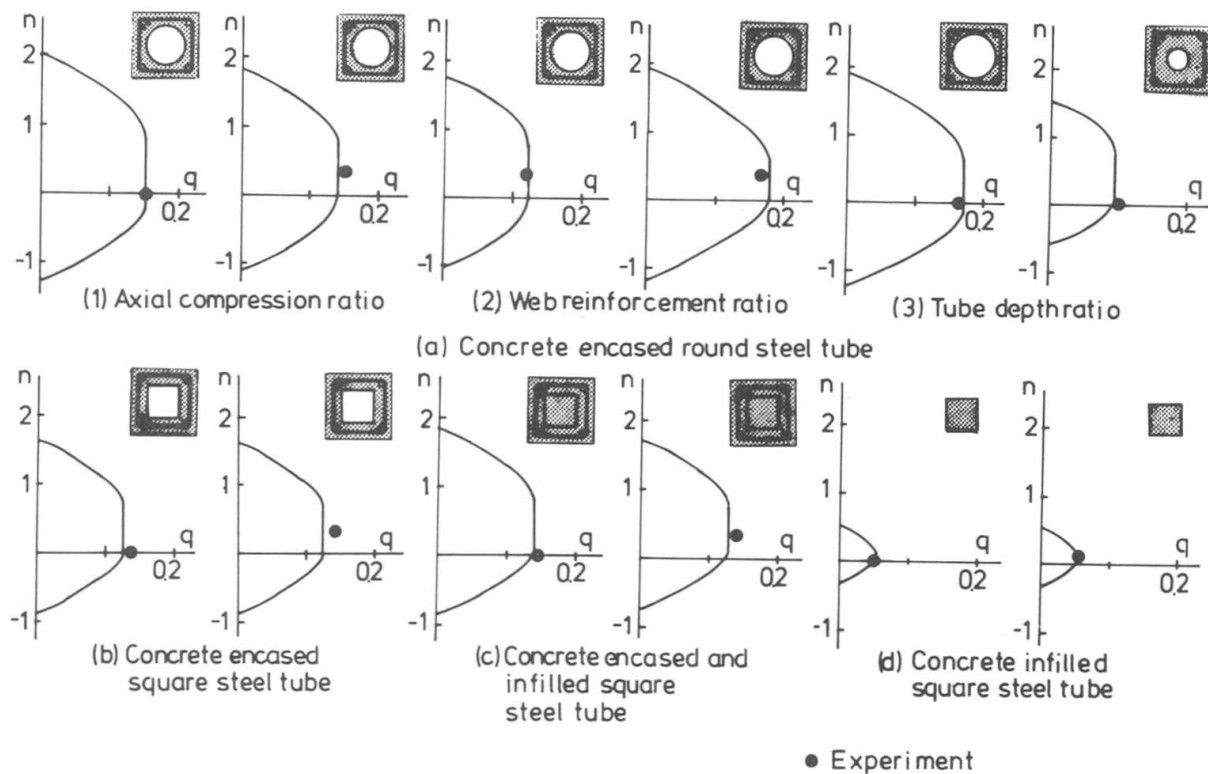


Fig.7 Comparison of theoretical prediction with test results for selected example of composite column with steel tube (Ref. 7)

