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## Concrete Columns Submitted to Cyclic Biaxial Bending

Colonnes en béton soumises à une flexion cyclique biaxiale

Stahlbetonstützen unter zweiachsialer zyklischer Biegebeanspruchung

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### SUMMARY

This paper describes a theoretical evaluation of the fatigue strength behaviour of concrete plastic hinges, subjected to a constant axial load and cyclically varying biaxial bending, with controlled curvatures. The low cycle fatigue strength is evaluated taking into account the «spalling» of concrete, the damage around cracks and the instability of compression bars. Various numerical examples are reported.

### RESUME

L'article traite d'un procédé analytique pour l'évaluation de la résistance à la fatigue des rotules plastiques en béton, sujettes à un effort normal et à une flexion biaxiale, avec déformations imposées. La résistance à la fatigue est évaluée en considérant l'épaufrure du béton, l'endommagement tout autour des fissures et l'instabilité des barres comprimées. Différents exemples numériques sont présentés.

### ZUSAMMENFASSUNG

Der Beitrag beschreibt ein analytisches Bemessungsverfahren für die Erfassung der Ermüdungsfestigkeit plastischer Betongelenke unter zyklischer Belastung mit Normalkraft und zweiachsialer Biegung. Der Widerstand wird unter Berücksichtigung der Abspaltung der Betonüberdeckung, der Schädigung in den Rissbereichen und der Instabilität der Druckbewehrung bestimmt. Es werden verschiedene Zahlenbeispiele gegeben.



## 1. INTRODUCTION

Referring to frames with sidesway it is well known that, in presence of seismic loads, columns must be considered among the most seriously involved structural elements. Because the seismic wave-directions are widely variable, it is necessary to consider the columns subjected to biaxial bending, cyclically varying during ground motion.

In general, because it is not economical to design columns in elastic range, it is advisable to make them able to absorb and dissipate seismic energy by ductile behaviour. If we examine the behaviour of a column in elasto-plastic range, it is evident that its performance relies on low cycle fatigue strength of plastic hinges which develop near the joints.

A high ductility factor for monotonic loads, i.e. a high ratio of ultimate curvature ( $\chi_{ult}$ ) to yielding curvature ( $\chi_y$ ), is not sufficient to assure that a structure can survive to exceptional seismic actions and, as a consequence, to cyclic loads. As a matter of fact it is well known that high ductility factor means good capacity to dissipate seismic energy during one cycle of loading, but it is not implied that, after many loading cycles, the energy dissipation capacity is still satisfactory. The period during of the ductile behaviour for sufficient numbers of cycles is necessary.

This problem seems very important for the definition of the validity of the well known structural models of hysteretic behaviour (for example Takeda model), and for the determination of the "residual safety" of a concrete building which survived to an earthquake.

This research concerns an analytical study of concrete beam-column elements, subjected to imposed cyclic curvatures; in addition this paper contains the stress-strain curves for the longitudinal reinforcement, which first reaches the yielding point. Then we examine the monotonic ductility variation according: to the variation of neutral axis slope, to different steel contents, to various axial load levels, to different distribution of longitudinal steel bars and to some ratios  $\chi_{max}/\chi_{ult}$ . For the same parameters we have also determined the number of cycles before failure. On the basis of the above-mentioned results we have drawn the so-called low strength fatigue curves. This paper ends with some remarks about the behaviour of concrete elements subjected to complex imposed loading histories.

In the references we recall only the notes strictly relating to these subjects; in particular we refer to the note by Ceccoli-Benedetti-Bianchi [13] where you can find the fundamental criteria adopted for a preliminary analytical development.

## 2. DESCRIPTION OF THE PROCEDURE

This study has been developed choosing: (a) appropriate constitutive laws for steel and concrete, (b) geometric features of sections, (c) loading histories.

