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## Non Conventional Finite Element Model for Strengthening of Masonry

Modèle à éléments finis pour le renforcement de murs en maçonnerie

Ein Finites-Element-Modell für die Verstärkung von Mauerwerk

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## SUMMARY

Damaged masonry walls and their reinforcement are studied using a plane stress finite element model specifically derived to simulate non-linear behaviour and anisotropy in an advanced state of cracking, as well as for modelling the addition of bonded or unbonded steel reinforcement.

## RESUME

Les murs en maçonnerie endommagés et leur renforcement sont étudiés au moyen d'un modèle à éléments finis en état plan de contraintes, établi pour simuler le comportement non-linéaire et l'anisotropie dans un état avancé de fissuration, ainsi que pour tenir compte de l'association avec des armatures d'acier, adhérents ou non-adhérents.

## ZUSAMMENFASSUNG

Das beschädigte Mauerwerk und seine Bewehrung werden mit Hilfe eines Modells aus finiten Elementen im ebenen Spannungszustand untersucht. Das Modell berücksichtigt nicht-lineares Verhalten, die Anisotropie im fortgeschrittenen gerissenen Zustand wie auch die Bewehrungseinlagen mit oder ohne Verbund.



## 1. INTRODUCTION

Conventional linear-elastic finite element techniques cannot adequately describe the behaviour (nor assess the safety) of damaged masonry walls, because the non-linearity and the anisotropy created by cracking and by the material deterioration play an essential role both in the distribution of action among walls, and in the state of stress within the walls. A plane stress model already developed by the Authors and simulating cracking has proved to be in good agreement with wall tests in axial and diagonal compression performed at the University of Pavia. [4]

The adopted approach, using bulk modulus and shear modulus (both variable in non linear field) seems to be convenient:

- load/displacement equations are decoupled;
- stress tensor and strain tensor invariants can be used;
- K and G have a clear physical meaning.

The aim of the present study is to check the validity of the approach in the case of coupled shear walls in an advanced state of cracking and deterioration, and to extend it to the assessment of strengthened masonry walls by improving the model with bonded or unbonded reinforcing steel bars.

## 2. THE MODEL UTILIZED IN SIMULATING MASONRY

### 2.1 General considerations

The model utilizes an eight node plane stress element. It is an isotropic model, but takes into account the anisotropy due to cracking, by changing with continuity its elastic properties. Cracking is considered as smeared over the entire element. The non-linear constitutive equations are written making use of the bulk modulus (K) and the shear modulus (G). [1][2][6]

### 2.2 Criteria for crack formation

Two different criteria have been considered for cracking. The first one is based on the attainment of tensile strength in the principal direction. The second one is based on the attainment of the tensile strength normal to the loading direction, due to the interaction between bricks and mortar (coaction criterion).

### 2.3 The bulk modulus K

The bulk modulus (K) assumes only two values:  $K_1$  before, and  $K_2$  after cracking. The second one is quite frequently less than zero: it means that the Poisson modulus is greater than 0.5. This happens because cracks are considered as a part of masonry, so that the volume of an element may grow under a mean compressive stress. [9]

### 2.4 The shear modulus G

The formulation used for G is already known for some geotechnical problems:

$$G = G_0 + \gamma_1 \sigma_m + \gamma_2 \sqrt{J_2}$$

in which  $\sigma_m$  is the third of the first invariant of the stress tensor and  $J_2$  is the second invariant of deviatoric stress tensor.

$G_0$ ,  $\gamma_1$ ,  $\gamma_2$  are constants to be determined by tests on a given type of masonry.

### 2.5 The input parameters

Nine input parameters are needed for the definition of the constitutive equations of the material:  $K_1$ ,  $K_2$ ,  $G_0$ ,  $\gamma_1$ ,  $\gamma_2$ ,  $\sigma_{cm}$ ,  $\sigma_{cb}$ ,  $C_1$ ,  $C_2$ .

$\sigma_{cm}$  is the cracking tensile stress of masonry

$\sigma_{cb}$  is the cracking tensile stress of a brick

$C_1$ ,  $C_2$  are constants (functions of the properties of bricks and of masonry) utilized in the coaction cracking criterion.

