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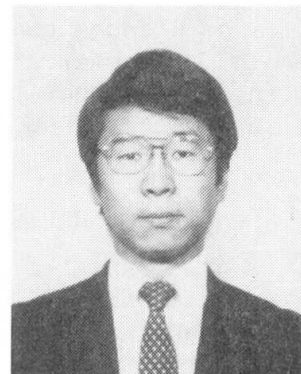
**Probabilistic Modeling of Ship Collision with Bridge Piers**  
Modèle de la collision d'un navire contre les piles d'un pont  
Wahrscheinlichkeitsmodell der Schiffskollision mit Brückenpfeilern

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## SUMMARY

This paper presents a model to estimate the probability of ship collision with bridge piers constructed over a strait or bay. The model includes, at the operational variables, the design variables such as the span length and the pier diameter, the traffic volume and the fairway width. Some numerical examples of the collision probability for these variables are also presented. In computation, statistical data collected in Japan are used.

## RÉSUMÉ

L'article présente un modèle pour estimer la probabilité de collision d'un navire contre les piles d'un pont construit au-dessus d'un détroit ou d'une baie. Les variables de conception telles que la portée du pont, le diamètre des piles, le volume du trafic et la largeur de la voie maritime sont pris en considération. Quelques résultats numériques de la probabilité de collision sont calculés, sur la base de données statistiques au Japon.

## ZUSAMMENFASSUNG

Ein Modell wird vorgeschlagen, um die Wahrscheinlichkeit einer Schiffskollision mit Brückenpfeilern in einer Meerenge oder Bucht abzuschätzen. Die Bauparameter einer Brücke, wie zum Beispiel die Spannweite und der Pfeilerdurchmesser, und der Verkehrsumfang von Schiffen werden hier als die operativen Variablen behandelt. Ferner werden einige numerische Ergebnisse der Kollisionswahrscheinlichkeit für diese Variablen anhand der statistischen Daten in Japan gezeigt.



## 1. INTRODUCTION

Ships carrying hazardous material such as oil and LNG tend to increase in number and size. On the other hand, a number of maritime structures such as oil-platforms and bridges over a bay or strait recently become to be constructed. Therefore, once a ship happens to collide with those structures, there will be tremendous losses and damages.

Taking these situations into account, not only the structural safety but also the ships' navigational safety should be considered in planning and design of those structures.

This paper discusses a model to estimate the collision probability of ships with a bridge pier. The model has been developed based on the model proposed previously by the authors ([1], [2]). In order to give useful information for planning and design of bridges constructed over a bay or strait, this model includes the span length, the pier diameter, the fairway width and the marine traffic characteristics as the operational variables. Numerical examples by the proposed model are also presented and discussed.

## 2. FACTORS INFLUENCING SHIP COLLISIONS

The authors ([1]) classified the factors which influence the collision of ships with obstacles in or near the fairway such as the bridge piers and other offshore structures as shown in Table 1. These factors can be divided into two groups: operational factors and non-operational ones.

In the operational factors, fairway width, curvature and obstacles are related to location and design of the structure. In case of bridges, the fairway width and obstacles are represented by the span length and the pier cross-sectional diameter, respectively, and the curvature partly depends on the location of the bridge. Since this paper aims to obtain the probability of ship collision with a bridge pier, the factors of fairway length, fairway crossing and fairway side shape are not considered explicitly. Curvature of the fairway is assumed to be infinite. That is, the fairway is assumed to be straight under given bridge location. However, it should be notified that these fairway characteristics can not be omitted in case of the probability of collisions between ships.

In the non-operational factors, navigator's and natural conditions are implicitly taken into consideration as the random variation of the distance where the ships start their give-way motions to avoid the collision with the bridge pier.

