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Non-linear Analysis and Strengthening of a Reinforced Concrete Building

Calcul du renforcement d'un bâtiment au moyen de l'analyse non-linéaire

Nicht-lineare Analyse für die Bemessung der Verstärkung von Stahlbetonbauten

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SUMMARY

Severe disorders appearing in a reinforced concrete framed building led to the necessity of determining its safety level. This aim was achieved both with experimental tests and with numerical analyses, linear and non-linear. Non-linear analysis proved to be very useful in determining the moment redistribution capability of the structure and realistic values of displacements.

RESUME

Les dommages assez graves apparus dans une structure à cadres en béton armé ont nécessité la détermination de son niveau de sécurité. Des essais et des analyses numériques linéaires et non-linéaires ont été réalisés. L'analyse non-linéaire a permis de déterminer la capacité de redistribution des moments à l'intérieur de la structure, ainsi que de calculer les valeurs réalistes de déplacements.

ZUSAMMENFASSUNG

Das Auftreten von schweren Schäden in einer Konstruktion aus Stahlbeton mit Rahmencharakter macht die Bestimmung ihres Sicherheitsniveaus notwendig. Zu diesem Zwecke wurden experimentelle Untersuchungen sowie numerische Analysen linearer und nicht-linearer Art durchgeführt. Die nicht-lineare Analyse erwies sich als äusserst nützlich um die Umlagerungskapazität der Konstruktion und die auftretenden Verschiebungen zu bestimmen.



1. INTRODUCTION

One of the fields in which non-linear analysis of structures is most useful is certainly the evaluation of the degree of safety of existing damaged r.c. buildings.

In fact, in these cases it is important to obtain a realistic image of the distribution of action effects and displacements, so that decisions concerning repairs can be taken according to the "true" behaviour of the structure. Repairs of damaged buildings are extremely costly; if the redistribution of action effects is well known, through non-linear analysis, the added margin of safety permitted by structural ductility can be understood and sometimes exploited, and the amount of repairs necessary to restore an adequate safety level can be reduced.

Besides, non-linear analysis permits to determine realistic values of displacements at service load level.

In the following paragraphs the investigations concerning a university building, in which severe disorders were produced by poor design and execution are described. The importance of the application of non-linear analysis in understanding the structural behaviour is emphasized.

2. USE OF NON LINEAR ANALYSIS IN ASSESSING THE SAFETY AND SERVICEABILITY OF EXISTING R.C. STRUCTURES

As it is well known, the analysis of r.c. structures at ultimate limit state is performed in a rather conventional and unrealistic way if linear analysis is used. In fact the critical sections of the structure are verified if

$$M_{act d} \leq M_{yu}$$

where M_{yu} = yielding moment of the section

and $M_{act d}$ = acting design moment from linear elastic analysis.

This procedure is on the safe side in most cases and is normally used to design new structures, but has the following shortcomings:

- The acting moments are computed disregarding the redistribution produced by cracking and plastic behaviour of reinforced concrete. Therefore there is a contradiction in the fact that, while the yielding moment is calculated considering non-linear constitutive laws of materials, the acting moment is computed assuming that the behaviour of elements made of the same materials is perfectly linear.
- The ultimate limit state which is conventionally adopted is a "first yield" limit state, that is no plastic rotation of critical sections is permitted, while the ultimate limit state as defined by modern codes, such as the CEB Model Code [1] corresponds to the reaching of either a failure mechanism or the rotation capacity in a critical section, thus fully exploiting the available plastic redistribution, which can be very high in ductile structures.

As a consequence, non-linear analysis need be performed to assess the behaviour of a given structure in a realistic way.

3. EXAMPLE STRUCTURE - DESIGN CRITERIA - EXTENT AND NATURE OF DAMAGE

The aforesaid considerations were applied to the case of a recently built r.c. building, where damages occurred due to excessive deflections (Figs.1 and 2). The building is schematically shown in Fig.3: it is composed of 3 floors, the ground floor being used as lecture-hall. The others were partitioned in order to be used as studies and laboratories.

The floors, 15 meters square, are orthogonal grids of r.c. beams. These beams were designed as simply supported, despite the fact that no discontinuities exist between the beams and their supporting columns.