A more exhaustive description of the model can be found in [8]



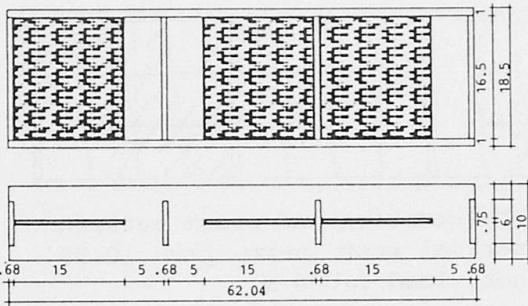


Fig.1 - Geometrical characteristics of Hendry's specimen (inches).

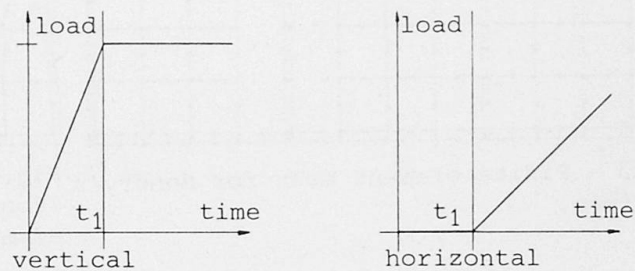


Fig.2 - Numerical loads laws

### 3. MODELLING OF THE ORIGINAL TESTS

#### 3.1 Characteristics of the experimental tests

The original experimental tests [3] were performed on coupled plain masonry walls confined by a thin reinforced concrete frame (Fig.1). The masonry structures were built by one-sixth-scale bricks and a ratio 1:4 cement and sand mortar. The horizontal load was increased under constant vertical precompression (Fig.2) in each test. Different tests had different vertical precompression.

#### 3.2 Problems in numerical simulation

In Fig.3 the very simple mesh used is shown. Not all the needed parameters were available from tests. The set was completed using values obtained from other experimental tests carried out on plain masonry. The constants used are:

$$\begin{aligned} K_1 &= 7000 \text{ N/mm}^2 \approx 1000000 \text{ psi} \\ K_2 &= -7000 \text{ N/mm}^2 \approx -1000000 \text{ psi} \\ G_0 &= 1400 \text{ N/mm}^2 \approx 200000 \text{ psi} \\ \gamma_1 &= 20000 \\ \gamma_2 &= 12000 \\ \sigma_{cm} &= 0.18 \text{ N/mm}^2 \approx 25 \text{ psi} \\ \sigma_{cb} &= 2.8 \text{ N/mm}^2 \approx 400 \text{ psi} \\ C_1 &= 1 \\ C_2 &= 0.4 \end{aligned}$$

The corresponding Young modulus in the elastic stage is about  $4000 \text{ N/mm}^2$  ( $562500 \text{ psi}$ ). It is important to underline that the most relevant parameters in these tests are  $G_0$ ,  $\gamma_1$ ,  $\gamma_2$  because change of shape is the governing phenomenon. The load history (Fig.2) allows the masonry characteristics change as a function of the vertical load before the application of the horizontal load. In particular, the shear modulus increases since the ratio between  $\gamma_2$  and  $\gamma_1$  is so that a monoaxial compression decreases the shear deformability of the material. Then, during the application of the horizontal load,  $G$  begins to decrease, as the tests show.

#### 3.3 Experimental and numerical results

In Fig.4 the shape of one simulated specimen at  $10700 \text{ N}$  ( $2500 \text{ lb}$ ) is shown and the elements in which at this step the principal tensile stress is greater than  $\sigma_{cm}$  are indicated with the direction of hypothetical cracks. The experimental test shows a very similar crack pattern. In Fig.5 it is possible to compare the experimental and the numerical force-displacement curves. The principal differences are:

- experimental curves seem to start with a sensible difference in shear modulus; this fact is not evident in numerical applications;
- for the highest precompression load two experimental curves are available: the numerical one follows one of them at low horizontal load; the other one at higher horizontal load; it seems nevertheless overestimate the stiffening effect of precompression.

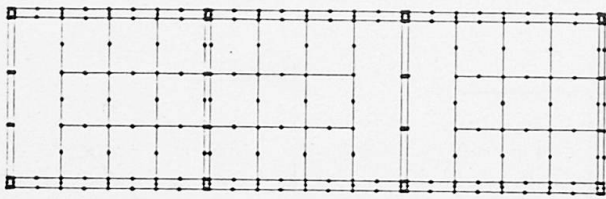


Fig.3 - Finite element mesh for Hendry's specimen.

