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Planning for Quality — Concepts and Numerical Tools

Planification de la qualité — Concepts et méthodes numériques

Qualitätsplanung — Konzepte und numerische Methoden

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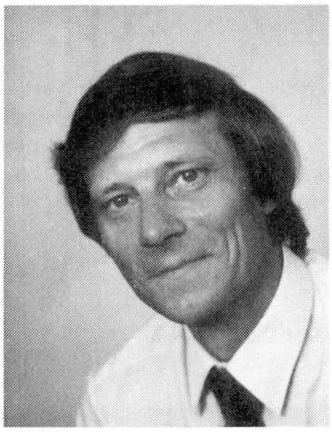

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SUMMARY
Planning for quality plays an important part in quality assurance of buildings. Yet, it has been to a large extent a rather pragmatic subject since no formal feasible tools to arrive at optimal solutions were available. In the paper an attempt is made to structure the problem and model its ingredients, particularly the occurence and detection of human errors throughout the building process.

RESUME
La planification de la qualité joue un rôle important dans l'assurance de la qualité des constructions. Cependant, le sujet est souvent traité de façon assez pragmatique, puisque des méthodes formelles et pratiques en vue d'une solution optimum ne sont pas à disposition. Dans ce travail, on a essayé de structurer le problème et de quantifier ses composantes, surtout l'existence et la découverte des erreurs humaines dans le processus de construction.

ZUSAMMENFASSUNG
Qualitätsplanung spielt eine wichtige Rolle bei der Qualitätssicherung von Bauwerken. Sie läuft vielfach jedoch ziemlich pragmatisch ab, da keine formalen, praktikablen Methoden für optimale Lösungen zur Verfügung stehen. In der Arbeit wurde versucht, das Problem zu strukturieren und die Einzelheiten zu modellieren, insbesondere das Auftreten und Entdecken von menschlichen Fehlern während des Bauprozesses.

1. INTRODUCTION

The most ambitious definition of *quality* of a technical facility is that it is at its optimum utility to its user possibly under external constraints with respect to the probability of reaching certain adverse high consequence states. A fairly modest interpretation of *quality assurance* is assurance, in the sense of documentation, that a given set of pre-selected specifications has been met by pre-defined compliance rules with a reasonable degree of confidence. In the following we shall adhere to the first view on a definition of quality with all its consequences for its assurance. If utility is measured in monetary terms (in what else?) quality assurance, therefore, should optimally balance costs and benefits or at least minimize costs. These may include costs for pre-investigations and siting studies, for design, construction and their control, for inspection and maintenance during use resp. for non-possible use during repair, possibly for demolition and removal after the anticipated time of use or when the structure becomes obsolete but, most important, for failures of the system in their different forms. From experience it appears essential that the whole life-cycle of a constructed facility is covered. Planning for quality thus is not only the assessment of the various means to achieve quality, the organization of the verification of the various measures, the creation of an appropriate professional, psychological and financial climate and a reasonable time-schedule; it is also the optimal allocation of the available resources in the various quality relevant measures. Accordingly, the subject of "Planning for Quality" might be split into two areas.

I. the phenomenological description of the ingredients of quality assurance in the wide sense

II. the mathematical formulation and numerical solution of aims, tools and bases of quality assurance for sound decision-making.

Yet, at least for the building sector, only a few studies are available in both areas.

Practice of quality assurance appears to be widely based on intuition and speculation and only occasionally as in the control of the production of materials on more or less carefully assembled and evaluated experience. General systematic approaches apparently do not exist. Perhaps most revealing, the subject but particularly planning for quality is hardly teachable at present. This is what the author wishes to make clear before attempting to elaborate on a few aspects selected out of a much larger group of elements constituting the overall problem.

2. BASIC STRUCTURING OF THE PROBLEM

Once the "infra-structure" of quality assurance for a given job is known and settled the remaining steps are to assess the logical structure of the overall system and, then, allocate the efforts in the most optimal way. It is to be underlined that the first qualitative and the second quantitative step are highly interactive. The second step results in decisions about the final setting of the specific quality assurance measures which then may be up-dated during the course of the work.

In order to get hold of the problem the concept of *hazard scenarios* is introduced (1). This concept is not the only one possible and, in fact, appears unfit for certain complex problems.

It is used here merely to demonstrate the potential of a discrete consideration. A *hazard scenario* will be understood as a more or less complex "scenario" of events one or a few of which play the role of defining it but also guide its probabilistic formulation. Such leading events could, for example, be a critical construction phase, the structure under normal service conditions, extreme values of one or several types of external or internal actions upon the structure with due consideration of the other simultaneously acting forces, the presence of abnormal environmental conditions as, for example, clay lenses in the soil, but also the existence of one or more types of errors, flaws or omissions during planning, design, construction, use and, possibly, demolition of the structural facility. Because a definition of what is an error, blunder, flaw, omission or just negligence has many facettes and appears hardly clear cut we will assume it undirected and avoidable under the circumstances, i.e. with due regard to the type and amount of skill and effort appropriate for the work. What is important for our purpose is that this understanding of a hazard scenario as effective upon a structure can be displayed in terms of the well-known event - or fault (failure) trees and, thus, also allows for straight - forward mathematical treatment and numerical manipulations. It is a discrete representation of reality in most cases but might be as such the only one feasible for numerical treatment. We shall call the set of hazard scenarios complete if it contains the scenarios which are reasonably imaginable. "Unimaginables" are not and principally cannot be handled. Conversely, not considering a relevant scenario during planning and design may already be interpreted as an error. We shall not discuss this case further but it will be clear from the following how to deal with it.