Fig.1 Typical floor

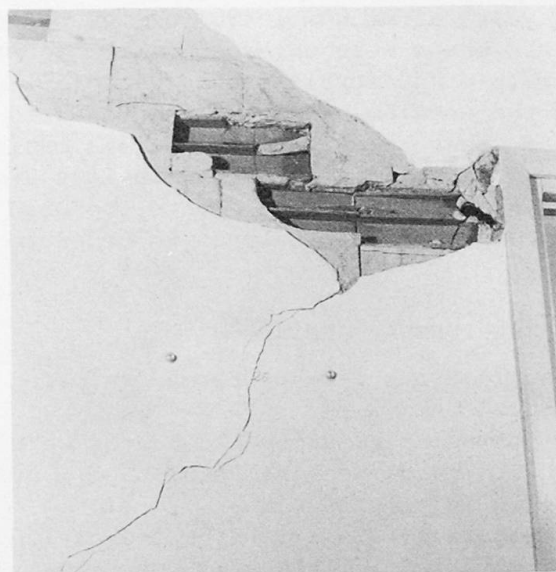


Fig.2 Cracking in partitions

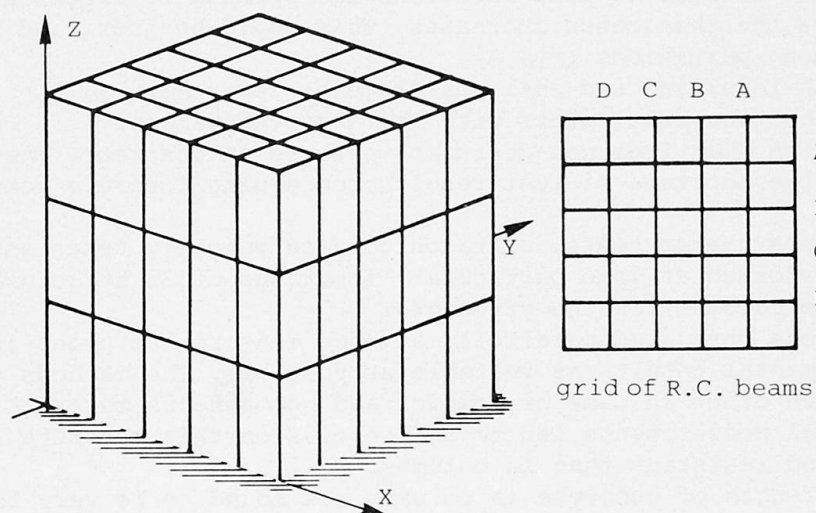


Fig.3 Simplified topology of the structure

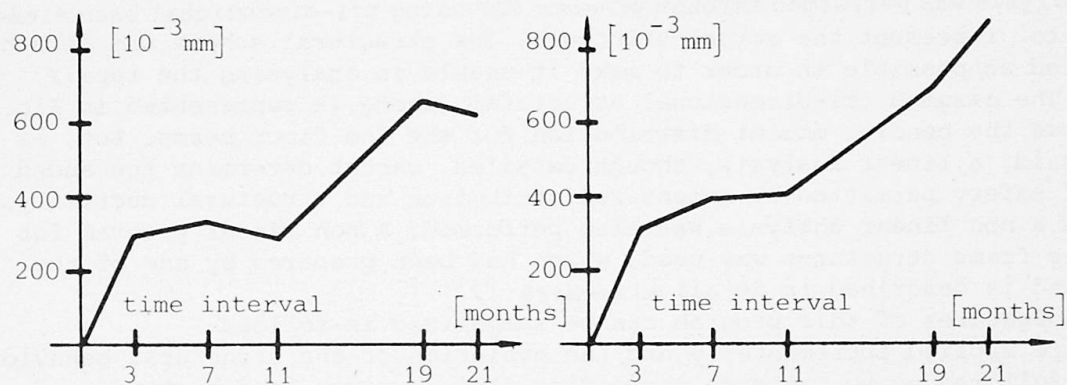


Fig.4 Gauge length variation across two cracks (versus time)



Few years after completion of the building large and increasing deflections of the beams were observed, which led to cracking in partitions (Fig.2). A group of 30 strain-gauge lengths were chosen across the cracks in partitions and the opening of cracks was measured and plotted versus time. Fig.4 shows some typical diagrams making clear the increasing trend of the deformation, if seasonal variations are suitably taken into account. The increase in damage led to the need of accurate experimental measurements and computational analyses in order to assess the safety conditions.

