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## Calculation of Damaged and Repaired or Strengthened Concrete Structures

Calcul de structures en béton endommagées et restaurées

Berechnung beschädigter und instandgesetzter Stahlbetonbauten

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### SUMMARY

This article presents an analytical method for an accurate calculation of the stiffness coefficients of a column, damaged or repaired and/or strengthened after damage. Analytical models are used for the description of the damaged region of the column. The jacket-column system is analysed using compatibility conditions for the deformations of the jacket and the column. Several practical conclusions and an example showing the influence of the damaged or repaired and strengthened member on the response of the overall structure, are given.

### RESUME

L'article présente une méthode analytique pour le calcul précis des coefficients de rigidité d'une colonne, endommagée ou restaurée/renforcée après l'endommagement, à l'aide d'un programme pour l'ordinateur. La description des régions endommagées de la colonne est réalisée à l'aide de modèles analytiques. Le système colonne-chemise est analysé à l'aide des conditions de compatibilité des déformations de la colonne et de la chemise. Quelques conclusions sont présentées et un exemple montre l'influence d'un élément endommagé ou restauré sur le comportement de la structure entière.

### ZUSAMMENFASSUNG

Der Beitrag gibt ein analytisches Verfahren für die genaue Berechnung des Steifigkeitskoeffizienten einer beschädigten oder instandgesetzten und verstärkten Stütze mit Hilfe eines Computerprogramms. Für die Beschreibung der beschädigten Stelle der Stütze werden analytische Modelle benutzt. Das System Ummantelung-Stütze wird mit Hilfe der Kompatibilitätsbedingung für die Verformungen des Mantels und der Stütze analysiert. Praktische Schlussfolgerungen und ein Beispiel zeigen den Einfluss des schadhafte und instandgesetzten Bauteiles auf das Verhalten der ganzen Konstruktion.



## 1. INTRODUCTION - PHILOSOPHY OF DESIGN OF DAMAGED AND REPAIRED-STRENGTHENED REINFORCED CONCRETE STRUCTURES.

One of the most important steps during a repair and/or strengthening procedure for a damaged reinforced concrete (R.C.) structure, is the estimation of the overall (residual) bearing capacity of the structure, regarding both the vertical and the horizontal actions, either immediately after damage, or after repair and/or strengthening. For such an estimation, and in order to take into account the real mode of behaviour of the structure, the calculation of the characteristics of the individual members is needed. In general, damaged building elements are suffering a certain decrease of their stiffness, while, repaired and/or strengthening building elements, may be subjected to a stiffness increase. Sometimes, members reestablished to their initial (before damage) stiffness level (e.g. epoxy injection in a crack) may still have a reduced stiffness due to very fine but extensive invisible cracks which are impossible to restore. Nevertheless, in most cases, a considerable redistribution of action effects, is expected in the structure.

Redistribution may be estimated for the whole structure by means of ordinary structural analysis methods, on condition that member characteristics are known. On the other hand, the calculation of the stiffness characteristics of the structural elements is important, due to modification of the dynamic characteristics (fundamental period etc.) of the structure, and thus of the level of a future dynamic loading (e.g. earthquake loads).

To take into account, during the analysis of a R.C. element, the effects of local defects and the deterioration such as cracks, loss of strength or loss of section, is in general a difficult task. In the present article the general principle of a new (modified) stiffness for the damaged region is considered. Such a stiffness modification for the damaged region of the element may appear either to the stiffness versus deflection, mainly for beams and columns (initial value  $EJ$ , new value  $(EJ)'$ ; variation  $\Delta(EJ)$ ,  $E$ =Modulus of Elasticity,  $J$ =moment of inertia), or to the stiffness versus axial shortening, mainly for columns (initial value  $EA$ , new value  $(EA)'$ ; variation  $\Delta(EA)$ ,  $A$ =section), or to the stiffness versus angular deformations, mainly for walls and columns (initial value  $GA$ , new value  $(GA)'$ ; variation  $\Delta(GA)$ ,  $G$ =shear modulus). The first of these three cases, is considered in this article.