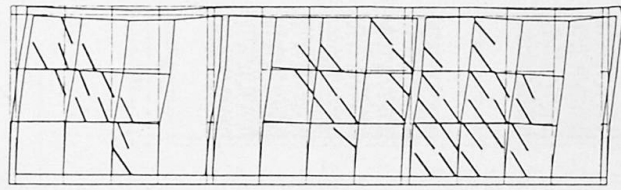
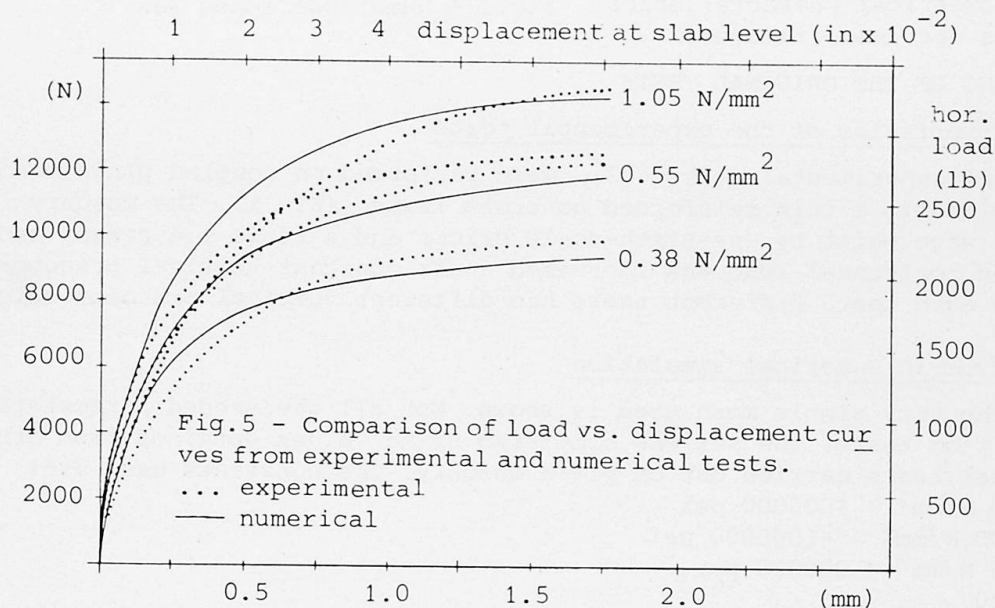


Fig.4 - Deformation and crack pattern from numerical test (vert. prec.  $0.55 \text{ N/mm}^2$ , hor. load  $11000 \text{ N}$ ).



#### 4. MASONRY STRENGTHENED BY STEEL BARS

##### 4.1 Element used for reinforcing bars

The behaviour of reinforcing bars is simulated by truss elements having a bi-linear constitutive law. Two kinds of mesh are used: the first one has just one truss element for each reinforcing bar; in the second mesh each bar is simulated by several trusses connected to the wall in intermediate nodes (Fig.6). From a physical point of view one may think to an external bar fixed at the ends to the masonry or to a reinforcement connected with continuity to the masonry by bond (concrete, shot-concrete, epoxy).

In both cases the reinforcement alone has no stiffness; in the second case each element works as a stiffening of the single connected plane stress element.

##### 4.2 Bond

Bond between bars and masonry is taken into account only in a global way, it means that the corresponding nodes are simply connected one to each other. The use of bond elements is not necessary for two reasons:

- the global behaviour is studied, and not the local stress situation between bars and masonry.
- masonry in tension is already taken into account in the formulation of the constitutive law.

The apparent increase of stiffness of bars due to the stress transmission to masonry is obtained by simply increasing their Young modulus. [12]



#### 4.3 Loads

As seen before, in the case of plain masonry confined by a R.C. frame, horizontal loads were applied at an edge of the upper slab.

In the case of not confined plain masonry this is not possible.

There are two kinds of problems. The first one does not depend on the model: even if a linear elastic model is used loads do not redistribute themselves in the same way as having a rigid slab above the walls, because it becomes of some importance the compressive or tensile deformation of the wall which the load is applied to. In that case it is better to apply the load to the truss connecting the walls. If the material constitutive equations are non-linear also this procedure is not suitable because the two elements directly connected to the truss immediately degrade.