Formally, failure of the system due to failure in any of the hazard scenarios is failure of a series system, i.e. a system fails once any of its links fails or it fails in any of the scenarios considered. Let the index r count the number of hazard scenarios for which the leading event is an human error. Also, index with s the hazard scenarios with other leading events. Then, the failure probability with F_n denoting the failure event of the n-th scenario

$$P_f = P\left(\bigcup_{\{N\}} F_n\right) \leq \sum_{\{N\}} P(F_n) \tag{1}$$

in which the union operation runs over all subsets $\{N\} \subset \{0,1,\ldots,s,\ldots,S,0,1,\ldots r,\ldots,R\}$. For convenience, let s=r=0 denote the case of some normal service condition and no error existing. Damage statistics indicate that it is generally important to consider hazard scenarios where the leading event is a combination of events. Although the occurence event of such a combined event may be associated with much smaller probabilities than any single event the probability of structural failure given their joint occurence can be large and, therefore, the contribution to eq. (1) can also be large. The upper probability bound in eq. (1) is exact, if the events F_n are or are made disjoint. In the contrary case one may wish to improve it. Such an improvement will be given later on.

Fortunately, the failure event F_n associated with the n-th hazard scenario can be broken down further because these events appear to have a similar structure in almost all cases. In fact, for system failure we now have the intersection of various events. The first of those events is the occurence event. The last event clearly is failure of the structural system. In between these events the failure of some protective apparatus usually is placed. For system failure all these failure events must take place. In other words, they form a parallel system. For example, assume that a certain design error occurs. Then the events necessary for system failure are {(Occurence of the error <u>and</u> non-detection during some checking <u>and</u> structural failure) <u>or</u> (Occurence of the error <u>and</u> detection <u>and</u> non-correction of error <u>and</u> structural failure)}. We have:

$$F_n = \{O_n \cap \bar{D}_n \cap V_n\} \cup \{O_n \cap D_n \cap \bar{R}_n \cap V_n\} \tag{2}$$

with an obvious short-hand notation. "$\cap$" stands for an "and"-connection (intersection) whereas "$\cup$" denotes an "or"-connection (union of events).