Namely, the results of a research project, referring to the problem of modification of the stiffness of a reinforced concrete structure after damage or after repair and/or strengthening, undertaken by the authors in the National Technical University of Athens, are presented. For the accurate calculation of the new stiffness of these structures, an analytical method for the calculation of the equivalent stiffness of the individual structural elements is proposed, based on properly selected models. Although the method presented is general, attention is focused on the columns, which are usually the most sensitive parts of the structure. Columns are considered either damaged, or strengthened with the common method of jacketing. Slip between jacket and columns, is also taken into account. The analysis is performed using the transfer matrix method of linear structural analysis, adequately suitable for the case. The calculations for the problem are performed through a general computer program, especially written for the case, and including all analytical models for the damaged area as well as the interaction between jacket and damaged element in case of repair and/or strengthening. It is to be noted here that linear structural analysis, neglecting nonlinearity, leads to more or less approximate solutions. But it remains a very powerful tool for the designer, for the systematic analysis of structural problems. Especially for the case of slightly damaged members, with non serious inelastic damage, but also for complicated problems, as those described above, where due mainly to the great number of parameters involved-it is not still possible to formulate a general mathematical model of the problem taking into account all parameters of

nonlinearity and leading to numerical results, ready for a practical application.

2. STIFFNESS MODIFICATION OF COLUMNS AFTER DAMAGE,

In order to calculate the new stiffness of a damaged column, the damaged and undamaged parts of the member are distinguished. The length of each part is expressed as a percentage  $\mu$  of the total height  $h$  of the column. For the undamaged parts, the geometrical characteristics remain obviously the same, as before damage. For the damaged parts, the modification of these characteristics must be estimated. The most important of them, the stiffness  $(EJ)$  is now a percentage  $\lambda$  of the initial stiffness  $(EJ)$ .

Considering an undamaged column AB of height  $h$ , section height  $d$ , section width  $b$  and stiffness  $EJ_0$ , let the well known stiffness coefficients be:

$K_{0DD}$ ,  $K_{0RD}$  for unitary displacement  $\delta=+1$  at the end B (top) and  $K_{0DR}$ ,  $K_{0RR}$  for unitary rotation  $\phi=+1$  at the end B, as shown on figure 1.

For a damaged column, the elements of the stiffness matrix  $[K]$  can be expressed as percentages  $\rho$  of the equivalent values of the elements of the stiffness matrix for the undamaged column. Using the transfer matrix method, which is adequately suitable for this case of various successive parts composing the whole column, the percentages  $\rho$  are expressed as functions of the parameters  $\lambda, \mu$  of the parts and the ratio  $h/d$ . These expressions have been formulated for the general case of  $n$  successive damaged or undamaged parts, in closed algebraic formulas. Using these formulas for the case of a column damaged only at the bottom A over a length  $\mu \times h$ , the three coefficients  $\rho$  are given on figure 2 as a function of the ratio  $h/d$ , for various values of the parameter  $\lambda$  of the damaged part. Even for  $\lambda=1$ , the coefficients  $\rho$  are different than 1, due to the influence of the shear forces' contribution.