Table 1 Factors Influencing Collisions

Operational		Non-Operational		
Channel Characteristics	Traffic characteristics	Navigators Characteristics	Ship Characteristics	Natural Conditions
1. Fairway Width	1. Ship Size Distribution	1. Quality	1. Ship Size	1. Tidal
2. Fairway Length	2. Sailing Velocity Distribution	2. Illegal Sailing	2. Speed Performance	Stream
3. Depth	3. Total Traffic Volume	3. Bad Watching	3. Steering Performance	2. Wave
4. Curvature	4. Traffic Volume Ratio in Different Directions	4. With or Without Pilot	4. Stopping Performance	3. Sight Distance
5. Fairway Crossing	5. Crossing Traffic Volume		5. Radar Equipment	4. Wind Direction
6. Navigation Mark	6. Wake Position Distribution			5. Wind Force
7. Obstacles	7. Headway Distribution			6. Weather
8. Channel Side Shape				7. Time

### 3. PROBABILITY MODEL OF SHIP COLLISION

#### 3.1 Modeling Process

As shown in Fig. 1, the basic consideration starts with modeling the give-way motion of a ship of particular size,  $B_k$ , sailing at the position  $X_k$ . The collision of this ship with the bridge pier is defined as the event of failure of the give-way. This is given by the function of  $B_k$ ,  $X_k$ , the pier diameter,  $D$ , fairway width,  $W$ , span length,  $L$ , and the distance  $l_k$ , between the ship and the center of the pier section when the ship starts give-way motion. Hereafter, the distance,  $l_k$ , is called as the give-way starting distance (GWS-distance). Since the GWS-distance can be regarded as a random variable, the event of failure of give-way becomes a random event. The probability of occurrence of this event is defined as the "failure probability of give-way", and denoted by  $P_f$ . On the other hand, the sailing position,  $X_k$ , can be considered as a random variable whose probability density function (p.d.f.),  $\phi_x(X_k|W, Q)$ , is specified by the fairway width and the traffic volume per hour,  $Q$ . Thus the expected failure probability of give-way,  $P_{ef}$ , is given as a function of  $B_k$ ,  $D$ ,  $W$ , and  $Q$ . Based on this probability, probability of ship collision,  $P_c$ , is obtained.

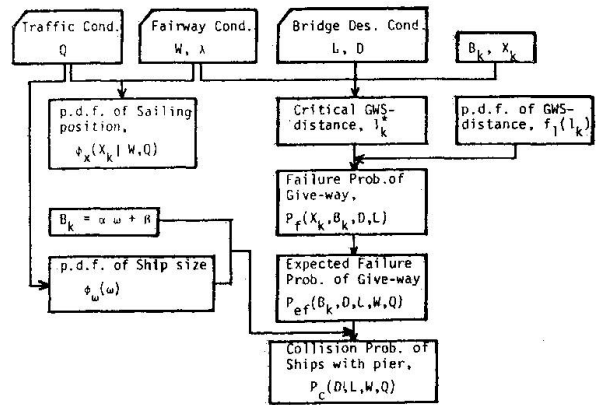


Fig. 1 Modeling Process of Collision Probability

#### 3.2 Failure Probability of Give-way

As discussed in the previous section, modeling the give-way motion is the basis of a mathematical treatment of ship's collision with a bridge pier. Suppose a ship of particular size,  $B_k$ , sailing at the position,  $X_k$ , with the velocity,  $V_k$ , takes a give-way motion (Fig. 2a). In general the give-way motion includes altering course by steering, speed-down, anchoring, and so forth. However, the present model considers only the steering motion because speed-down, anchoring and other motions are quite rare comparing with the steering motion. Let  $l_k$  be the distance between the ship and the pier in the  $y$ -coordinate when the ship starts the give-way by the angle,  $\theta$ , of altering course (see Fig. 2b). The distance,  $d(t)$ , between the center of the ship and the pier when time  $t$  is passed after starting the give-way motion is given by

$$d(t) = \left[ \left( X_k - V_k t \sin \theta - \frac{L}{2} \right)^2 + \left( l_k - V_k t \cos \theta \right)^2 \right]^{1/2} \quad (1)$$

in which

$$L = W + (2\lambda + 1)D \quad (2)$$

This is reduced on the assumption that the ship can be approximated by the circle of diameter,  $B_k$ , which denotes the