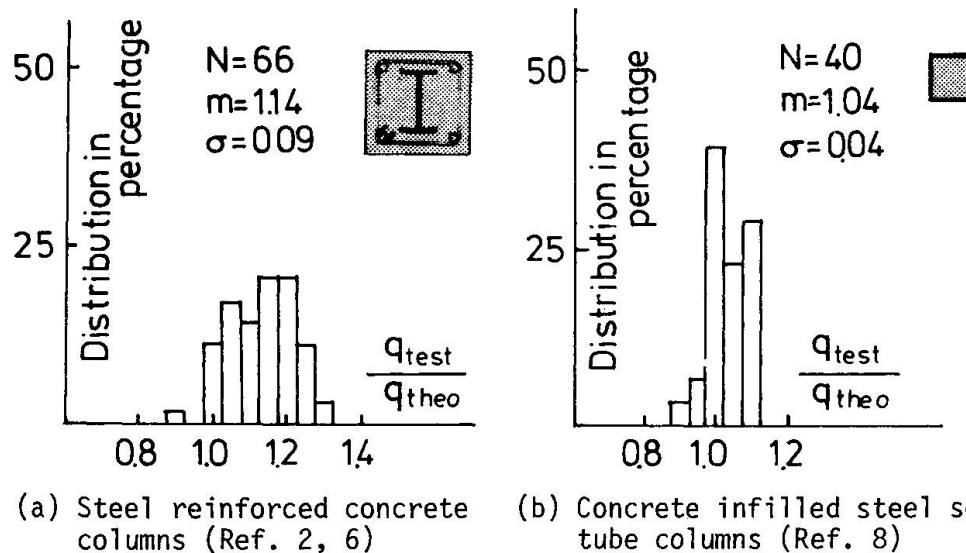


Fig.8 Histogram of the ratios of test values to theoretical value

## 6. COMPARISON BETWEEN THE ANALYTICAL INTERACTION CURVES AND EXPERIMENTAL RESULTS

Figure 7 shows the comparison of analytical interaction curves with results obtained from the test of composite columns with steel tube under load reversal. In Fig. 8, the ratio between the analytical estimates and experimental results (66 steel reinforced concrete specimens (Ref. 2, 6) and 40 concrete infilled steel tube specimens (Ref. 8)) is plotted in histograms. Those figures indicate the effectiveness of the analytic procedure proposed in this paper.

## 7. CONCLUSIONS

The following conclusions can be drawn from this study.

1. The ultimate shear strength of composite structural components can be estimated as the sum of the shear resisting forces of the following four portions: that is, 1) the reinforced concrete portion with the effective width of  $b'$  ( $= b - b_f$ ), 2) the steel portion, 3) the concrete portion encased by the steel, and 4) the concrete portion outside the steel flanges.
2. The procedure as well as the formula provided by AIJ can estimate the shear strength of previous tests with reasonable accuracy.

## ACKNOWLEDGEMENT

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#### APPENDIX DESIGN FORMULA OF ULTIMATE SHEAR STRENGTH OF COMPOSITE COLUMNS IN AIJ STANDARD

SRC standard is now being revised, and a new edition will be published in 1986 in which a provision will contain design formula for the ultimate shear strength of composite columns. The shear strength of composite column  $Q_u$  is considered to be equal to the sum of the strengths of steel portion  $sQ_u$  and that of reinforced concrete portion  $rcQ_u$  and is as follows.

$$\begin{aligned}
 Q_u &= sQ_u + rcQ_u \\
 sQ_u &= \min(sQ_{fu}, sQ_{su}) \\
 rcQ_u &= \min(rcQ_{fu}, rcQ_{su}) \\
 rcQ_{su} &= \min(rcQ_{su1}, rcQ_{su2}) \\
 rcQ_{su1} &= b \cdot r_j \cdot (0.075 \cdot F_c \cdot \alpha + 0.5 \cdot \rho_w \cdot \sigma_{wy}) \\
 rcQ_{su2} &= b \cdot r_j \cdot (0.15 \cdot F_c \cdot b'/b + \rho_w \cdot \sigma_{wy})
 \end{aligned}$$

where, the notations are as follows,

- $b$  = width of columns (cm)
- $b'$  = effective width of reinforced concrete portions (cm)
- $r_j$  = distances between centroids of tension and compression stresses in reinforced concrete (cm)
- $F_c$  = concrete compressive strength ( $\text{kgf/cm}^2$ )
- $\alpha$  =  $4/(rcM/rcQ \cdot d + 1)$  and  $(1 \leq \alpha \leq 2)$
- $\rho_w$  = hoop ratio
- $\sigma_{wy}$  = yield stress of hoop ( $\text{kgf/cm}^2$ )