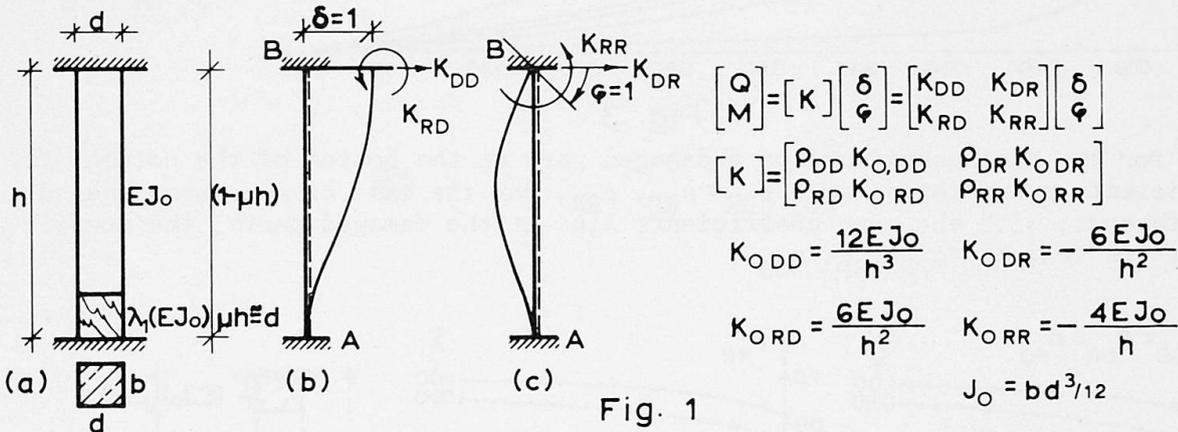


Fig. 1

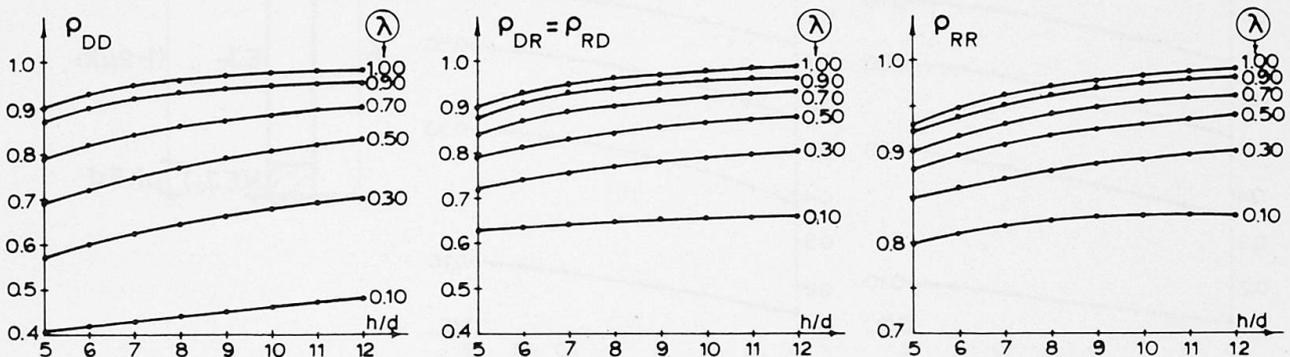


Fig. 2

The conclusion from these formulas and diagrams is that the values of the three coefficients  $\rho$  for each pair of values  $h/d$  and  $\lambda$  are significantly different and thus, it is not possible to assume a constant diminution of the stiffness of the

column, in order to take into account the diminuation  $\lambda$  of the damaged part.

The modification of the column stiffness has a direct influence on the stiffness of the overall structure. As an example the simple frame of the figure 3 is considered. The geometrical characteristics are given on the figure. The two columns BA and CD are damaged at the bottom, with the same stiffness diminuation  $\lambda$ . On fig.3, the variations of moments M, shear forces Q and fundamental period T, are given as a function of the parameter  $\lambda$  and as percentages of the corresponding values of the undamaged structure. Obviously, for  $\lambda=1$ , all diagrams are zero, while for  $\lambda=0$ , they reach the value of a similar frame with joints A and D absolutely free to rotation.

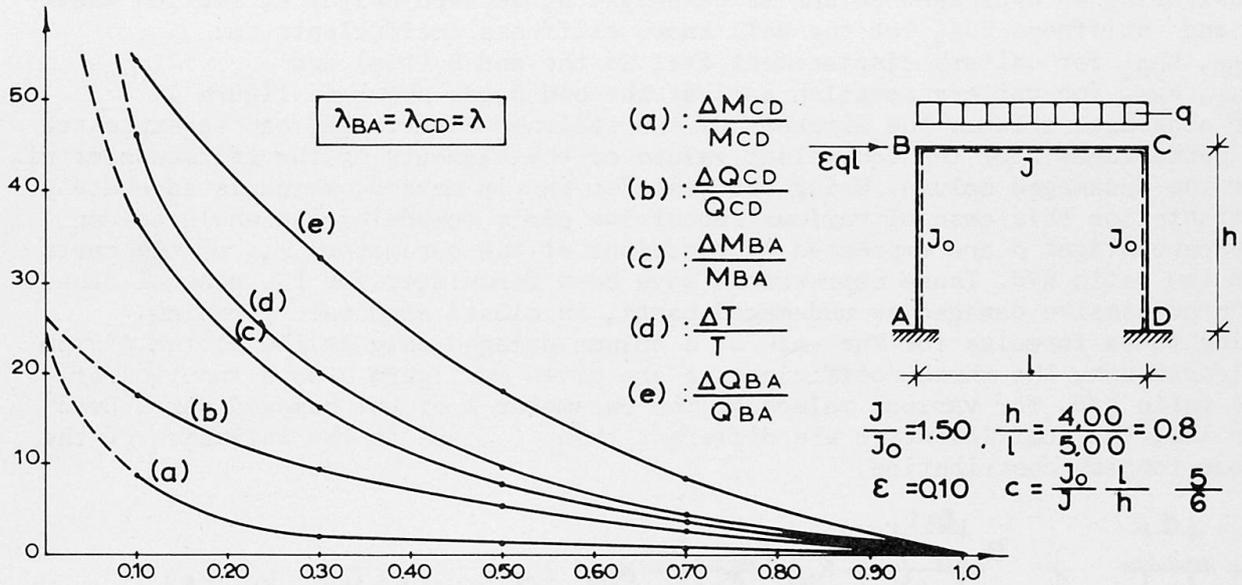


Fig. 3

While for the case considered of a damaged part at the bottom of the column, the coefficients  $\rho$  are three:  $\rho_{DD}, \rho_{DR} = \rho_{RD}, \rho_{RR}$ , for the case of a column damaged at both ends, with the same coefficients  $\lambda, \mu$ , at the damaged parts, the coefficients are two:  $\rho_{DD} = \rho_{DR} = \rho_{RD}, \rho_{RR}$ .