With reference to point (a), for steel we have substantially employed the well known constitutive law proposed by Ramberg and Osgood [1], in the form suggested by Goldberg and Richard [3] and modified by Giuffrè and Pinto [5] (Fig. 1a). The adopted model allows to take into account the "Bauschinger effect", but not the strain-hardening. The concrete envelope stress-strain curve is the one suggested by Kent and Park [6], but in the modified version by Okamoto. Starting from this model we have introduced some modifications, apportioned by the same Kent and Park, to consider the confinement of concrete by the hoops.

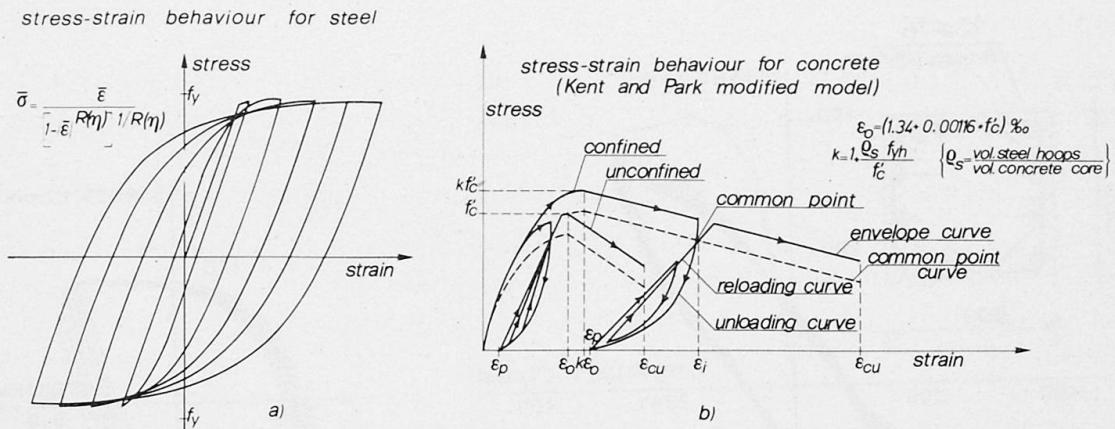


Fig. 1

The unloading branches begin on the before stated envelope curve and, with parabolic law, reach the residual plastic strain according to Karsan and Jirsa [4]; the reloading is imposed by a linear branch, passing through the "common point". The ultimate strain of the confined concrete, according to Corley [11], Bo and De Stefan [9], has been assumed as 7%; the concrete spalling begins when a compressive strain of 3% is reached. The concrete damage, following the reversal strain, strictly connected to the shear transfer, has been introduced considering a progressive reduction of the envelope curve amplitude up to 5%, for each deformation cycle.

The value of the buckling load of the compressed bars, no more continuously bound, is evaluated considering the length of bond slip gradually increasing, according to Tassios and Yannopoulos [10]. At the end of bond slip length the bar is assumed partially clamped; the constraint is furnished by the elastic subgrade corresponding to the still intact concrete. The critical load is defined on the basis of the Shanley theory [2] and we consider the post-buckling of the bars.

Referring to point (c) we notice that, univocal references lacking, the variation of impressed curvatures has been considered of the sinusoidal type, with amplitude less than  $\chi_{ult}$ . We have also checked some different curvature variations, about which we are dealing at point 5.

On the basis of the previously described assumption, the hysteretic loops of plastic hinges may be determined by incremental procedure. At each step of impressed curvature, the equilibrium is reached by iterative loops for the cross section, decomposed in elemental areas. The behaviour of the plastic hinge under biaxial bending has been analyzed considering several slopes of the neutral axis. The amplitude of the bending moment connected to each curvature value, has been determined vectorially adding the moment components having directions coinciding with neutral axis and its normal (1).

### 3. DETAILED ANALYSIS OF THE BEHAVIOUR OF A PLASTIC-HINGE

Figure 2a shows the  $M-\chi$  plot for a plastic hinge (whose section is represented in

(1) The code developed by the Authors is implemented on the Digital VAX 11/780 of C.C.I.B. (Bologna), Engineering School.

