Decision Models for the Diagnosis and Treatment of Structural Defects

Modèles de décision pour le diagnostic et le traitement de dommages structuraux

Entscheidungsmodelle für die Diagnose und Behandlung von Bauschäden

R.F. WARNER
Prof. of Civil Eng.
Univ. of Adelaide
Adelaide, Australia

J.N. KAY
Sen. Lecturer in Civil Eng.
Univ. of Adelaide
Adelaide, Australia
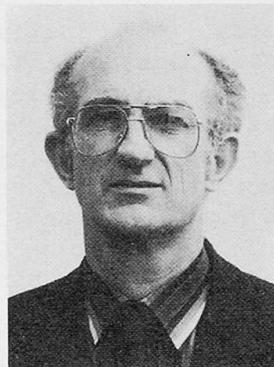
SUMMARY

Statistical decision theory provides a useful basis for deciding whether or not to proceed with corrective work on a possibly defective structure, and for choosing from a range of options for carrying out such work. Minimum expected cost, adjusted to allow for risk aversion, is used as the decision criterion. The importance of additional data gathering is discussed.

RESUME

Une théorie, basée sur des statistiques, permet de décider de la réparation ou du renforcement d'une structure défectueuse et de choisir la méthode à utiliser dans un cas particulier. Les critères de décision incluent le coût minimum prévisible pour un niveau de risque donné. La nécessité de récolter des informations complémentaires est discutée.

ZUSAMMENFASSUNG

Die statistisch fundierte Entscheidungstheorie gibt eine nützliche Grundlage für den Entscheid, ob und wie eine Reparatur an einer schadhaften Konstruktion ausgeführt werden soll. Als Entscheidungskriterium dient das Verhältnis der zu erwartenden Minimalkosten zum erlaubten Risiko. Die Bedeutung zusätzlicher Dateninformationen wird diskutiert.

1.   ASSESSMENT OF STRUCTURAL DEFECTS

Structural defects are defects which adversely affect the safety and serviceability of a structure. They have a wide variety of causes, including design error, construction error, material deterioration, steel corrosion, accidental damage, foundation movement, overload, and fire damage.

Before corrective work on a defective structure can be undertaken, some form of assessment is needed in order to identify the defects and diagnose their causes, and hence provide a prognosis of the future performance of the structure in both the defective and the repaired conditions. Unfortunately, systematic procedures for proper structural assessment are usually both costly and time-consuming [1]. For example, diagnostic charts prepared for concrete buildings [2] show that a small number of common symptoms are produced by a large range of different defects, so that careful, costly and time-consuming investigatory procedures are needed to diagnose and assess a structure accurately.

In practical situations, the initial decisions concerning the repair of a defective structure often have to be taken promptly on incomplete information without the benefit of a proper, accurate assessment. In these circumstances, difficult, risky and expensive decisions have to be taken, with the possibility of two types of assessment error being made. These may be referred to as Type I and Type II errors, because of their resemblance to the errors of statistical hypothesis testing and quality control [1].

A Type I error is made if the structure is assessed, incorrectly, as being defective.

A Type II error arises if the structure is assessed as being adequate when in fact it is defective.

Type II errors can endanger property and possibly life and are therfore potentially less acceptable than Type I errors, which may incur unnecessary, but possibly costly, repair work.

After an inspection and initial assessment has been made of a defective structure, it is necessary to identify the available courses of action and the range of possible consequences of each course of action, so that the most appropriate action can be chosen. While the nature and severity of the structural defect will greatly influence the choice of specific repair procedures, the following options will usually be included among the available courses of action:

(a)   do nothing, the structure being assessed as adequate to fulfil its
      intended function;
(b)   do not undertake corrective work, but monitor the structure for further
      signs of defect or deterioration;
(c)   carry out a thorough structural assessment, if necessary after taking it
      out of service for safety reasons;
(d)   undertake corrective work while maintaining the structure in-service;
(e)   take the structure out of service temporarily for safety reasons and
      carry out corrective work;
(f)   take the structure out of service indefinitely and investigate the
      alternatives of repair, reconstruction, demolition and replacement.