The adopted solution is to distribute also the horizontal load to all the nodes at the upper edge of walls (Fig.7).

#### 4.4 Simulated meshes

In Fig.8, on the left hand, the different meshes simulated are shown. In order to check the validity of the new model, the structure of Fig.1 has been simplified by considering the masonry panels without the r.c. frame. The two walls are connected by a rigid truss. Two principal reinforcing systems are used: the vertical one should improve the flexural behaviour; the horizontal one, coupled to the vertical, should improve the shear behaviour. Each horizontal bar has a section of  $5 \text{ mm}^2$ , each vertical bar has a section of  $8 \text{ mm}^2$ ; the horizontal reinforcement is therefore 2 per mille of the masonry section, and the vertical one is 4.5 per mille.

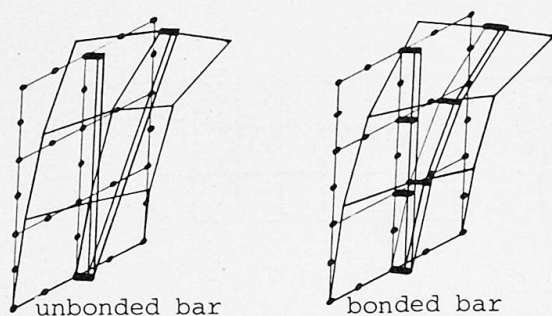


Fig.6 - Different behaviour for different simulation of bars.

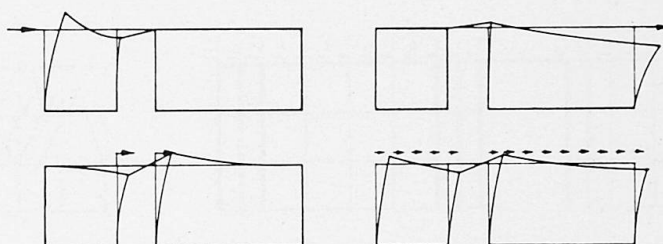


Fig.7 - Loads and deformation.

### 5. NUMERICAL RESULTS

#### 5.1 Deformation and crack pattern

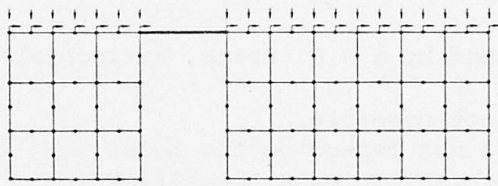
In figure 8, on the right hand, deformation and cracks of each specimen are drawn in front of the corresponding mesh. Plain masonry has both flexural and shear cracks; the deformed shape too shows that also the flexural behaviour is very important when the greatest part of tensile stress is not taken by a R.C. frame.

Cracks at the upper edge of the walls are due to the direct application of loads; the same reason justifies the large displacements at the same edge.

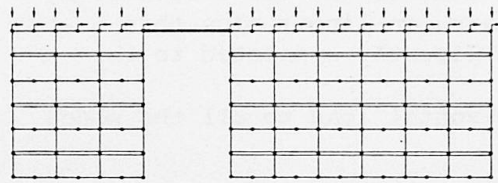
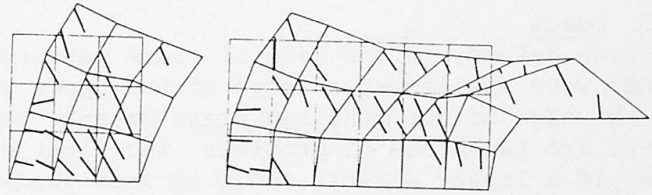
The element more interested by this behaviour is at the right upper corner, in fact it is the only one not confined for tensile stresses. When a horizontal reinforcement is added, the deformation is less; no significative difference is observed in the crack pattern. With vertical reinforcement only, the displacements become even smaller and flexural cracks disappear.

On the other hand, there is no sensible benefit for the shear cracks.

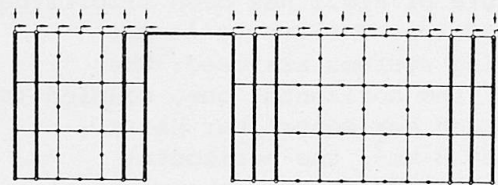
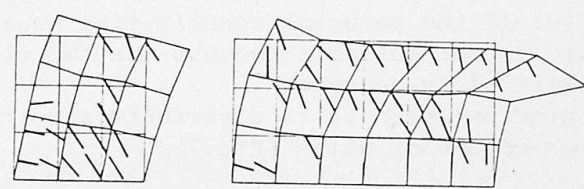
A considerable improvement is obtained, both in deformations and crack patterns, when the two kinds of reinforcement are put together. In this case also the shear cracks are much less significant.