4. EXPERIMENTAL CHECKS

Investigations were performed in order to ascertain the reasons of the observed behaviour.

The following experimental checks were carried out:

- strain-gauge measurements;
- levelling of the slabs;
- penetrometer tests of the foundation soil;
- non-destructive tests on concrete.

The conclusions drawn from there are summarized here.

The mentioned strain-gauge measurements across cracks were carried on and showed two different trends. At some locations the opening of cracks seems to decrease, at others the phenomenon increases. This could be justified by stress redistribution among partitions (Fig.4).

The results of slab levelling and soil penetrometer tests exclude all connections of foundation settlements with the observed damages.

Essential information came from non-destructive tests on concrete, that were made necessary by the shortage of test results concerning concrete compressive strength.

Surface hardness sclerometer tests, ultrasonic pulse velocity tests and pull-out tests were performed at some particular locations of 33 structural elements (beams and columns) in the structure.

Those testing methods were used parallelly at each measurement point in order to make the experimental results as reliable as possible. The methods are consistent with each other in case of compact and homogeneous material.

All the experimental measurements led to the conclusion that concrete in beams was more compact and resistant than in columns.

The compressive strength of concrete in columns was found to be very low, therefore in the computations a 12 MPa design compressive strength was assumed, coinciding with the minimum allowed by C.E.B. Model Code [1].

5. LINEAR AND NON-LINEAR ANALYSES

Linear analysis was performed through program SAP using tri-dimensional beam elements in order to represent the actual structure. The structural scheme was chosen as detailed as possible in order to make it usable in analyzing the repair effects. The assumed tri-dimensional structural scheme is represented in Fig. 3. Fig.5 shows the bending moment distribution for the 2nd floor beams. But, as already said, a linear analysis, though detailed, cannot determine the added margin of safety permitted by moment redistribution and structural ductility. Therefore a non linear analysis was also performed. A non linear program for r.c. plane frame structures was used, which has been prepared by one of the authors and is described in detail elsewhere [2][3].

The basic features of this program can be summarized as follows:

- Loads are applied incrementally and the evolution of the structural behaviour can be followed up to collapse proceeding along a given load history.
- Non linear effects can be simulated:
 - a) Cracking and "tension stiffening"