2. EXPECTED COST MODELS

A convenient framework for choosing the most appropriate of the available courses of action for a defective structure may be provided by statistical decision theory. One of the simplest decision criteria is based on expected values. It can be used to deal with defective structures, and can also be modified to allow for risk aversion, which is an important consideration in engineering decision making.

In a given situation, the courses of action to be considered will be referred to as A(1), A(2),..A(i),.. . For practical purposes it is convenient and usually adequate to lump the possible outcomes of any action A(i) into a finite number of distinct possibilities. For example, if action A(1) is do nothing (option (a) above), then possible outcomes would range from satisfactory structural behaviour over the entire design life of the structure to sudden, catastrophic collapse while in-service. The latter outcome would be an example of a Type II error of assessment. The finite set of outcomes of action A(i) will be expressed as S(i,1), S(i,2),..S(i,j),..S(i,Ni). The total cost of outcome S(i,j), including the cost of the original action A(i), will be expressed as C(i,j).

It is not possible to predict accurately which of the possible outcomes will in fact follow a particular action. Imperfect knowledge of the structural system, the approximate nature of structural theories, random variations in system parameters and uncertainty with regard to real world demands on the structure (loads, temperature gradients, etc) all prevent precise predictions from being made. On the other hand, the structural engineer can usually estimate the relative likelihood of each outcome, which can be expressed in the form of a probability. The probability, as perceived by the engineer, that outcome S(i,j) will follow from action A(i) can be expressed as P [S(i,j)|A(i)] or simply as P[S(i,j)] where

$$\sum_{j=1}^{Ni} P\,[S(i,j)] \;=\; 1.0 \tag{1}$$

The expected cost of any course of action A(i) is defined as follows:

$$EC\,[A(i)] = \sum_{j=1}^{Ni} P\,[S(i,j)] \cdot C(i,j) \tag{2}$$

According to the expected value criterion, the preferred course of action is the one which has the minimum expected cost.

It will be recognised that the probabilities P[S(i,j)] are likely to be educated guesses in many cases. However, the use of these numbers in a formal calculation process is more rational and more reliable than the conventional approach of intuitive selection.

3. EXAMPLE OF EXPECTED COST CRITERION

Extensive inclined torsion-type cracks developed in a prestressed concrete building during construction and were diagnosed as being caused by temperature warping of the floor slabs. Warping of the slabs had imposed torsional deformations in the supporting beams. Concern was felt for the safety of the building, because the cracked beams, being fully prestressed, contained no transverse reinforcement. Unusual loads (blast, earth tremor, etc) which might well occur at some future time, could result in partial collapse. Four courses

Table 1 :   Calculation of Minimum Expected Cost and Utility
(Cost shown in millions of dollars)

	Expected Cost calculation			Expected Utility calculation	
	(1)	(2)	(3)	(4)	(5)

A(1)   Cost = 0

	(1)	(2)	(3)	(4)	(5)
S(1,1)	0.50	0.00	0.00	0.0	0.00
S(1,2)	0.30	0.70	0.21	-2.4	-0.72
S(1,3)	0.10	1.00	0.10	-3.6	-0.36
S(1,4)	0.10	10.00	1.00	-100	-10.00

Σ = 1.31   Σ = -11.08

A(2)   Cost = 0.01

	(1)	(2)	(3)	(4)	(5)
S(2,1)	0.50	0.01	0.05	0.0	-0.00
S(2,2)	0.42	0.71	0.30	-2.5	-1.05
S(2,3)	0.04	1.01	0.04	-3.6	-0.14
S(2,4)	0.04	10.01	0.40	-100	-4.00

Σ = 0.79   Σ = -5.19

A(3)   Cost = 1.10

	(1)	(2)	(3)	(4)	(5)
S(3,1)	0.90	1.10	0.99	-4.0	-3.60
S(3,2)	0.10	1.80	0.17	-7.0	-0.70
S(3,3)	0.00	-	0.00	-	0.00
S(3,4)	0.00	-	0.00	-	0.00

Σ = 1.16   Σ = -4.30

A(4)   Cost = 0.80

	(1)	(2)	(3)	(4)	(5)
S(4,1)	0.95	0.80	0.76	-2.8	-2.70
S(4,2)	0.05	1.50	0.08	-5.8	-0.29
S(4,3)	0.00	-	0.00	-	0.00
S(4,4)	0.00	-	0.00	-	0.00

Σ = 0.84   Σ = -2.99

Notes :

(1) = P[S(i,j)]   ; (2) = C(i,j)   ; (3) = P[S(i,j)].C(i,j)

(4) = U(i,j)   ; (5) = P[S(i,j)].U(i,j)

of action were identified:

A(1) :   do nothing;
A(2) :   take no immediate action but monitor the building;
A(3) :   provide permanent protective propping around the columns,
          thereby causing partial impairment of the building;
A(4) :   introduce additional external prestress to correct the defect.