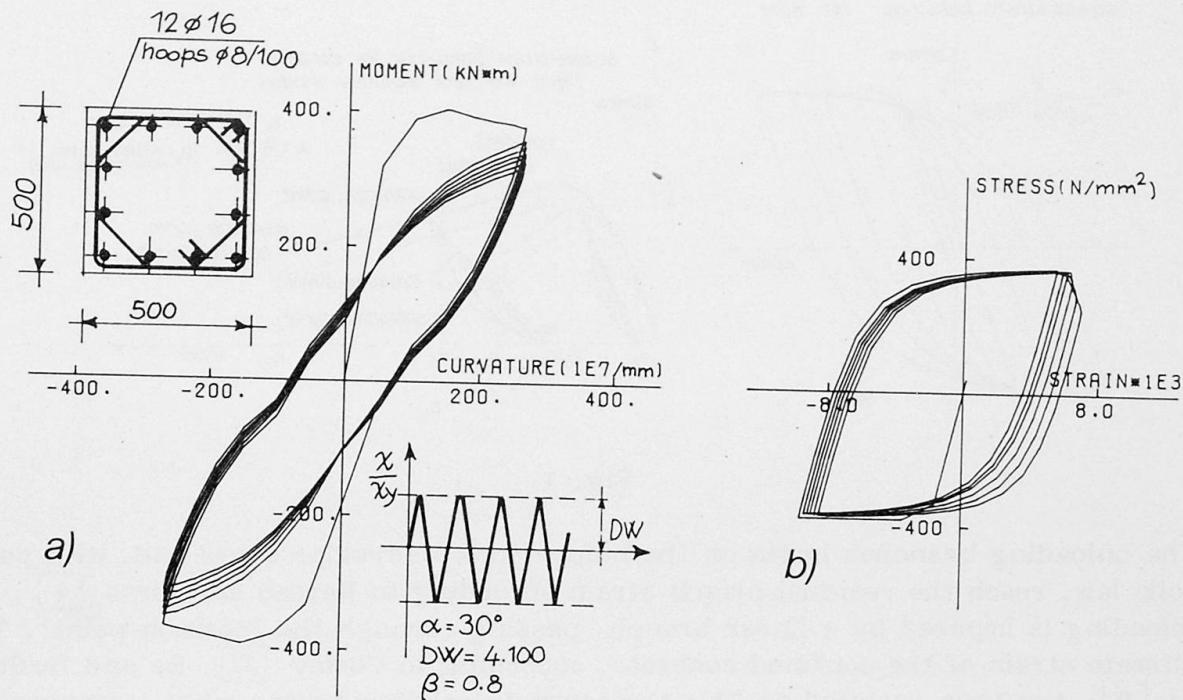


Fig. 2

the same figure) subjected to biaxial bending, with  $\alpha = 30^\circ$  and  $N = 0.2 b h f'_c$  ( $f'_c = 25$  N/mm $^2$ ). The maximum curvature reached at each cycle is  $0.8 \chi_{ult}$ . Figure 2b shows the  $\sigma$ - $\epsilon$  plot for the bar which first reaches the yielding point.

The  $M-\chi$  plot, at first linear as materials are stressed in the elastic range <sup>(2)</sup>, shows a pronounced decay of load carrying capacity when steel is yielded. Spalling stops the increase of the resisting moment and, because of the triangular shape of the compressive zone, the decrease of the moment carrying capacity is gradual. After the first quarter of cycle, the bending moment decreases according to the decreasing of the curvature; at this stage the  $M-\chi$  plot is essentially governed by the steel in tension, because, during unloading, the concrete load carrying capacity is quickly lost.

When bending moment changes sign, at a first stage moment capacity is high because concrete in the previous tensioned zone is reloaded and steel in the previous compressed zone is unloaded. Subsequent decreasing stiffness follows from spalling and softening of  $\sigma$ - $\epsilon$  curve of concrete.