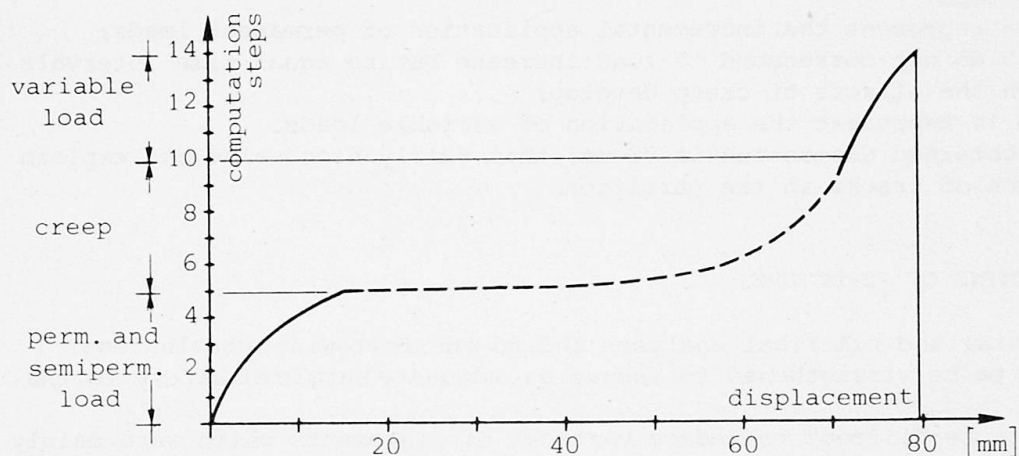
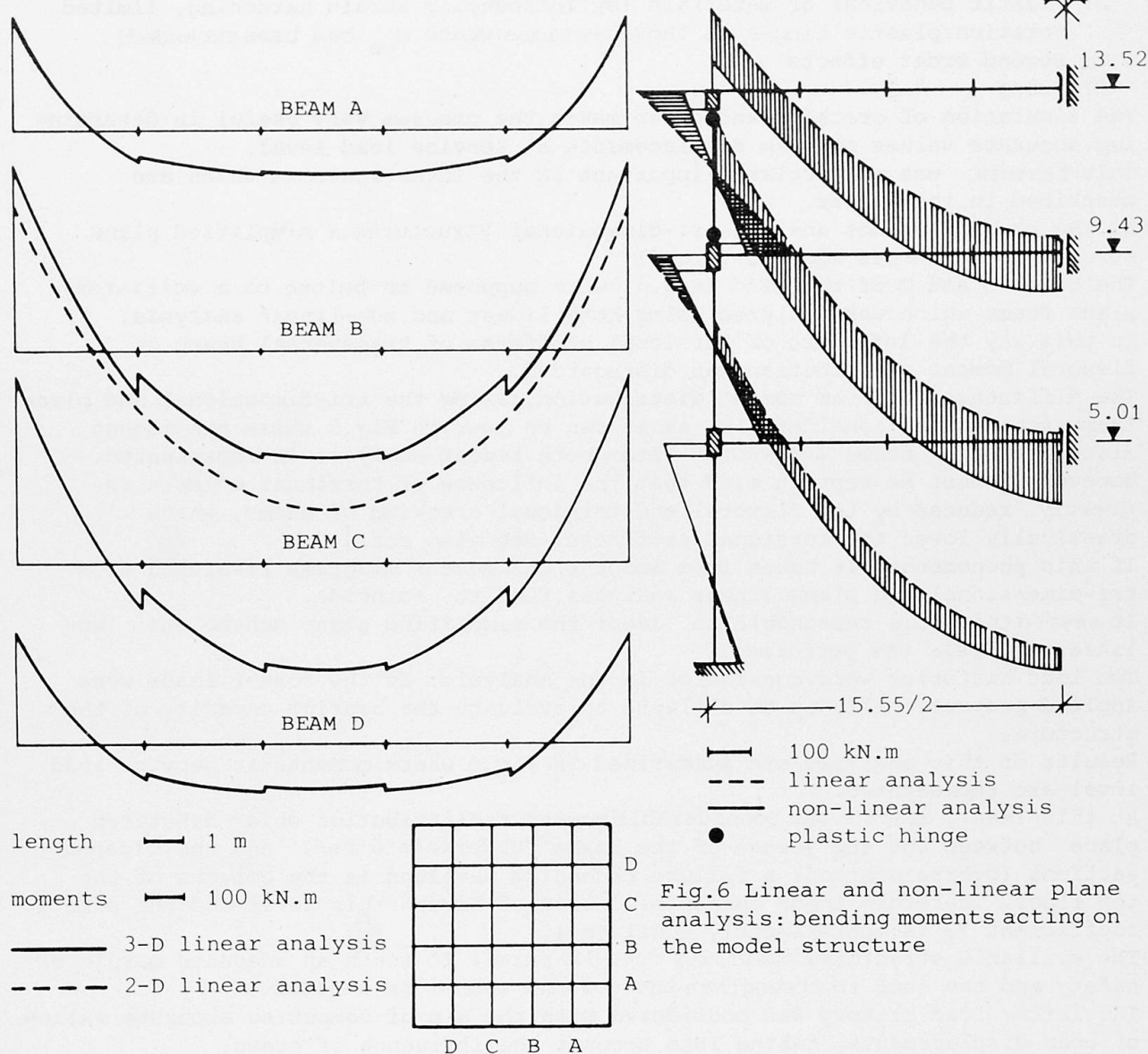


Fig.5 Tri-dimensional linear analysis: bending moments acting on 4 beams of 2nd floor grid

Fig.7 Evolution of vertical displacement at mid-span of a 2nd floor beam



- b) Plastic behaviour of materials (by introducing strain hardening, limited rotation plastic hinges in those sections where M_{yu} has been exceeded).
- c) Second order effects
- d) Creep

The simulation of cracking and creep makes the program very useful in determining accurate values of beam displacements at service load level.