As a result of action (A1), the possible outcomes were considered to be as follows:

(a) S(1,1) : the structure survives satisfactorily;
(b) S(1,2) : further cracking occurs, especially in the roof system,
             and the client is forced to undertake repair work;
(c) S(1,3) : non-catastrophic local failure occurs as a result
             of an adverse combination of external forces;
(d) S(1,4) : sudden collapse occurs due to a rare combination of external loads.

The estimated probabilities of these outcomes and their expected costs are given in Table 1 in columns (1) and (2). The possible outcomes following action A(2) are considered to be similar to those for A(1); ie S(2,1) = S(1,1), etc. However, the probabilities are different, as indicated in column (1) of Table 1. Actions A(3) and A(4) are considered to eliminate the possibility of outcomes (c) and (d). This is taken into account in Table 1 with zero probabilities as appropriate. In column (2) of Table 1 the costs C(3,1) and C(3,2) include an estimate of the reduced value of the building due to partial impairment. According to the expected costs calculated in column (3) by means of Eq 2, the preferred course of action is A(2).

In this simplified example, the range of possible outcomes listed above as items (a) to (d) happen to be the same for each of the courses of action considered, and a simplified notation S(1), S(2), etc could obviously have been used. In many situations, the range of outcomes will depend on the course of action and the notation adopted here reflects this more general situation.

Two aspects of the expected cost approach as used in this example require further consideration and comment. The first concerns the aptness, or otherwise, of the expected monetary value criterion; the second arises from difficulties experienced by many engineers in choosing appropriate probabilities for use in the calculations. Each aspect will be considered in turn in the following sections of this paper.
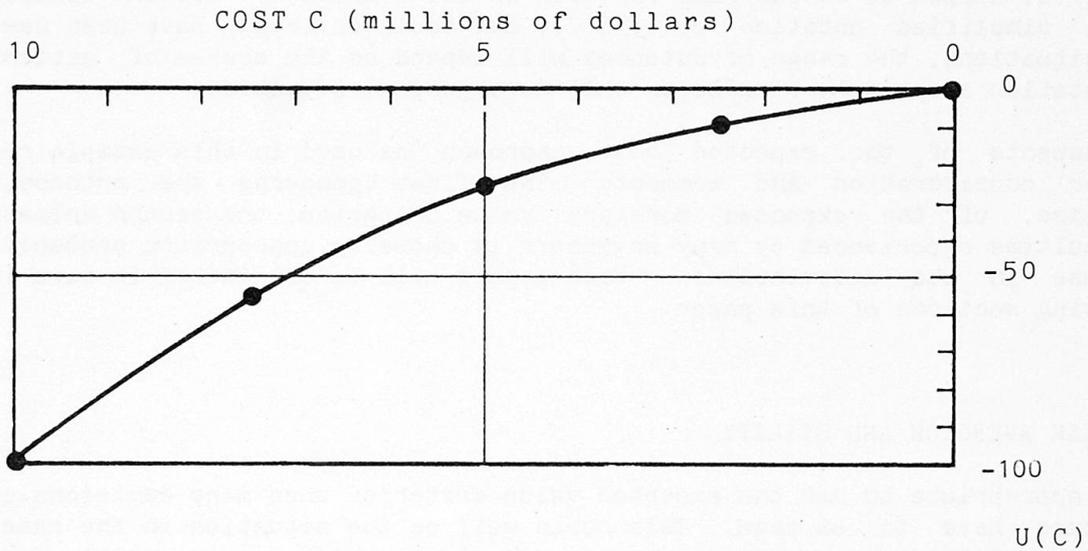