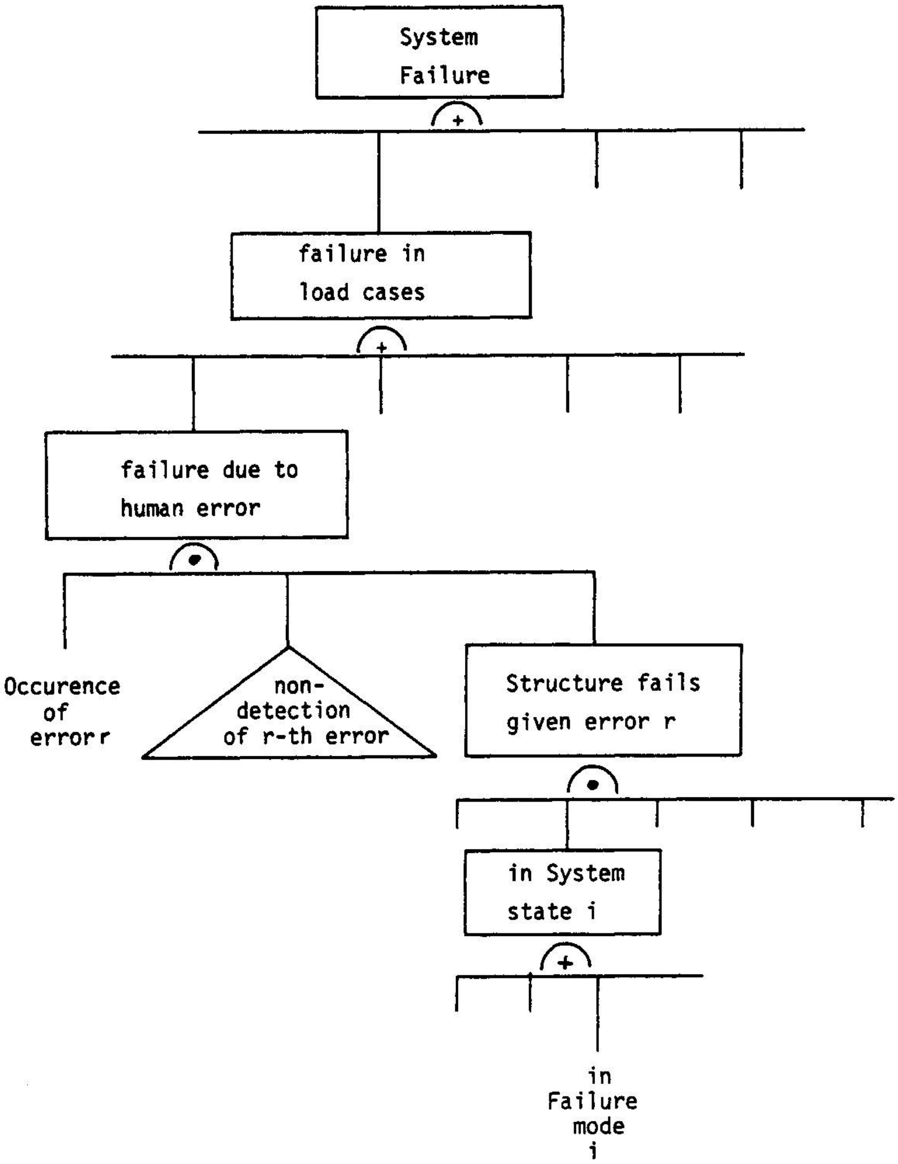

Figure 1: Fault (failure) tree of overall system

Both the detection event and the structural failure event given the error can be split down further. For the moment this is done here only for the structural system which typically can be modelled by a tie set of failure events (series systems in parallel) where in the subsequent formula the failure modes in any redundant system state are indexed by j while the system states are indexed by i.

$$V_n = \bigcap_i \bigcup_j V_{n,ij} \qquad (3)$$

If, for simplicity, we neglect the second event in the union of eq. (2), eq. (1) can be written after insertion of eq. (3) as:

$$P_f = P\left(\bigcup_{\{N\}}(O_n \cap \bar{D}_n \cap (\bigcap_i \bigcup_j V_{n,ij}))\right)$$

$$\leq \sum_{\{N\}} P\left(O_n \cap \bar{D}_n \cap (\bigcap_i \bigcup_j V_{n,ij})\right) \qquad (4)$$

in which the inequality does not only hold with respect to the upper bound approximation already introduced in eq. (1) but also since the redundant system states i usually are limited to a few interesting ones and hence, its failure probability is overestimated. This "system" is represented by its fault tree in figure 1. The corresponding block diagram is easily constructed from the fault tree. Very much the same structure of a fault tree is obtained for exceptional loading situations. As an example take the hazard of a ship colliding with a bridge pier. In this case the protective element could be either ship-owned devices to prevent the ship veering out of course or a jetty securing the piles. Those protective elements if fulfilling their intended function can be used to diminish the occurence probability or at least the magnitude of the loads.

Similarly, the investigation of the logical structure of the overall system could have been done by the use of event-tree methodology, if one considers all sets in {N} as "initiating" events which ultimately could lead to failure. For both types of analysis the formal reduction to minimal cut sets of failure events facilitates their numerical evaluation (see [2] for a suitable algorithm).

3. COMPUTATION OF FAILURE PROBABILITIES

We now turn to the determination of the probabilities in eq. (4). Due to interesting developments in so-called first-order reliability methods in the very past, the computational part is no more a serious problem and further developments are expected both with respect to simplifications and advanced numerical techniques. The idea of first-order reliability methods is surprisingly simple and will be scetched for the computation of the failure probability of a structural component. Let $V = \{g(\underline{X}, \underline{\pi}) \leq 0\}$ define the failure domain of a component in the space of uncertainty variables $\underline{X} = (X_1, \ldots, X_n)^T$ such as strength of materials, loads, geometrical dimensions or (randomized) agreed-upon prediction errors of the physical model in use. π is a vector of parameters collecting, for example, structural dimensions, material grades or certain parameters describing the effort of quality assurance measures. For convenience of notation it is dropped in the sequel. Transform the vector $\underline{X}$ into an independent standard normal vector $\underline{U}$ such that $P(g(\underline{X}) \leq 0) = P(g(\underline{U}) \leq 0)$ [3]. If the failure surface $G \equiv g(\underline{U}) = 0$ is linearized by a plane whose normal vector is pointing to the coordinate origin and fits the surface in a point closest to the origin (or, alternatively where the multinormal density $\varphi_n(\underline{u})$ obtains its maximum on G), the failure probability is

$$P_f \approx \Phi(-\beta) \qquad (5)$$

where β (=safety index) is this distance of the most likely failure point (β-point) to the origin and Φ the univariate standard normal integral. This estimate can be further improved and be made asymptotically correct, if the curvatures of G in the β-point are taken into account. [4,5]

If, on the other hand, the probability of a parallel system with m components is to be computed one has [6]:

$$P_{f,P} \approx \Phi_m(-\underline{\beta}; \underline{\underline{R}}) \qquad (6)$$

where Φ_m is the m-variate normal distribution function, $\underline{\beta}$ the vector of componental safety indices and

$$\underline{\underline{R}} = \{\rho_{ij}\} = \{\underline{\alpha}_i^T \underline{\alpha}_j\}$$

the matrix of correlation coefficients between the componental state variables with $\underline{\alpha}_i$ the vector of direction cosines of the i-th approximating hyperplane.