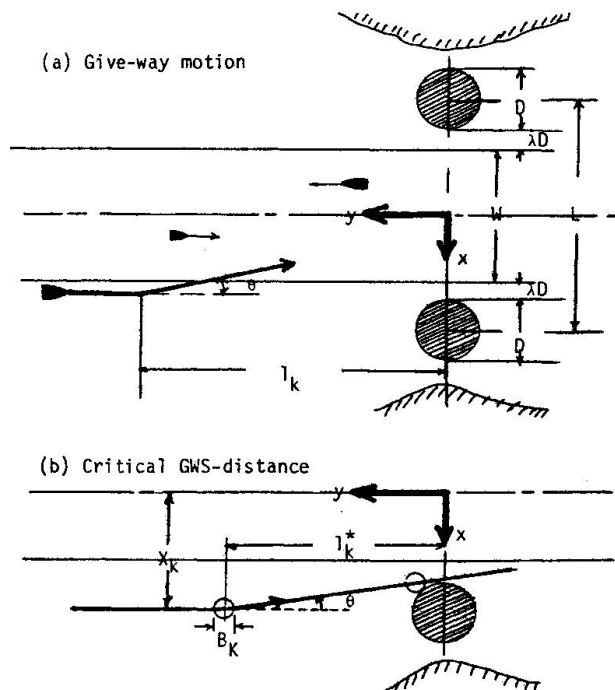


Fig. 2 Give-way Motion and Critical GWS-Distance



width of the ship. Eq.(1) gives the minimum distance,  $d^*$ , as

$$d^* = l_k \sin\theta + (X_k - \frac{L}{2})\cos\theta \quad (3)$$

Then the collision of the ship with the pier is defined as the event that the distance,  $d^*$ , is less than equal to the "collision diameter,  $D_{ck}$ ", which is defined by

$$D_{ck} = (B_k + D)/2 \quad (4)$$

that is,

$$d^* \leq D_{ck} \quad (5)$$

Applying Eqs.(3) and (4) to Eq.(5), the critical distance,  $l_k^*$ , is given by the following equation :

$$l_k^* = D_{ck} \operatorname{cosec}\theta + (X_k - \frac{L}{2})\cot\theta \quad (6)$$

The critical GWS-distance,  $l_k^*$ , means that the ship will collide with the pier if she starts the give-way motion at the distance to the pier less than  $l_k^*$ . In the practical situation, ships start the give-way motions at the various distance depending on the conditions of their own instruments and navigators and others. Based on the authors' observational data at Obatake in Japan, the GWS-distance,  $l_k$ , follows lognormal distribution as shown in Fig. 3. Thus the p.d.f. of  $l_k$  is approximated by

$$f_l(l_k) = \frac{1}{\sqrt{2\pi} l_k \sigma_l} \exp\left[-\left(\frac{\log l_k - \mu_l}{\sqrt{2} \sigma_l}\right)^2\right] \quad (7)$$

In the above equation, the mean  $\mu_l$  and the standard deviation  $\sigma_l$  could be a function of the conditions discussed previously. For instance, the observational data by the Ministry of Transportation of Japan ([3]), the GWS-distance under head on situation between two ships is the function of their velocities and sizes. However, enough data of GWS-distances are not collected to identify the function statistically. Therefore, in this paper the GWS-distance  $l_k$  is assumed to follow a lognormal distribution with constant mean and standard deviation.

Taking into account that  $l_k$  follows the lognormal distribution, the failure probability of give-way,  $P_f$ , is calculated as follows :

$$P_f(X_k, B_k, D, L) = \operatorname{Prob.}[l_k \leq l_k^*] = \begin{cases} \frac{1}{2} [1 + \operatorname{ERF}(\frac{I_{ck}}{\sqrt{2}})] & \text{for } X_k \leq X_k^* \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

where  $\operatorname{ERF}(\cdot)$  is the error function,  $I_{ck}$  and  $X_k^*$  are given by

$$I_{ck} = \frac{\log l_k^* - \mu_l}{\sigma_l} \quad (9)$$

and

$$X_k^* = \frac{W + 2\lambda D - B_k}{2} \quad (10)$$

### 3.3 Collision Probability

Ships can be expected to take their sailing position on the fairway at their will. However, their positioning could be affected by the fairway conditions and the traffic condition. Inoue ([4]) reported that the sailing position is affected by the fairway width and the traffic volume per hour modified by the ship length, and that the sailing position follows the normal distribution as shown in Fig. 4.