The  $M-\chi$  plot shows a slight pinching which is characteristic of plastic hinges subjected to "moderate" axial load. According to Park and Paulay [7], this is due to the fact that the section is completely cracked. The equilibrium of internal forces is assured by the compressive stresses which, owing to steel hysteresis, are applied to the reinforcing bars still subjected to positive strains.

The second cycle of the imposed curvatures shows a clear softening of the resisting moment due to the lower answer of the compressed concrete, to the spalling and to the progressive damage. During the subsequent cycles we note an increasing re-

(2) The step from intact section to cracked section does not appear because of the small number of points selected in each loading cycle.

duction of the maximum moment owing to the buckling of the compressed bars, as it appears on  $\sigma$ - $\epsilon$  plot.

#### 4. ANALYSIS OF THE NUMERICAL RESULTS

The geometrical and mechanical characteristics of the plastic hinges we have examined are described in table 1. Preliminarily we have studied the behaviour of the various plastic hinges under gradually increasing controlled curvature.

Table 1

Unit	$b \times h$ (mm $\times$ mm)	$f'_c$ (N/mm <sup>2</sup> )	$f_y$ (N/mm <sup>2</sup> )	n	$\phi_{sl}$	$\rho_{sl}$	$d_{sh}$	$\phi_{sh}$	$\rho_{sh}$
1	$300 \times 300$	25	380	3	12	1.00	100	8	1.20
2	$300 \times 300$	25	380	3	14	1.37	100	8	1.20
3	$300 \times 300$	25	380	4	12	1.51	100	8	1.24
4	$300 \times 300$	25	380	4	14	2.05	100	8	0.95
5	$300 \times 300$	25	380	4/5	14	1.19	100	8	0.95
6	$500 \times 500$	25	380	4	16	0.96	100	8	0.73

$n$  = number of longitudinal bars along each side;  $\phi_{sl}$  = longitudinal bar diameter;  $\rho_{sl}$  = % of longitudinal reinforcement;  $d_{sh}$  = hoop spacing;  $\phi_{sh}$  = hoop diameter;  $\rho_{sh}$  = % of transverse reinforcement.

Figs. 3 and 4 show the  $M-\chi$  plots for unit 2 and 5 under axial load equal to  $0.2 b h f'_c$ . For unit 2 we note a progressive reduction of maximum bending-moment and ultimate curvatures when  $\alpha$  increases: both the phenomena are related to the trapezoidal or triangular shape of the compressed zone, and the consequent reduction of internal lever arm. When  $\alpha = 0$  (simple bending) the spalling is much more evident than when  $\alpha \neq 0$  (biaxial bending); in fact the damaged concrete part in the former case is much more noticeable than in the latter. For unit 5 the  $M-\chi$  plots are strongly influenced by the side ratio and, as foreseen, the less stiffness the more ductility.

In fig. 5, relating to unit 1, we show the number of cycles preceding failure and the ductility factor when the slope  $\alpha$  of neutral axis is varying. Increasing  $\alpha$ , the ductility decreases, but the worsening of behaviour during seismic loads is only apparent because, at the same time, the number of cycles increases. The reduction of the ductility factor is essentially connected to a quicker reduction of  $\chi_{ult}$  com-