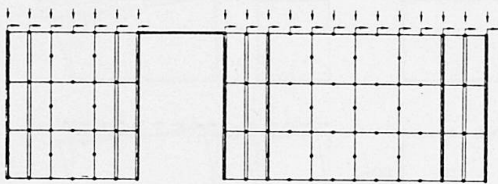
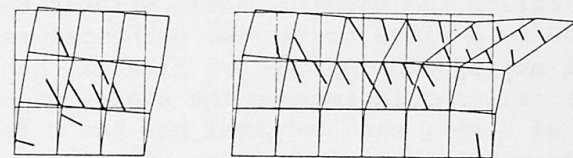
Plain masonry



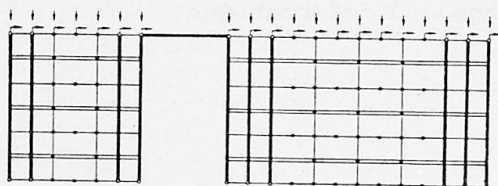
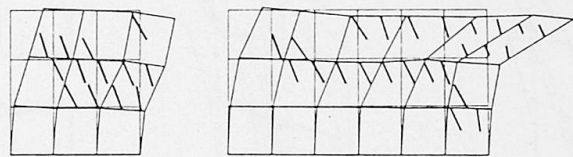
Horizontal bonded reinforcement



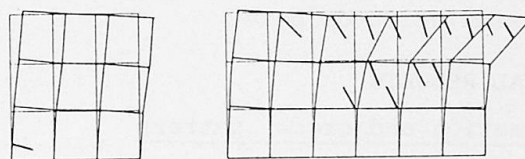
Vertical unbonded reinforcement



Vertical bonded reinforcement



Vertical unbonded horizontal bonded reinforcement



Vertical bonded horizontal bonded reinforcement

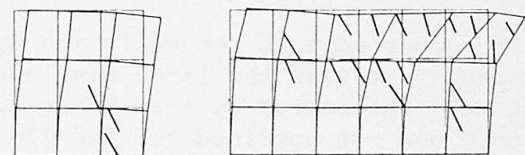


Fig.8 - Deformation and crack pattern for different kinds of reinforcement (horizontal load 8750 N).

### 5.2. Load-displacement curves

In Figure 9 the curves horizontal displacement vs. horizontal load for the different cases of reinforcement are plotted and compared.

The vertical load acting on walls is the same for all cases and has the intermediate value among the three values considered on the wall with R.C. frames.

The diagrams show the maximum horizontal displacement in each case (upper right node of the wider wall).

The following points need to be underlined:

- the sensible difference of displacements also at low load level is due to the immediate deterioration of the elements at the upper edge;
- bonded bars behave better than unbonded because they can locally delay the formation of cracks, but difference is not very large for the higher internal hypostaticity of the mesh;
- for the same reason, an increase of steel area does not affect sensibly the curves (none of the bars reached the yielding stress even when the masonry was strongly deteriorated);
- curves do not show the most important effects of reinforcement: improvement of out-of-plane strength, improvement of ductility and containment of cyclic deterioration. [4][7][10][11]