This feature was particularly important in the investigations which are described in this paper.

As the program cannot analyze tri-dimensional structures, a simplified plane structural scheme was adopted (Fig.6).

The beams B and C of the grid (Fig.3) were supposed to belong to a multistory plane frame which was analyzed using both linear and non-linear analysis.

In this way the influence of torsional stiffness of transversal beams on flexural moment distribution was disregarded.

The differences in beam moment distribution, using the tri-dimensional and plane frame schemes are considerable, as it can be seen in Fig.5, where the moment distribution in beams deriving from both linear analyses is represented.

However it must be kept in mind that the influence of torsional moments is greatly reduced by the flexural and torsional cracking of beams, which drastically lower the torsional stiffness. See also ref. [4].

If this phenomenon is taken into account the moment diagrams resulting from tri-dimensional and plane linear analyses tend to coincide.

It seemed therefore reasonable to adopt the simplified plane scheme when non linear analysis was performed.

Two load histories were considered in the analysis. In the former, loads were applied proportionally up to collapse to evaluate the bearing capacity of the structure.

Results of this analysis are summarized in Fig.6 where moments at service load level are represented.

At this level, despite the considerable moment redistribution which has taken place between the end joints of the beams (underreinforced) and the midspan sections (overreinforced) a failure mechanism develops in the columns of the top floor. Therefore loads cannot be increased beyond this level and the safety coefficient γ_f is approximately equal to 1.

The available structural ductility does not permit to reach an adequate margin of safety and the need to strengthen the columns cannot be avoided.

The latter load history was considered with the aim of computing accurate values of beam displacements, taking into account the influence of creep.

First permanent and semipermanent loads were applied; then creep was simulated using the procedure described in [3]; at last variable loads were applied incrementally up to service load level.

The values of vertical deflections are represented in Fig.7, in function of the computation steps:

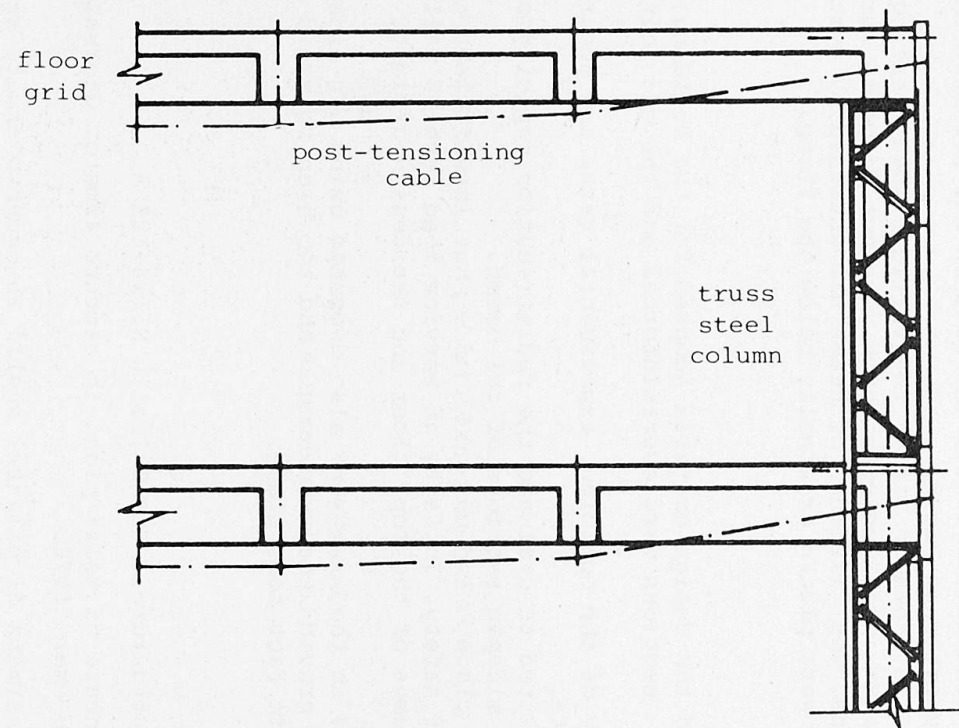
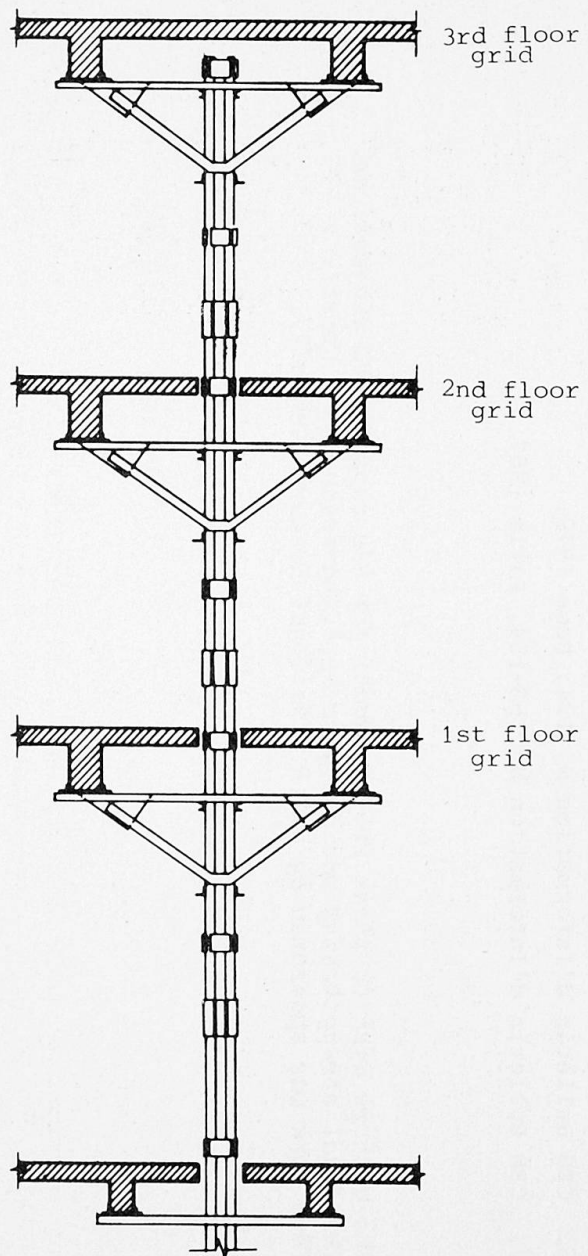
- steps 1 to 5 represent the incremental application of permanent loads;
- steps 6 to 9 do not correspond to load increase but to equal time intervals during which the effects of creep develop;
- steps 10 to 14 represent the application of variable loads.

The maximum obtained deflection is 79 mm; this fairly high value may explain the formation of cracks in the partitions.

6. STRENGTHENING OF STRUCTURE

The experimental and numerical analyses led to the following conclusions.

- Columns had to be strengthened to insure an adequate margin of safety to the structure.
- Beams had to be "lifted" to reduce vertical displacements which were mainly responsible for the formation of cracks in partitions.



△ Fig.9 Final strengthening scheme (truss steel columns and post-tensioning cables)

◁ Fig.8 Temporary strengthening scheme to support grid beams



A temporary support was first designed to reduce the load on the columns; the scheme of this support is represented on Fig.8.

The final strengthening scheme is shown on Fig.9: the vertical load bearing function was committed to truss steel columns which were put in place surrounding of the existing columns. The "lifting" of beams was obtained by post tensioning steel cables which were placed externally below the floor.

7. CONCLUSIONS

The previous paragraphs showed the design criteria adopted for the building: as a consequence the mid-span sections were overreinforced and the end joints underreinforced.

To assess the degree of safety of the building, experimental tests and accurate linear analyses were inadequate.

Only non linear analysis permitted to evaluate the redistribution capacity of the structure between end and midspan sections of the beams.

The redistribution which took place is indeed considerable, but insufficient to guarantee an adequate level of safety. In fact, at service load level a failure mechanism develops in the columns of the top floor and the design ultimate limit state cannot be reached.

Maximum vertical displacements in the beams were also computed using non linear analysis. These displacements proved to be considerable and too high to avoid the damages in partitions, which in fact took place.

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