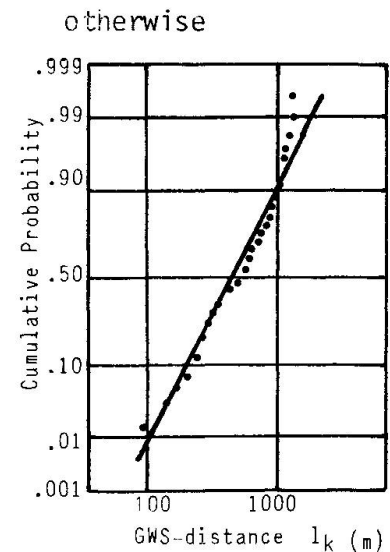


Fig. 3 Prob. Distribution of GWS-distance,  $l_k$ .

According to his conclusions, the mean sailing position,  $\mu_x$ , in a two-way traffic fairway of width,  $W$ , can be approximated by

$$\mu_x = a W \quad (11)$$

where  $a$  is a constant defined for the given fairway conditions, that is,

$$a = 0.2 \text{ with centerline mark}$$

$$a = 0.1 \text{ without centerline mark}$$

and the standard deviation of sailing position,  $\sigma_x$ , in a certain direction on the two-way traffic fairway is

$$\sigma_x = -7.170 + 0.105W + 2.168Q_L^* \quad (12)$$

in which  $W$  is measured in meter and  $Q_L^*$  is the traffic volume per hour modified by the ship length when the standard ship length,  $L_S^*$ , is employed as  $L_S^* = 35$  m. Eq.(12) means that sailing position in  $x$ -direction (see Fig. 2b) tends to spread outer side of the fairway as the width and the traffic volume increase.

The modified traffic volume  $Q_L^*$  is calculated by the traffic volume,  $Q'$ , per hour in a certain direction and its ship size distribution. Based on the data presented by Fujii ([5]), the ship length in the traffic volume  $Q'$  follows the lognormal distribution. Namely, denoting  $\omega$  as the natural logarithm of ship length  $L_S$ ,

$$\omega = \log_{10} L_S \quad (13)$$

the p.d.f. of  $\omega$  is given by

$$\phi_\omega(\omega) = \exp\left[-\frac{1}{2}\left(\frac{\omega - \mu_\omega}{\sigma_\omega}\right)^2\right] / \sqrt{2\pi} \sigma_\omega \quad (14)$$

Using Eq.(14), the modified traffic volume  $Q_L^*$  in Eq.(12) can be calculated by

$$Q_L^* = Q' \int_0^\infty 10^\omega \phi_\omega(\omega) d\omega \quad (15)$$

From Eqs.(11) and (12), the p.d.f. of  $X_k$  is given by

$$\phi_x(X_k | W, Q') = \exp\left[-\left(\frac{X_k - \mu_x}{\sqrt{2} \sigma_x}\right)^2\right] / \sqrt{2\pi} \sigma_x \quad (16)$$

Applying Eq.(16) to Eq.(8), the expected failure probability of give-way,  $P_{ef}$ , is

$$P_{ef}(B_k, D, L, W, Q') = \int_{X_k}^\infty \frac{1}{2} \left[ 1 + \operatorname{ERF}\left(\frac{I_{ck}}{2}\right) \right] \phi_x(X_k | W, Q') dX_k \quad (17)$$

This probability is the elementary probability in the sense that any one ship of size  $B_k$  is expected to have the probability  $P_{ef}$  to collide with the pier under the hourly traffic volume,  $Q'$ . Therefore, when  $Q$  traffic volume per year is expected and  $Q_k$  ships of size  $B_k$  exist in  $Q$ , the Probability,  $P_{sk}$ , that any ship of  $Q_k$  does not collide with the pier is

$$P_{sk}(D, L, W, Q', Q) = (1 - P_{ef})^{Q_k} = 1 - P_{ef} Q \phi_\omega(\omega_k) d\omega_k \quad (18)$$

Therefore, the probability that all of  $Q$  ships do not collide with the pier is

$$P_s(D, L, W, Q', Q) = 1 - Q \int_0^\infty P_{ef} \phi_\omega(\omega_k) d\omega_k \quad (19)$$