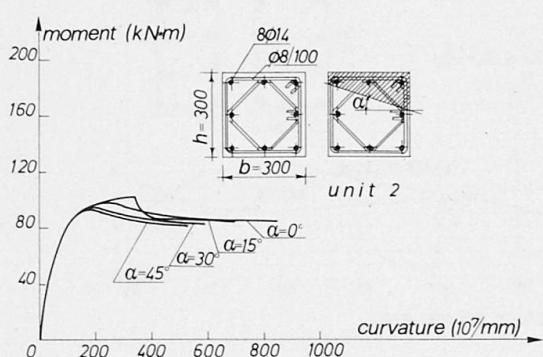


Fig. 3

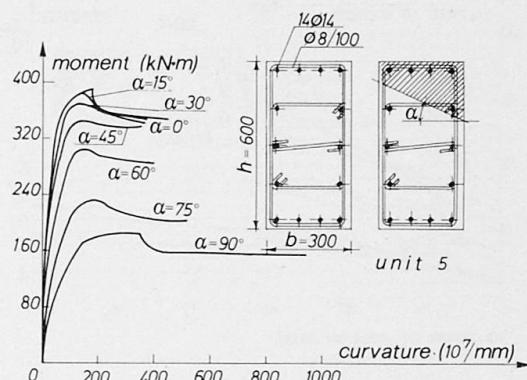


Fig. 4

pared with  $\chi_y$ . In other words the reduction of  $\chi_y$  versus the increment of  $\alpha$  is due to the triangular shape of the compressed zone, but the reduction of  $\chi_{ult}$  is more pronounced because, in addition to the above mentioned triangularization, we also have the reduced efficiency of compressed concrete, whose stress-path follows the failing branch. The increment of number of cycles preceding failure is related to the gradual buckling of the compressed bars.

When the axial load increases, monotonic ductility factor decreases, but not always when dealing with cyclic loads. For  $\beta = (\chi_{max}/\chi_{ult}) \leq 0.5$  and  $\alpha > 15^\circ$  the fatigue strength increases because the increment of axial load reduces the negative influence on the triangular shape of compressed zone. When  $\alpha < 15^\circ$  the triangularization of the compressed zone is not important and for  $\beta > 0.5$  from the beginning we have an extended spalling; at this moment a remarkable lowering of the neutral axis is necessary, but beyond a certain level the widening of the compressed zone does not counterbalance the loss of the cover concrete and we have an increment of the compressive strains. Similar trends are evident for the remaining units.

It may be interesting to notice that increasing the number of bars along each side, when  $\alpha < 15^\circ$ , we have an increase of the number of cycles preceding failure (fig. 6). The plots relating to unit 5 (fig. 7), taking into account the particular shape of the section, are slightly different, but it is confirmed that generally, when ductility reduces, we have an increment of the number of cycles before failure and vice-versa.

The results of the numerical developments can be handled to draw the so-called fatigue curves; these curves, concerning unit 1 and for different  $\alpha$  values, are shown in fig. 8. Considering our forcibly limited analysis, the fatigue curves must be considered indicative only, but it is clear that, with adequate experimental tests, they will be useful to evaluate the ductility factor necessary during an earthquake with a fixed number of deformation cycles. About the subject it is worthwhile no-