If the system is given as a cut set of failure events which always can be achieved by appropriate set operations we have [6,7]:

$$P_f = P(\cup\cap F_{ij}) \begin{cases} \leq P_1 + \sum_{i=2}^{n} \left(P_i - \max_{j<i} \{P_{ij}\} \right) \\ \geq P_1 + \sum_{i=2}^{n} \left(\max\{0, P_i - \sum_{j<i} P_{ij}\} \right) \end{cases} \qquad (7)$$

with $P_i = \Phi_{n_i}(-\underline{\beta}_i; \underline{\underline{R}}_i)$ and $P_{ij} = \Phi_{n_i + n_j}(-\underline{\beta}_i; -\underline{\beta}_j; \underline{\underline{R}}_{ij})$

In this case, $\underline{\underline{R}}_{ij}$ collects the correlation coefficients between any two components i and j of the parallel systems.

The crucial question in applying this methodology to the general system formulation as expressed in eq. (1) is to model both the occurence event and the detection event appropriately and particularly such that dependencies among the different redundant failure events are properly taken into account. Remember that the trivial bounds for redundant systems, i.e. the lower bound as the product of the individual probabilities and the upper bound as the smallest componental probability are almost useless for our purpose. The upper bound ignores any redundancy. The lower bound dramatically overestimates the effect of redundancy in most cases.

4. EFFECT OF "NORMAL" QUALITY ASSURANCE MEASURES

The spectrum of quality assurance measures is too wide to be exhaustively considered herein. Therefore, a few examples of frequent quality assurance measure may suffice to demonstrate how to model their effect on component or system probabilities with emphasis to cases which have been considered by the author elsewhere. For the moment, we exclude those measures aiming at error detection and removal. These will be dealt with in the next section.

The remaining measures include, for example, previous investigations to update usually relatively diffuse prior information on uncertain quantities such as loads or strength of materials or to identify and locate anomalies, normal quality procedures and, perhaps, some type of proof-testing. And, of course, the selection of safety elements in normal design procedures belongs to this category. Specifically, the parameter vector π may include partial safety factors $\underline{\gamma} = (\gamma_1,\ldots,\gamma_q)$ which, given the loads, define the safety margin between loads resp. load-effects and the resistances. Formally, this can be written as

$$P_f = P\ (g(\underline{X};\ \underline{\gamma},\ \underline{d}) \leq 0) \tag{8}$$

$\underline{d}$ is a vector of design parameters such that the design failure probability achieves a certain prescribed value, i.e.

$$P_{f,t} = P\ (g(\underline{X};\ \underline{x}^*,\ \underline{d}) \leq 0) \tag{9}$$

in which according to [8]

$$x_i^* = \gamma_i\ x_{iN} \tag{10}$$

with x_{iN} some nominal value of X_i. P_f generally is increasing (decreasing) for decreasing (increasing) γ_i's.

The majority of the other quality assurance measures modifies the stochastic nature of one or more uncertain variables. For example, let the variable X, e.g. a climatical load, have density f_X but depend on a parameter (mean yearly maximum) λ which varies from location to location. Prior to any specific inestigation the predictive density of X is

$$f_X\ (x) = \int_\lambda f_X\ (x|\lambda) f'_\Lambda\ (\lambda)\ d\lambda \tag{11}$$

where f'_Λ is the prior distribution on the uncertain parameter. Most likely, the nominal value specified in loading codes is defined in this distribution as a certain fractile. However, a near-by weather station can provide specific local data and via Bayes rule, the posterior density of λ given the observations $\underline{z} = (z_1,\ldots,z_n)$ is

$$f''\ (\lambda|\underline{z}) \propto l\ (\underline{z}|\lambda)\ f'\ (\lambda) \tag{12}$$

where $l(z|\lambda)$ is the likelihood of $\underline{z}$ given λ. Many special but useful results for eq. (12) resp. (11) can be found in the books on Bayesian statistics. With increasing effort parameter, here the sample size n, one can narrow down the variability of X to its natural local uncertainty.

If active control measures are specified for the production of materials, this "natural" uncertainty arising from the unavoidable variations in the raw materials and in the manufacturing process itself can be substantially diminished. As a first example, take a production process whose outcome is modelled as an (autocorrelated) random sequence. If the process is observed at any k-th value at which it is adjusted to the target, it is clear that the variability of the production outcome is smaller than for the uncontrolled case and decreases for decreasing k. Generalisations and modifications of this simple scheme are the subject of the rich theory of (stochastic) control where many useful results can be found.