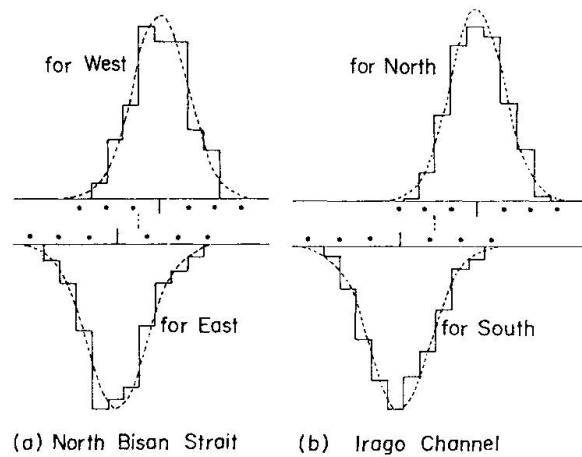


Fig. 4 Distribution of Sailing Position



In the above integration it should be noticed that  $P_{ef}$  is the function of  $\omega_k$ , because  $B_k$  has a unique relationship with the ship length  $L_k$ . Fujii ([5]) gives the relation as follows

$$B_k = \alpha \omega_k + \beta, \quad \alpha = 0.88, \quad \beta = -0.47 \quad (20)$$

Since  $P_s$  gives the probability that none of the ships of volume  $Q$  collide with the bridge pier, the probability that at least one ship collides with the pier is approximated by

$$P_c(D, L, W, Q', Q) = 1 - P_s = Q \int_0^\infty P_{ef} \phi_\omega(\omega_k) d\omega_k \quad (21)$$

where

$$Q' = Q / 8760$$

### 3.4 Average Number of Collision Ships

Since every ship of size  $B_k$  is expected to have the elementary probability of collision, the probability that  $N_k$  ships of  $Q_k$  will collide with the pier is given by the binomial distribution as

$$P_c(N_k) = \binom{Q_k}{N_k} (1 - P_{ef})^{Q_k - N_k} P_{ef}^{N_k} \quad (22)$$

This gives the average number,  $\bar{N}_k$ , of collision ships of size  $B_k$  as

$$\bar{N}_k = P_{ef} Q_k \quad (23)$$

Therefore, the average number,  $\bar{N}_c$ , of collision ships when total traffic volume  $Q$  per year is expected is given by

$$\bar{N}_c = Q \int_0^\infty P_{ef} \phi_\omega(\omega_k) d\omega_k \quad [\text{ships/year}] \quad (24)$$

This has the same form as Eq.(21). However, Eq.(21) is the approximate form of the probability. Therefore, it does not exceed unity even if  $Q$  becomes very large number, while Eq. (24) gives the average number of collision ships if it goes over unity.

## 4. NUMERICAL EXAMPLES AND DISCUSSION

In computation of numerical examples, the values of the parameters in the model are used as shown in Table 2. The angle of altering course,  $\theta$ , is based on the fact that the steering angle used by most of the ships in altering their courses is about 15 degree. The mean,  $\mu_1$ , and the standard deviation,  $\sigma_1$ , of GWS-distance are from the data observed at Obatake in Japan (see Fig. 3)

Table 2 Values of Parameters Used in Examples

$\theta = 30^\circ$	$\mu_1 = 6.15$
$\alpha = 0.88$	$\sigma_1 = 0.59$
$\beta = -0.47$	$\mu_\omega = 1.40$
$a = 0.2$	$\sigma_\omega = 0.15$

The statistical parameters,  $\mu_\omega$ , and  $\sigma_\omega$ , of the ship size distribution are assumed from those of the traffic in some straits in Japan (Fujii[5]).