4. RISK AVERSION AND UTILITY

It is appropriate to use the expected value criterion when many decisions of the one type have to be made. This could well be the situation in the case of a large organisation such as a government department which is responsible for the operation and maintenance of a large number of buildings or structures. The situation is rather different when the risk taker is a small organisation or an individual, facing an important one-off decision. Problems such as cash flow and possible loss of income create a high aversion to courses of action which might entail large costs, even when the associated probabilities are very slight. Strong aversion to risk is also likely to exist when low-probability outcomes entail loss of reputation or injury or loss of life. Professional reputation is an important consideration to the engineer, whether or not he is involved personally with the financial aspects of the problem.

OUTCOMES

P	COST C	U(C) (Eq.3)
1.00	0	0
0.90	2.5	-10
0.75	5.0	-25
0.45	7.5	-55
0.00	10.0	-100
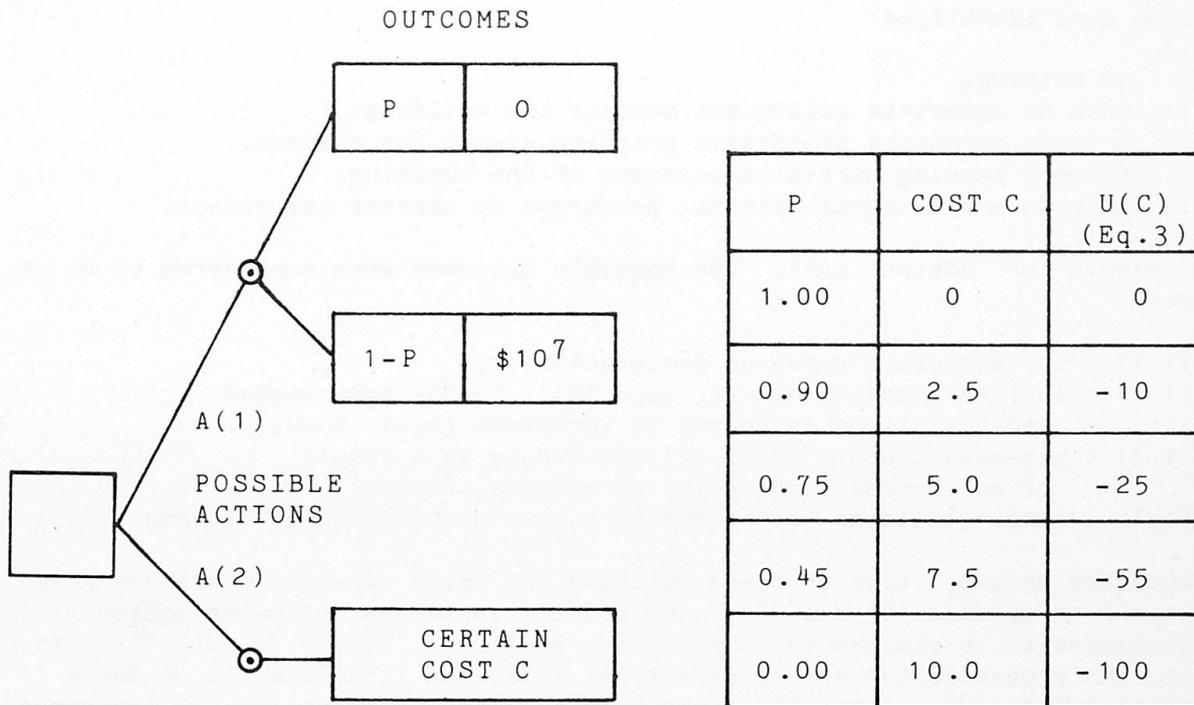
COST C is in millions of dollars
U(C) = Utility of Cost C   (see Eq.3)