As another example consider the inspection of flaws in welding of length L. Flaw occurence can be modelled by a Poisson process. The residual strength at the location of a flaw is X with distribution $F_X(x)$. It depends on the flaw size in a certain manner. Hence, if the distribution of flaw size is known so is that of X and for known stresses in the flaws the failure probability of the weld-line. Assume a certain inspection method. The probability of flaw detection increases with flaw-size, e.g. according to $P(D|X\leq x)=F_D(x)$. It follows that after inspection the flaw size distribution is

$$F_{X,D}(x) = F_X(x) \frac{\bar{F}_D(x)}{\int \bar{F}_D(x)dx} \qquad (13)$$

and the occurence rate of flaws is reduced from ν to $\nu\,(1-\int \bar{F}_D(x)dx)$ (see figure 2). The arguments are very similar if the material to be used is selected by continuous grading (see [9]) and also with respect to the effect of normal compliance control [10]. In particular, let the qualities offered be described by a vector of distribution parameters Θ whose prior (before compliance control) distribution is $f''(\Theta)$ and assume a certain compliance rule, i.e. acceptance for $z_n \in A$ where A is the acceptance region and z_n some function of the control sample. Then, the posterior distribution of $\underline{\Theta}$ is given by

$$f''(\underline{\Theta}) = \frac{L(\underline{\Theta})\ f'(\underline{\Theta})}{L(\underline{\Theta})\ f'(\underline{\Theta})d\underline{\Theta}} \qquad (14)$$

reflecting the filtering effect of such activities. L is the acceptance probability given $\underline{\Theta}$. Clearly, the amount of filtering depends on the type of acceptance criterion and the sample size.
In some cases, prototype or proof-testing may be chosen. If prototype testing serves to collect specific information the mathematical scheme for the description of its effects is exactly the same as for the previous investigations mentioned before. For proof-testing, one has to distinguish whether its purpose is to truncate the distribution of resistances [11] (e.g. when setting and prestressing earth anchors with overloading) or whether its purpose is similar to prototype testing with or without the up-dating of design and/or construction strategies. Some further interesting results may be found in [12] and [13].
There is no reason here to extend the list of examples of models for the effect of "normal" quality assurance measures. What was to be shown is the general concept which with few exceptions relies heavily on Bayes' theorem. Depending on the specific problem at hand one or the other or some joint measures can be most

appropriate. All of the foregoing formulations fit into the general framework set out in section 2 and are numerically amenable with the aid of the reliability methods scetched in section 3. The situation is somewhat different if human errors have to be considered.

5. SOME ERROR OCCURENCE AND DETECTION MODELS

It is useful to distinguish between at least three different types of errors depending on the effect they would have

i. Inadequate physical model: Suppose there is a "correct" or generally agreed physical model for the problem at hand and a corresponding failure surface $g_o(\underline{X})=0$ can be formulated. Any other model, denoted by $g_r(\underline{X})=0$, r=1,2,..., may be considered as an error yielding a different (conditional) failure probability (see figure 3). Such errors include typical design errors such as an incorrect idealization of the structural model (first-order linear elastic versus second-order linear elastic-plastic analysis), computational errors, ignorance of three-dimensional effects in structural behaviour, etc. In a certain sense, it also includes the omission of significant load scenarios.

ii. Inadequate uncertainty model: This primarily results in a wrong dimension of the uncertainty vector $\underline{X}$, i.e. certain structural parameters are mistakenly assumed deterministic or known but are uncertain and, therefore would need some precautions. Note that this type of error almost always results in a greater failure probability.

iii. Misclassification error: This error type comprises pure misclassification errors, for example, when classifying soils and, in misclassifying, using wrong (prior) information about one or several important properties. It also includes misspecification (misreading) of material grades and the like or simply delivery of a false grade (see figure 4).

Clearly, there are other types of errors, for example the omission of a regular protective device when designing the facility, the failure to inspect and maintain, or inadequate use of the structure. Although some of them may fall into the categories just mentioned with respect to their formal treatment, others might require further thoughts but will not considered herein.

In some cases it is possible to model the occurence of errors by a simple Bernoulli sequence, i.e. at each possibility it occurs with probability p but does not occur with probability 1-p. Hence, the number of errors in a total of tasks N is given by the Binomial (hypergeometric) distribution.