### 5.3. Loads redistribution

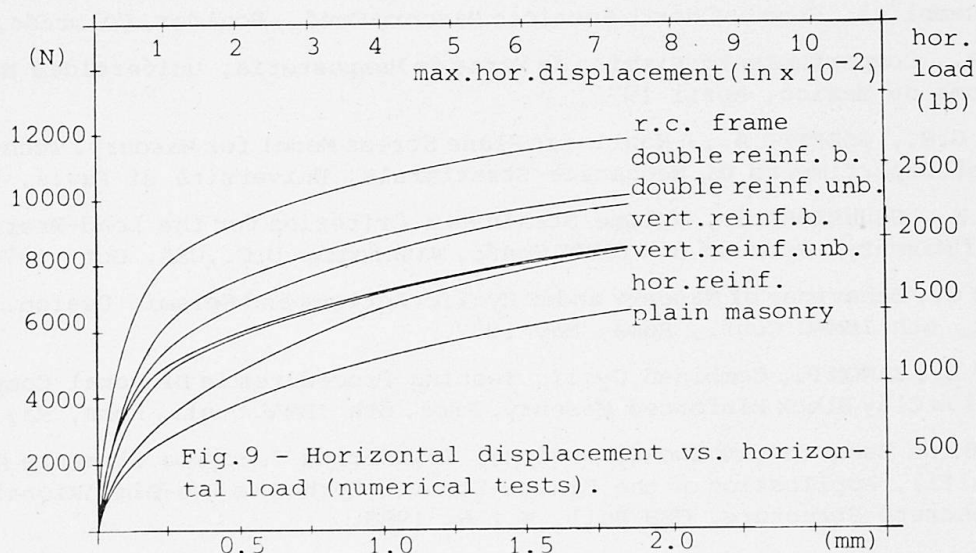
Fig.10 shows the effect of force internal redistribution in terms of the ratios between the force  $F_2$  taken by the second wall and the force  $F_1$  taken by the first wall, for different situations of applied loads and materials. It is important to underline the strong effect of a vertical precompression: the in-plane horizontal deformation due to Poisson effect causes a mutual action between the walls so that the  $F_2/F_1$  ratio is particularly affected by such phenomenon at low values of the horizontal load  $H$ .

This happens also with a linear material, but may be amplified with the present model.

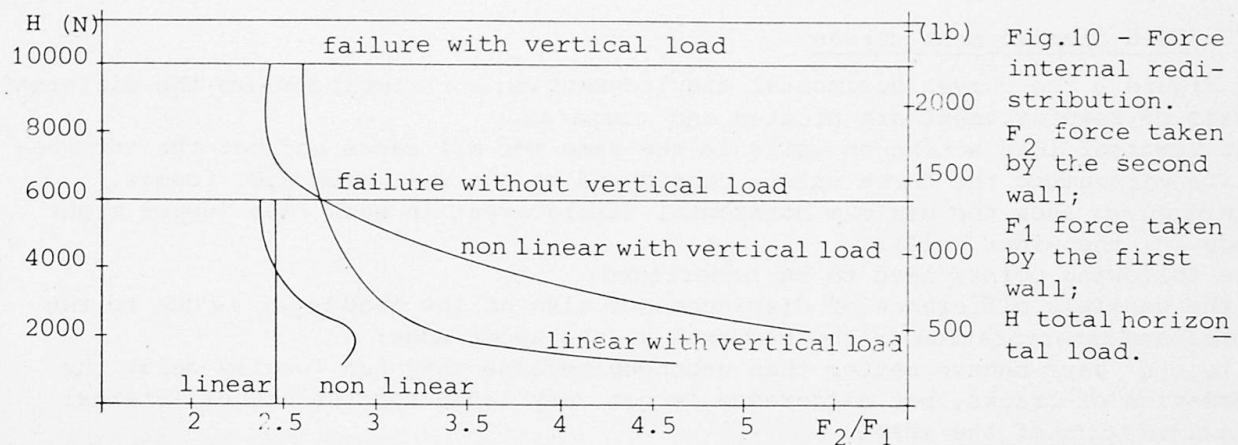
Curves referred to non-linear model show variations of the ratio  $F_2/F_1$  due to the different deterioration of the walls.

## 6. CONCLUSIONS

The proposed non linear plane stress model has proved to be suitable for the analysis of plain masonry shear walls, and of coupled shear walls confined by reinforced concrete frames. The effects of cracking and deterioration of plain masonry are satisfactorily described; nevertheless, the possible anisotropy of units is not yet taken into account. The redistribution of load among coupled shear walls is satisfactorily modelled.







The extension of the model to masonry strengthened by steel bars (both bonded and unbonded) has been performed in order to allow to study the effects of strengthening and its most suitable pattern. Vertical bars proved to be efficient in containing the horizontal cracking, but have a relatively small effect in containing shear deformation. Vertical and horizontal steel (even in very low percentage) sensibly improve the stiffness in the cracked stage and the strength. It has to be underlined that the model does not yet show the favourable effect of reinforcement in cyclic loading (improvement of ductility and containment of cyclic deterioration).

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