(a) Utility Function Decision Tree

(b) Utility Function

Fig. 1 :   Construction of Utility Function

Allowance can be made for any desired degree of risk aversion by making a non-linear transformation of monetary values into utility values [4,5]. An arbitrary range of utility values is chosen to correspond to the range of monetary values. In Fig 1b, utilities of zero and minus one hundred units are equivalent to zero dollars and ten million dollars, respectively. A simple hypothetical decision situation such as the one indicated in Fig 1a is then used to obtain other intermediate utility values which allow a smooth transformation curve to be plotted as in Fig 1b. In Fig 1a two possible courses of action are considered. If action A(1) is chosen, there is a probability P that a loss of zero dollars will occur, but a probability of (1 - P) of a loss of ten million dollars. For various specific values of P, the decision maker chooses the maximum certain cost C which he is prepared to pay for action A(2) in order to avoid A(1). As (1) and A(2) are of equal value the utility of C dollars is equal to the expected utility of A(1), ie

$$U(C) = (0.0).P + (-100).(1 - P) = 100 (P - 1) \qquad (3)$$

It is convenient to use readily visualised values of P. In the present example, P values of 0.45, 0.75 and 0.90 produced C values of 7.5, 5.0 and 2.5 million dollars, respectively, and these were used to plot the points and hence the curve in Fig 1b.

When the expected values are calculated in terms of utilities instead of dollars in columns (4) and (5) in Table 1, it is found that A(4) is the preferred course of action. This result is consistent with a strong aversion to risk.

In the construction of the utility function, it is important that the risk aversion characteristics of the risk taker be properly represented, as these may not coincide with those of the engineer or professional decision maker. A further advantage of introducing utilities is that non-monetary aspects of the possible outcomes can often also be allowed for.

5. SUBJECTIVE PROBABILITIES AND THE PURCHASE OF NEW INFORMATION

As already pointed out, many engineers would have difficulty in giving values to the probabilities $P [S(i,j)|A(i)]$. Indeed it could be argued that such values are in most cases subjective, since there is insufficient information available to produce such values, except by educated guessing.

In many situations an attractive course of action open to the decision maker is to make the structure safe temporarily and then acquire additional information by any of the means used in structural assessment [2] such as:

(a) study of available documentation concerning the design and
     construction of the structure;
(b) quantitative measurement and analysis of environmental conditions;
(c) non-destructive testing and inspection of structural components;
(d) sampling and laboratory testing of materials;
(e) retrospective analysis of structural behaviour to confirm an
     original diagnosis;
(f) load testing of the in-situ structure (proof testing).

It will be noted that of the six common courses of action recommended in Section 1 of this paper which might be taken in the case of a defective structure, two involve gathering further information about the state of the structure. However, a difficulty arises when the expected value approach is used to consider information gathering as a possible course of action: before the

expected value of this course of action can be calculated, the range of possible outcomes of the information gathering exercise must be identified and their probabilities also estimated.

In Table 1, actions A(1) and A(2) are unattractive because of the very high cost associated with the relatively unlikely outcomes S(i,3) and S(i,4). If a relatively inexpensive in-situ load test could be devised to check the possibility of local or general collapse, it would be very cost-effective. For example, if the cost of such a test is $100,000 then A(1) clearly becomes the best course of action if the test is likely to show that the structure can resist the unusual load combinations. Otherwise A(4) remains the best course of action.

A formal procedure for incorporating test results into the decision analysis is available through Bayes Theorem [4,5,6]. Suppose a test is conducted with potential results T(1), T(2),..T(r)...T(Nj). Each outcome S(i,j) of the decision process can be considered in turn and an estimate made of the probability that, given outcome S(i,j), test result T(r) would occur. This information, together with the original probabilities associated with the S(i,j), can be used with Bayes Theorem to determine an improved set of probabilities for the outcomes S(i,j), given a specific test result. The relation is:

$$P[S(i,j)|T(r)] = \frac{P[S(i,j)] \cdot P[T(r)|S(i,j)]}{\sum_{j=1}^{Nj} P[S(i,j)] \cdot P[T(r)|S(i,j)]} \qquad (4)$$

The denominator of the right hand side is simply a normalising term which ensures that

$$\sum_{j=1}^{Nj} P[S(i,j)|T(r)] = 1.0 \qquad (5)$$

This provides a means for deciding whether to test or not to test in situations where testing is one of the courses of action to be considered [6]. The probabilities and utilities may be manipulated to provide the expected cost of each of the testing alternatives and from these, together with the expected costs of the non-testing alternatives, the best course of action can be chosen.

6.   REFERENCES

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