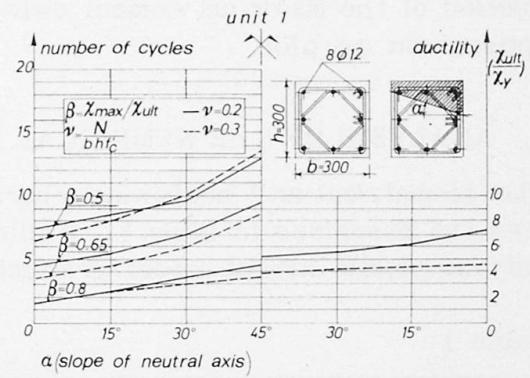


Fig. 5

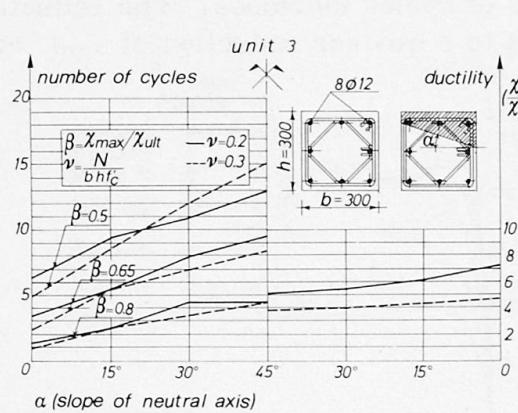


Fig. 6

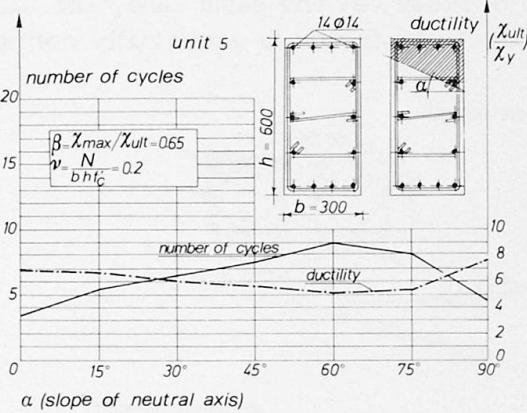


Fig. 7

ting that the drawn curves, taking into account the per- centual damage imposed for each cycle, do not admit an asymptote and however, with the same ductility factor, the number of cycles substanied under biaxial bending is always higher.

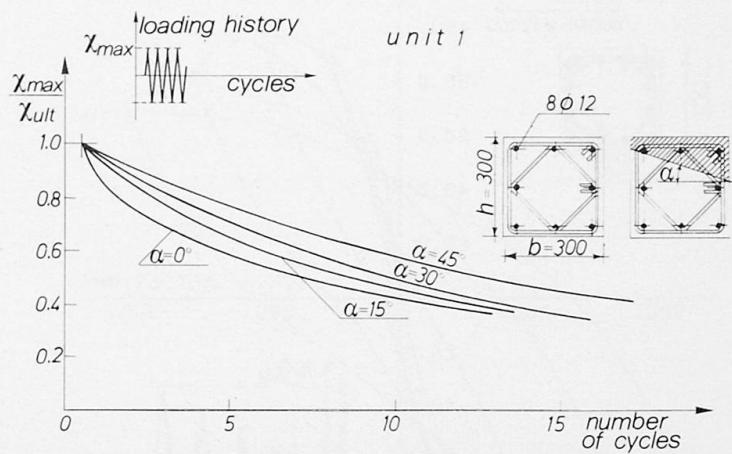


Fig. 8

## 5. IMPRESSED CURVATURES WITH A LAW NOT SIMPLY SINUSOIDAL

Referring to Bertero's considerations [8], we have analyzed, as an exemplification, loading histories able to define with greater reliability the static vicissitudes connected to the development of seismic phenomena.

In fig. 9 we have drawn the  $M-\chi$  and  $\sigma-\varepsilon$  plots for a plastic hinge subjected to a loading history characterized by maximum amplitudes different in the two directions; in brief the seismic effect is overimposed to an initial curvature due to the permanent loads. When  $\alpha = 45^\circ$  we get 10 cycles against 13 which we find for a sinusoidal loading history with an amplitude equal to half the sum of the maximum excursions; when  $\alpha$  increases we have a slight efficiency loss of the low cycle fatigue strength.