Fig. 5 gives the relation between average number  $\bar{N}_c$  of collision ships per year and the span length under a given fairway width and the traffic volume. From this figure, it can be understood that the number of collision ships will decrease as  $L$  increases. This is resultant from that the marginal space between the fairway edge and the pier becomes large as the span length increases. However, according to the authors' previous study ([2]), the average number of collision between ships per year does not change so long as the fairway width is constant. This is



shown by the dotted line in the figure. On the contrary, in Fig. 6 is shown the relation between  $\bar{N}_C$  and the fairway width under constant span length. In this case, the number of collision ships increases as the width increases. This might be felt strange. However, it should be notified that when the fairway width increases the marginal sea space decreases and in addition, ships tend to sail widely out of the fairway as shown in Eq.(12). While the number of collision between ships is reduced when the fairway width increases. This is also shown by the dotted line in the figure. The trade-off between them should be considered in planning and design of bridges over a strait or bay.

Fig. 7 gives the relation between  $\bar{N}_C$  and the traffic volume under a constant  $L$  and  $W$ . This can be intuitively understood.

## 5. CONCLUDING REMARKS

From these numerical results, it can be expected that under the traffic volume  $Q = 900$  ships per day and the span length  $L = 1$  km, and the fairway width  $W = 800$ m, ship collision with the pier is expected once in every two years.

The proposed model has many assumptions and simplifications. However, this model is expected to give useful information for planning and design of maritime structures such as bridges over sea, oil-platforms in the sea and so forth. Further developed model is under studied by the authors.

## 6. ACKNOWLEDGEMENT

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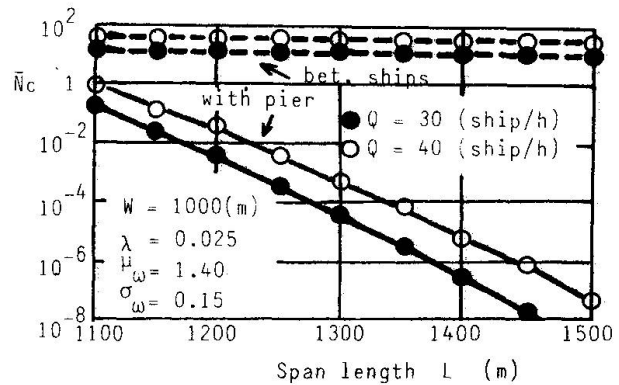


Fig. 5  $\bar{N}_C \sim L$  Relation

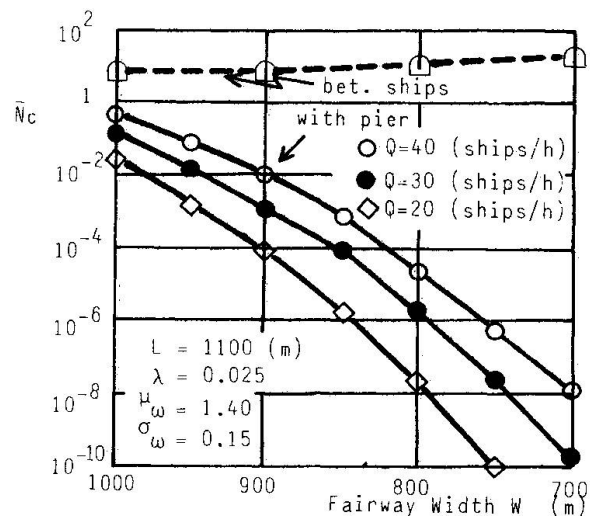


Fig. 6  $\bar{N}_C \sim W$  Relation

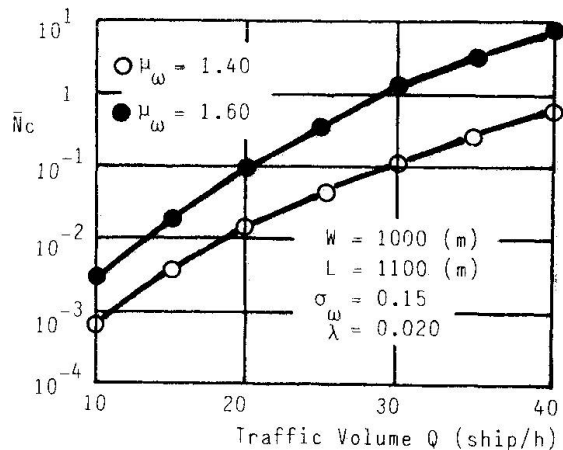


Fig. 7  $\bar{N}_C \sim Q$  Relation





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