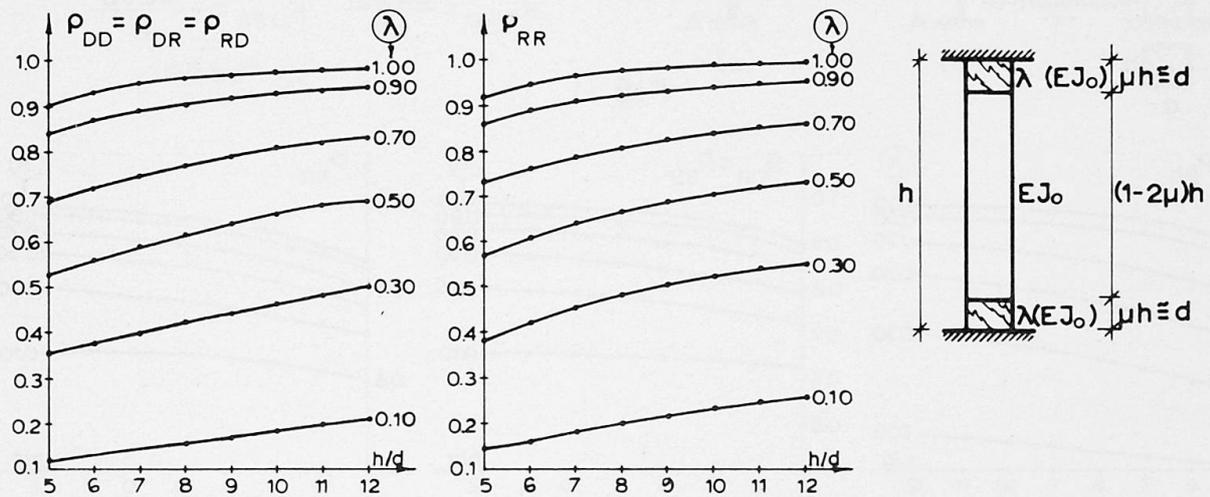


Fig. 4

On figure 4 the variation of the coefficients  $\rho$  as a function of the parameters  $h/d$  and  $\lambda$  for this last case are presented.

When the value of the parameter  $\lambda$  is known, the coefficients  $\rho$  can be calculated

through the aforementioned formulas. But practically, the problem is the estimation of this diminution  $\lambda$  of the stiffness of the damaged part(s). For this estimation an analytical model should be used,

3. ANALYTICAL MODELS FOR THE DAMAGED AREA,

The damage types are classified for various damage levels in groups and four models, one for each group, are used.

The first model describes the case of an horizontal crack of about 1mm width (fig. 5a). This type of damage is mainly due to local defects rather than to insufficient reinforcement. For that case, we assume a damaged length equal to 2 to 3 times of the crack opening on which the steel bars are operating as individual bars built-in the concrete. On condition of sufficient reinforcement, the column in that case reacts as about the undamaged column. As an example. the variation of the three coefficients  $\rho$  of the column are given on figure 6a, as a function of the ratio  $h/d$ . This example has been elaborated for a height of the damaged area  $0,002h$ ,  $\lambda=0,0005$  ( $\lambda$ =moment of inertia of steel bars versus moment of inertia of the whole column section),  $d/\delta=25$  ( $\delta$ =steel bar diameter),  $d'=0,90d$ , and steel to concrete ratio of moduli of elasticity  $n$  equal to 7.