Certain theoretical considerations in control resp. search theory suggest that errors occur according to a homogeneous Poisson process with intensity λ. However, the intensity (occurence rate) depends on parameters which in part vary from decision maker to decision maker. For example, one could assume

$$\lambda_i = \lambda_{io} \times g\left(\frac{t}{t_o}\right) \tag{15}$$

in which λ_{io} is the overall occurence rate of the error of type i, X a random variable with mean E[X]=1 and, possibly, varying between 0 and ∞ for the population of decision makers and g (t/t_o) a function expressing the variation of the occurence rate with the time pressure under which the given task has to be performed. If $g(t/t_o)=\Gamma(t/t_o)$, the function reflects the observation that for $t<t_o$, the time t_o being a reference time, the occurence rate can dramatically increase due to stress, for $t_o \leq t \leq 2t_o$ the occurence rate decreases below λ_{io} because the decision maker can afford much care but for $t>2t_o$ the absence of any pressure produces a larger occurence rate which is caused by increasing negligence and loss of motivation. Both the type of parameters and functions should, of course, only illustrate how the various factors influencing the occurence of errors could be modelled. It is here where much more research is needed. For simple intellectual tasks it is known that $\lambda_{io} \approx 10^{-3}$ but with greater differences between persons resulting, for example, in taking X_i as log-normally distributed with coefficient of variation $V_i=0.3$.

Now, let l_i be the number of tasks to be performed in a project, we have

$$P(N_i=1) \leq P(N_i \geq 1)=1-\exp[-\lambda_i l_i] \approx \lambda_i l_i \qquad (16)$$

for the occurence probability of errors of type i. For two errors of different type we may write

$$P((N_i=1) \cap (N_j=1)) \leq P((N_i \geq 1) \cap (N_j \geq 1)) \qquad (17)$$

$$= \prod_{k=i,j} (1-\exp[-\lambda_k(X) l_k] \approx \prod_k (\lambda_k(X) l_k)$$

assuming conditional independence between the occurences. But occurences depend on the variable X which now can be interpreted as a numerical measure for the intelligence, experience and carefulness of the decision maker. If several tasks are performed by the same decision maker the occurences of errors in any of these tasks clearly are dependent events. It remains to formulate the above findings such that they are suitable for a numerical treatment according to section 3. Let $P(O_i)=P(N_i \geq 1)$. Then, it is

$$P(O_i) = P(U_i \leq -\beta_i) = P(U_i + \beta_i \leq 0) \qquad (18)$$

with $\beta_i = -\Phi^{-1}[P(O_i)]$ the generalized safety index. But the right-hand side of eq.(18) is precisely of the form required. For two errors occuring one obtains for the "failure = occurence" event with eq. (17):

$$F_{ij} = \{O_i \cap O_j\} = \{U_{ij} - \phi^{-1}[\lambda_i(X)\lambda_j(X)l_i l_j] \leq 0\} \qquad (19)$$

where $X = F_X^{-1}[\phi(U)]$. It is seen that the randomness in the error occurences is modelled by an additional, auxiliary standard normal variable U_i resp. U_{ij}. Generalisations to the simultaneous occurence of more errors or a more complex dependence structure appears straightforward.

A similar approach can be used to model the detection of errors [16,17]. The theory of (random or classified) search suggests that the probability of detections grows approximately exponentially with the search effort. Hence, if E is the random amount of effort (time) to successful search of an error, we have

$$F_E(e) = P(\text{detection of error } i) = 1 - \exp[-\kappa \frac{e}{e_o}] \qquad (20)$$

in which κ is the detection rate, e the effort and e_o some reference effort. Let M be a quantity measuring the size of the error. Then, as an example, one could assume

$$\kappa = \frac{1}{n}(\Sigma(M_i - M_{oi})^2 + \kappa_o Y \qquad (21)$$

n is the possible number of errors of different type, M_i the size of the error, M_o the magnitude of M in the absence of an error, κ_o a basic detection rate and Y a qualification parameter with a similar interpretation as the variable X for the occurence rate. The variables X and Y usually are dependent. For example, a low error occurence rate may imply a low detection rate and vice versa reflecting the fact that human beings tend to rely on experts. Again, the failure (= non-detection) event can be formulated according to section 3, i.e.:

$$F_j = \{E_j > e\} = \{e - E_j \leq 0\} = \{e - F_E^{-1}[\phi(U_j)] \leq 0\} \qquad (22)$$

in which U_j is another auxiliary standard normal variable modelling the uncertainty in detecting errors.

The foregoing occurence and detection models should be viewed only as first attempts. Even then, the lack of suitable data is obvious and serious efforts must be undertaken to obtain at least subjective assessments for the parameters. What is important is that these crude models can guide data collections and consequently the up-dating and up-grading of the models and that they indicate a way to formulate dependencies between "system components" in parallel which as emphasized earlier is essential when analysing quality assurance systems.