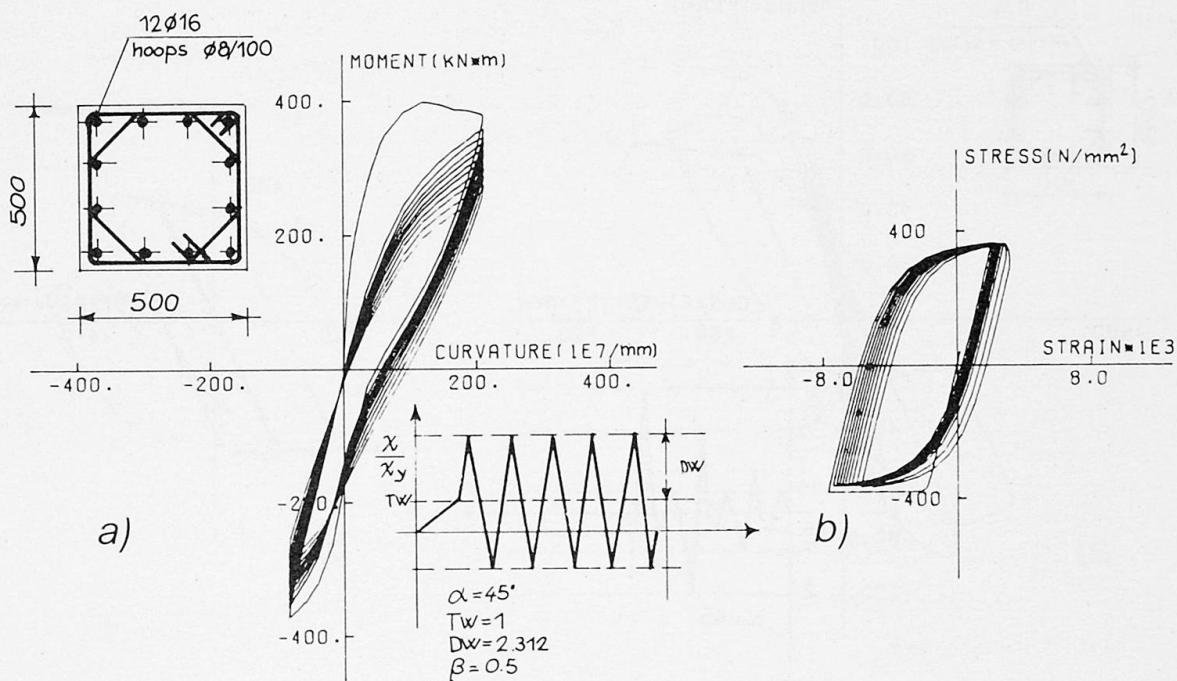


Fig. 9

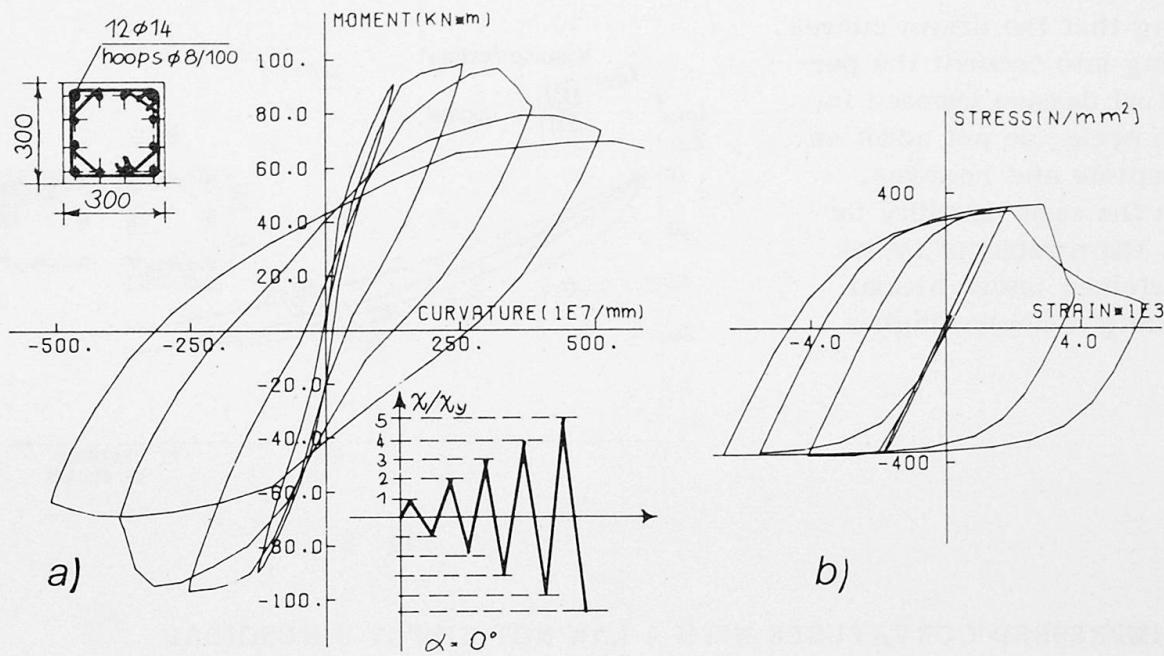


Fig. 10

In fig. 10 we consider loading histories varying according to a sinusoidal law with a gradually increasing amplitude. The number of cycles before failure results independent of  $\alpha$ ; when  $\alpha = 0$  we however note a widening of hysteresis loops, probably due to a more efficient contribution of the compressed bars which are distant from the neutral axis.