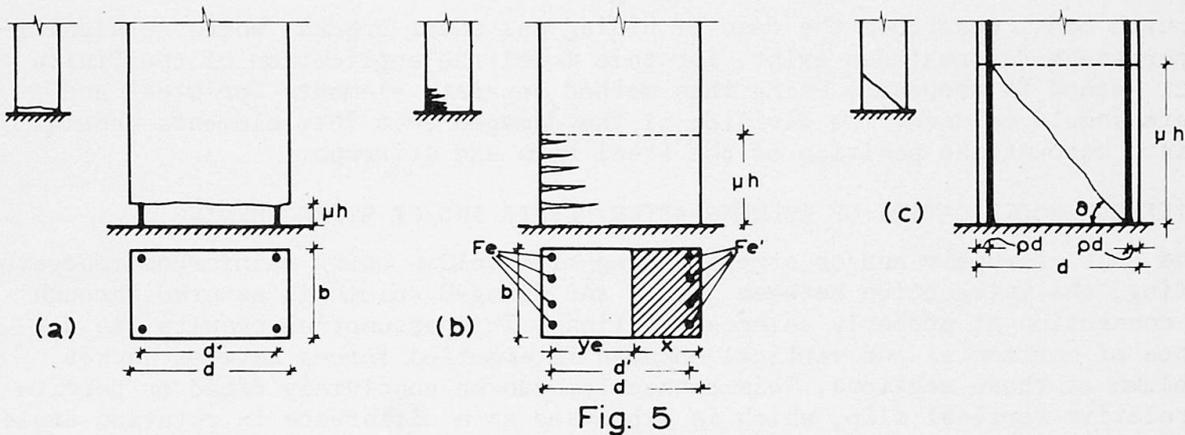


Fig. 5

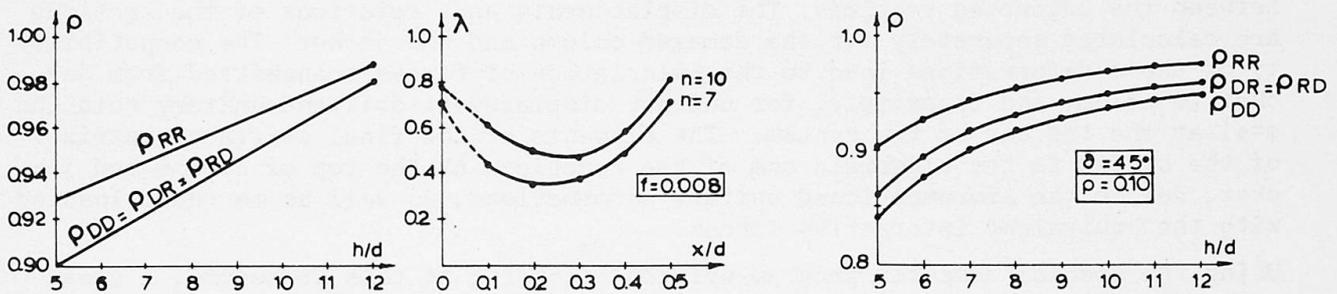


Fig. 6

The second model (fig. 5b), describes the case of frequent and large flexural cracks. For that case, the modification  $\lambda$  of the moment of inertia is calculated as a function of the height  $x$  of the compressive zone of concrete, taking into account all the steel bars. As an example, on figure 6b, the variation of  $\lambda$  is given as a function of the ratio  $x/d$ , for the case of minimum reinforcement  $f=0.008$  and two values of  $n=7$  and  $10$ . Further, for the calculation of the coefficients  $\rho$ , the diagrams of figure 2 can be used,

The third model (fig. 5c), describes the case of a diagonal shear crack of a width about 0,5mm, without permanent deformations. On the same figure the characteristics of the analytical model used are shown. On figure 6c the variation of the three coefficients  $\rho$  as a function of the ratio  $h/d$  are given for a column with a shear crack of the type considered at the bottom, without any other cracks,

The comparison of these figures with figures 2, shows that this case corresponds to a general diminution of the stiffness of the damaged area, equal about to  $\lambda=0,85$ . For this model, all coefficients  $\rho$  are the same for both diagonals of the damaged area. Nevertheless, it is possible that besides the main shear crack, other minor cracks in the damaged area, diminish the stiffness of the concrete. On figure 7, the values of the three coefficients  $\rho$  are given for various values  $\lambda$  of the diminution of the stiffness around the main crack.