The situation is a little bit less subjective for misclassification errors if there are prescribed classification procedures. Assume a building material or type of soil is classified into several classes. The uncertain quantity of interest is X but it depends on an uncertain parameter Θ_i whose distribution is associated with class i. Further, a random sample is drawn yielding the statistic $z(\underline{x})$ where $\underline{x}=(x_1,...,x_n)$ collects the observations. The classification (compliance) rule is such that if $z(\underline{x}) \in I_i$ the material is said to belong to class C_i. Any such procedure has two effects. On the one hand the distribution of X is modified through Bayes' rule, i.e. for the density of X:

$$f_X(x|C_i) = \int f_X(x|\Theta_j)\, f''(\Theta_j|C_i)\,d\Theta_j \tag{23}$$

with

$$f''(\Theta_j|C_i) \propto P(\{z(\underline{x})\in I_i\}|\Theta_j)f'(\Theta_j) \tag{24}$$

On the other hand, the prior probabilities for the classes are changed according to

$$p''_j \propto p'_j \int P(\{z(\underline{x})\in I_i\}|\Theta_j)\; f'(\Theta_j)d\Theta_j \tag{25}$$

The normalizing constants have been omitted. As usual the terms with (') are denoted by prior, with (") by posterior quantities. The latter probabilities are the probabilities of the joint event of occurence and non-identification of the j-th class. Unless the prior probabilities (occurence probabilities) are rather high for a particular class and the decision rule $z(\underline{x})\in I_i \Leftrightarrow C_i$ confirms this (for example, by the use of efficient statistics for large samples), one probably cannot discard the other classes. Prior probabilities can, however, be made close to zero for incorrect classes if other variables indicative for a class membership are used in the classification process.

A special case of the foregoing scheme is the quality control of materials. An error now is to misinterprete or hide the sample results or not to carry out quality control at all. Clearly, system failure can occur if this event happens which might very well be a small probability event but for physical failure one also has to assume a relatively diffuse unfiltered distribution of material qualities prone to be built into the structure and, hence, making failure rather likely. This possibility might be one of the reasons why for certain types of material and production a second independent barrier, the so-called external control (Fremdüberwachung) is introduced whose primary purpose is to check the regularity of normal control procedures and decisions and thus, making the probability of physical failure and lack of primary control and non-detection of absence of primary control hopefully close to zero. It should be an interesting exercise to actually compute the risk of non-detection of inadequate quality control given certain external control and naturally, production regimes. The probability of contradictory control decisions given positive decisions in internal control is easily calculated. The probability of non-acceptance of regular control on the basis of a decision rule such as "the control is regular if at most k contradictions are accounted in a total of n cases, otherwise it is irregular" given the value of the contradiction probability may simply be taken as the probability that a regular quality control procedure is non-existing.
This fortunate case is only mentioned here to point out that the level of modelling and analysis is, in fact, as far developed in certain fields as to allow such computations. These may then be used to decide on the necessity and/or efficiency of quality assurance measures, e.g. the planning of one or more error detection devices. In other fields not even the first step of modelling has been done and it appears to be this uncomfortable situation which presently even distorts the efforts to collect the right data information. You never observe anything you do not expect beforehand with reference to a famous saying of Albert Einstein! Most of the foregoing models for error occurence and error detection and certain modifications proposed elsewhere should be viewed in this sense. They may guide the aquisition of relevant data and, if falsified as adequate, suitably modified. But a first step is necessary.

6. QUALITY ASSURANCE BY SUITABLE DESIGN OF THE OVERALL SYSTEM

"Normal" quality assurance measures, such as prior investigations, proof-testing, quality control, proper design and its checking, etc., given a particular structure can, at least in principle, be modelled and quantified as outlined before. It should also be possible to quantify the relative efficiency of the measures. Sometimes, however, a change of the overall system, the structural lay-out or the construction methods can be much more efficient than any of the other measures. For example, one might wish to introduce additional, redundant control activities in order to reduce error probabilities. It is also generally true that structural reliability increases with the degree of static indeterminacy, with the degree of ductility of the relevant structural components and, most important, with decreasing stochastic dependence of the resistance quantities of the components. Further, for redundant structures it is generally not true that costly low variance - low mean materials are optimal. Only high variance - high mean materials can efficiently activate redundancy, if one takes for granted that low-variance-high mean materials always are uneconomical. It is not possible to consider here more such circumstances. When planning for quality one, in fact, has to concentrate on the aspects of redundancy, i.e. on the number and dependence structure of events below an "and" gate in our fault tree (or the number and dependence structure of the events along the branches of an event-tree). Planning for quality is also to provide an adequate number of "barriers" and to make their efficiency via proper organisations of tasks, distribution of responsibilities diversification in the delivery of materials, avoidance of undue time or economic pressure, etc., as independent as possible.

7. OPTIMISATION

In order to make our formulation sufficiently complete we still have to go one step further. The individual quality of a structure usually is not a binary property. i.e. perfect performance as opposed to total collaps. Each component may reach different states to which different benefits and losses can be assigned. The normal classification into three states, the states of so called full performance bounded by the serviceability limit state, the states of reduced performance resp. states requiring some maintenance and repair bounded by the ultimate limit state and the failure (collaps) states may be appropriate for most of the structural facilities but might be insufficient for more complex structures. Moreover, the losses may depend on the state of the structural system as a whole. The losses may particularly depend on the number and the type of components which have failed although the system has not yet reached the final collaps state. Just for illustration we shall assume states of the components and make the losses dependent on the system states such that the loss L is an increasing function of the system state. In particular, if the redundant system i fails the loss is H_i and the increment when passing from state i-1 into i equals $H_i - H_{i-1}$.