The plots in fig. 11, referring to cyclic sinusoidal curvatures gradually increasing

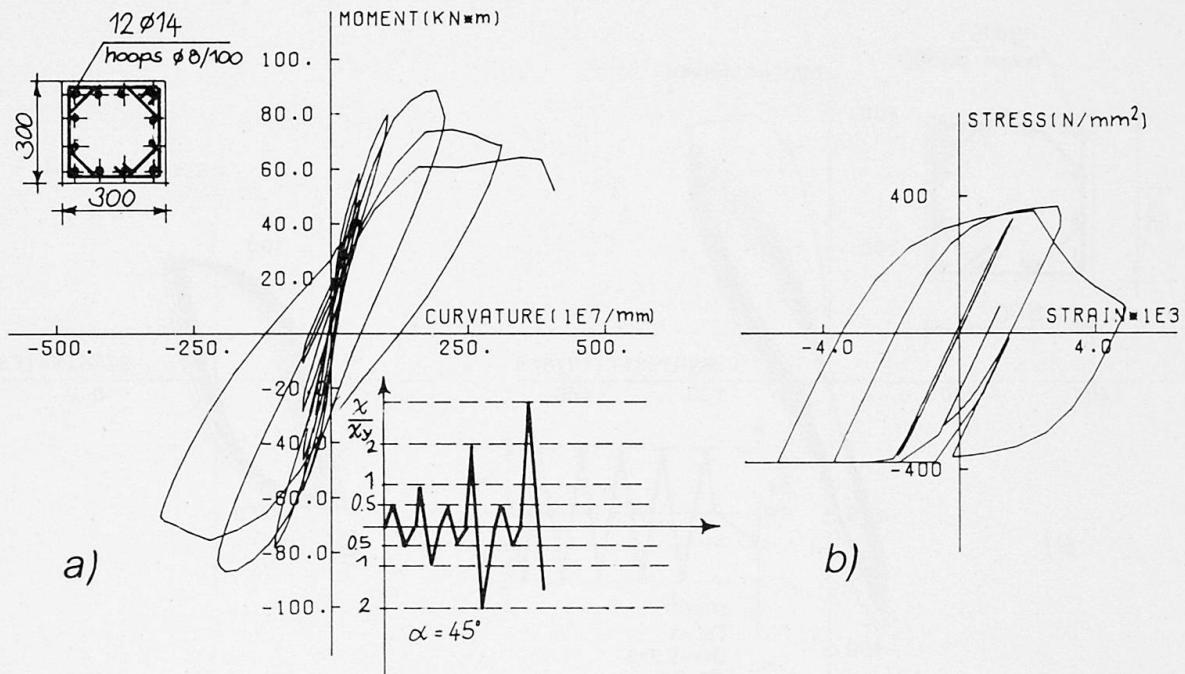


Fig. 11

and alternate to cyclic sinusoidal curvatures with a low amplitude, are difficult to comment in a few words.

## 6. SUGGESTIONS TO FURTHER RESEARCHES

We consider it very interesting to develop the study to draw the low strength fatigue curves referring to a large number of plastic hinges with different geometrical and mechanical properties. When we deal with loading histories of variable amplitude the problem is difficult and so it is necessary to introduce different criteria of study.

However on the one side some experimental checks of the results and, on the other side, refinements of the techniques of numerical simulation are necessary, particularly referring to shear influence and post-buckling of compressed bars.

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