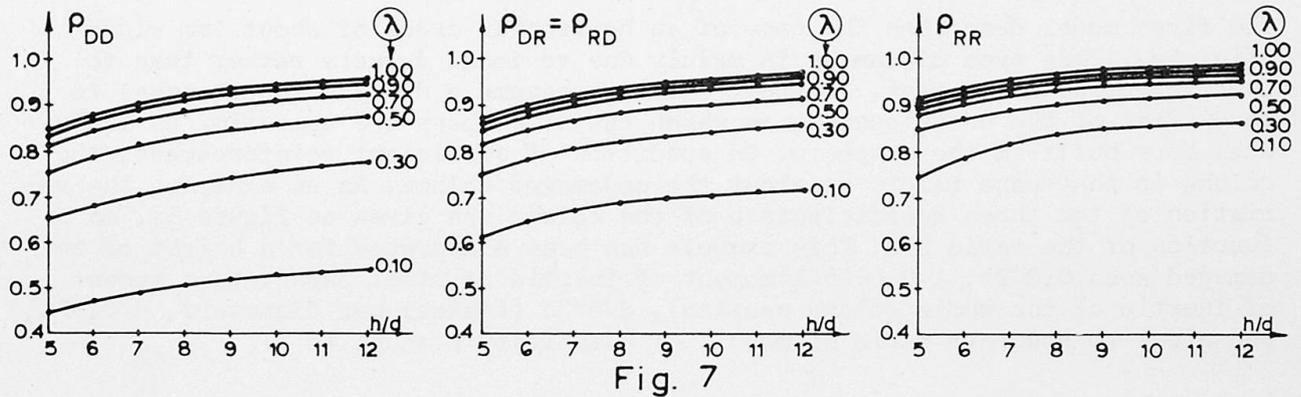


Fig. 7

The fourth model describes the case of bidiagonal shear cracks, where no significant permanent deformations exist. For this model the application of the Finite Element Method is proposed. Using this method separate elements for steel and concrete should be used. The division of the damaged area into elements should take into account the position of the steel bars and stirrups.

#### 4. STIFFNESS MODIFICATION OF COLUMNS AFTER REPAIR AND/OR STRENGTHENING.

For the case of repair and/or strengthening of a column using reinforced concrete jacketing, the interaction between jacket and damaged column is assured through their connection at properly selected sections. This assumption results the appearance of horizontal and vertical unknown interaction forces between jacket and column at these sections. This connection can be absolutely rigid or permitting relative vertical slip, which is expressed as a difference in rotation angle between the connected sections. The displacements and rotations of the sections are calculated separately for the damaged column and the jacket. The compatibility of these deformations lead to the calculation of forces transmitted from jacket to column and oppositely, for unitary displacement  $\delta=+1$  and unitary rotation  $\varphi=+1$  at the top end of the system. The elements of the final stiffness matrix of the system is the algebraic sum of the reactions at the top of column and jacket, due to the aforementioned unitary deformations, as well as to their loading with the equivalent interactive forces.

Using the special computer program written according to this procedure, a great number of cases have been investigated. It is to be noted here that for a jacket-strengthened column the value  $\lambda$  for the damaged column does not play an important role as the jacket mainly contributes to the final stiffness. So, while the differences between the various damage levels described through the equivalent models presented in the last paragraph are important for a damaged column, for the case of a jacket-strengthened column are of minor importance.

On figure 8, the variation of the first of the three coefficients  $\rho$  of the stiffness matrix is given indicatively as a function of the ratio  $h/d$  of the column, for various values of the ratios  $d/b$  of the dimensions of the column section and  $h/t$ , where  $t$  is the thickness of the jacket. Similar diagrams can be given for the other two coefficients  $\rho$ .

Generally speaking, the values of the coefficients  $\rho$  are fragmented upwards to the equivalent value of a full section with dimensions the external dimensions of the jacket (this value is higher than the value calculated with fully rigid