An objective function suitable for quality assurance orientated cost-effectiveness studies then is:

$$Z = C_o + C(\underline{e}) + \sum_k P(F_{n,k}(\underline{e})) \times (H_i - H_{i-1}) \qquad (26)$$

in which C_o are the cost independent of any quality assurance measures, $C(\underline{e}) \approx C(\underline{e}_o) + \Sigma c_t (e_t - e_{t,o})$ the cost for the selected quality assurance measures as an (approximately) linear function of the marginal cost C_t and the effort parameter e_t (e.g. safety factor, time spent in checking design calculations, sample size pf pre-investigation or compliance control of material procuction). The vector $\underline{e}_o$ may be taken as a base effort. Taking derivatives of Z with respect to $\underline{e}$ and setting it to zero yields the system of equations from which the optimal set of values $\underline{e}^*$ can be determined. Application of eq. (26) to the various overall system arrangements leads to the globally optimal system with optimal effort parameters. Optimization of eq. (26) can be made with or without constraints, e.g. constraints on so-called design failure probabilities which are those corresponding to some significant loading scenarios but no error scenario occuring. Proceeding in this manner quality assurance is by no means a simple task. Drastic simplifications can and must be introduced in practice.

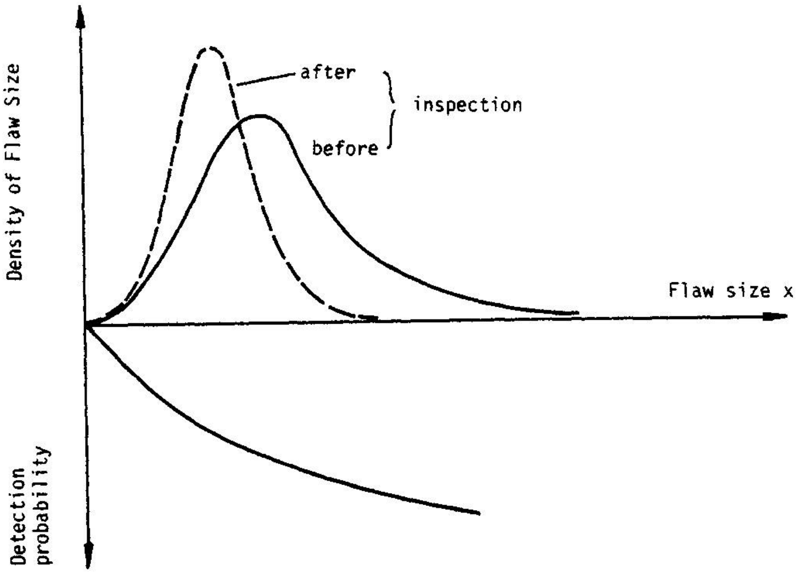

Figure 2: Flaw inspection and correction

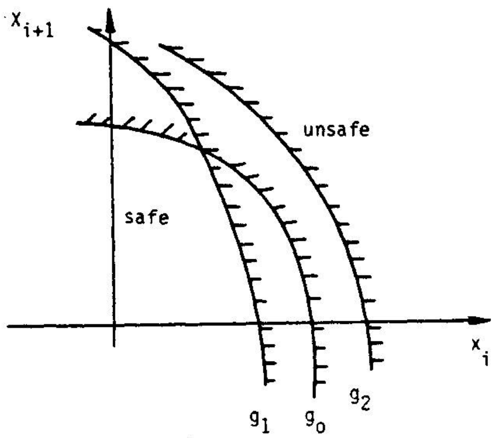

Figure 3: Failure domains for correct (g_o) and false (g_i, i=1,2,...) mechanical model

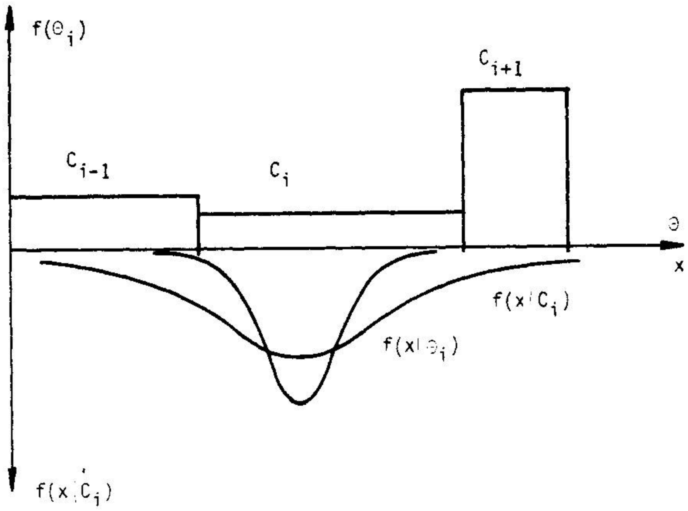

Figure 4: Prior information attached to classes

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