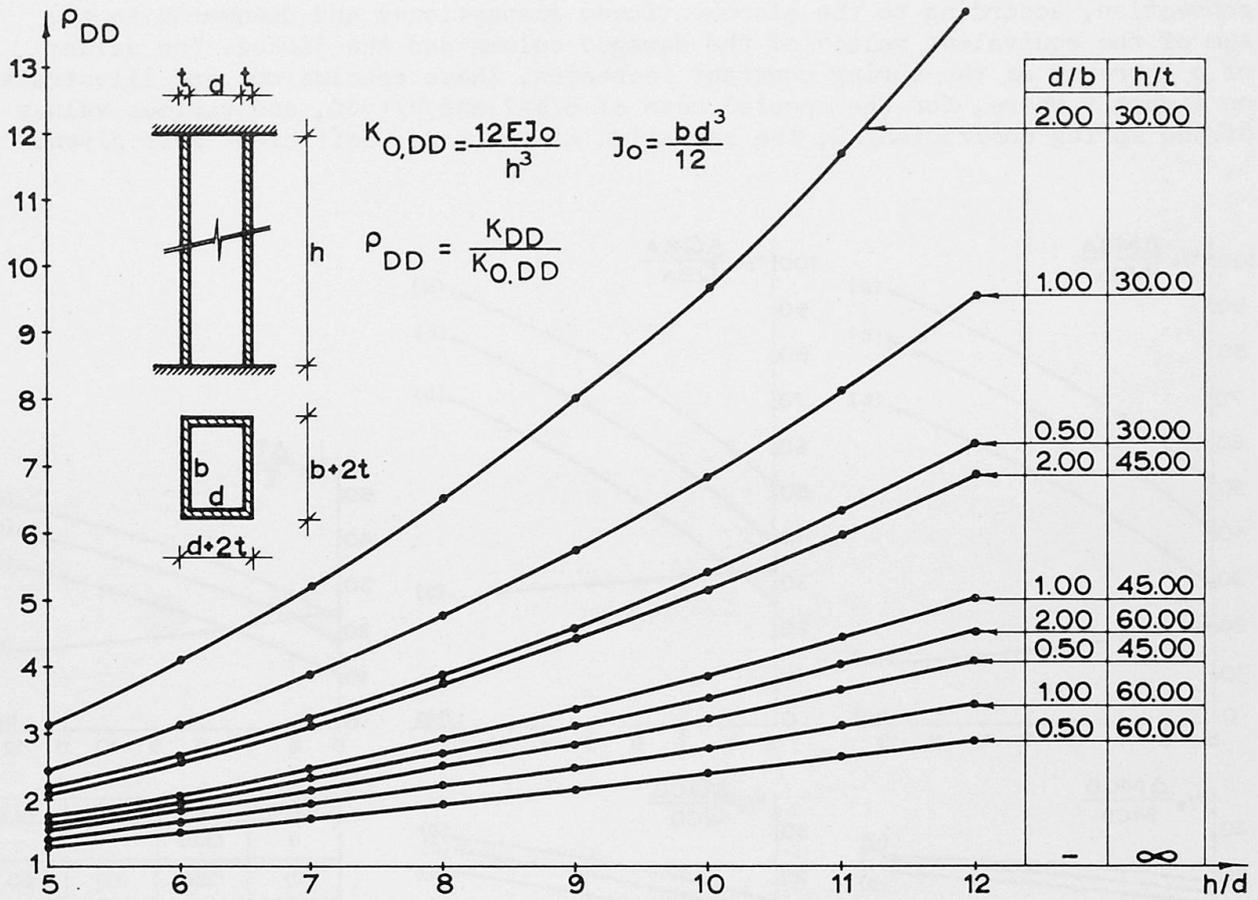


Fig. 8

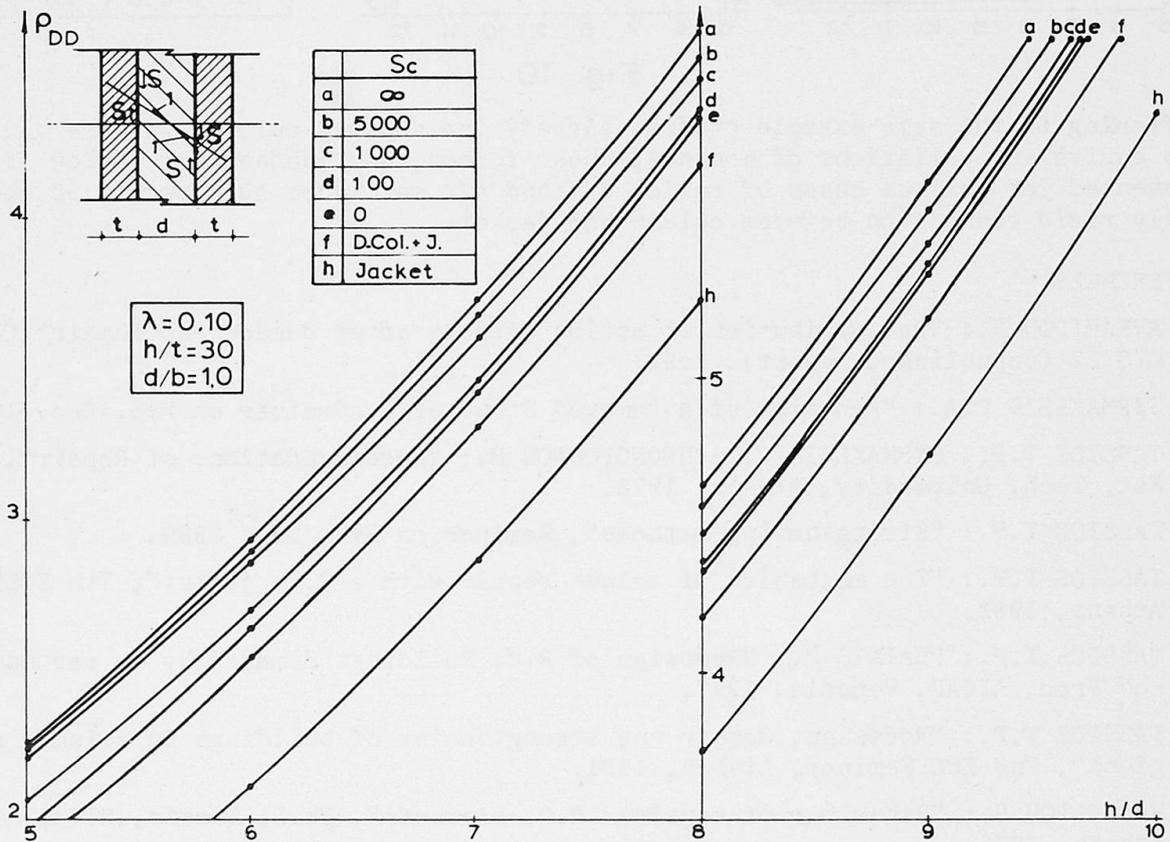


Fig. 9

connection, according to the aforementioned assumptions) and downwards to the sum of the equivalent values of the damaged column and the jacket. The values of  $\rho$  increase as the spring constant increases. These conclusions are illustrated on figure 9 where, for the special case of  $d/b=1$  and  $h/t=30$ , and various values of the spring coefficient  $S$ , the variation of the same coefficient  $\rho$  is given.

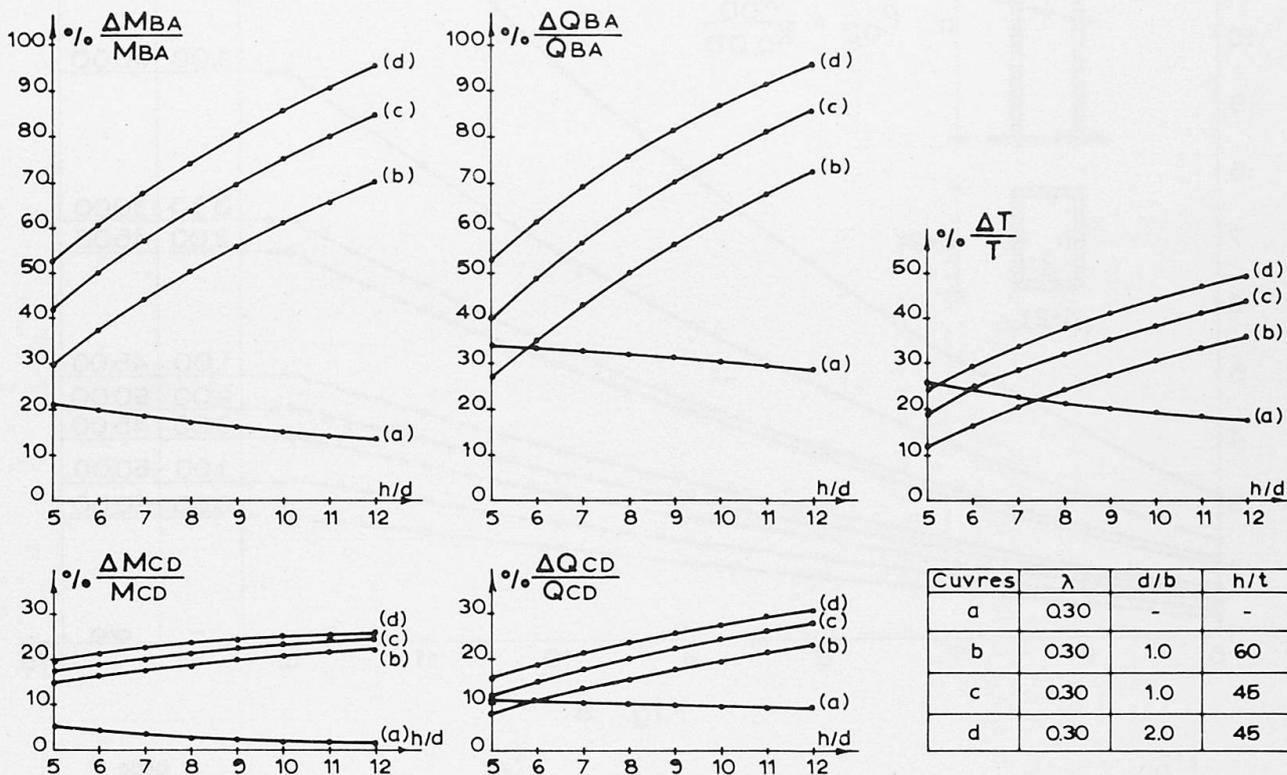


Fig. 10

Referring to the same example studied already for damaged columns (figure 3), the equivalent variations of moments, shear forces, and fundamental period are presented for various cases of ratios  $d/b$  and  $h/t$  on figure 10, for  $\lambda=0,30$  and fully rigid connection between column and jacket.

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