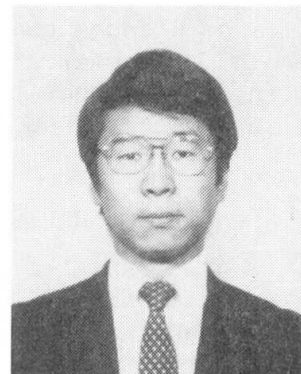# Probabilistic Modeling of Ship Collision with Bridge Piers

Modèle de la collision d'un navire contre les piles d'un pont

Wahrscheinlichkeitsmodell der Schiffskollision mit Brückenpfeilern

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SUMMARY

This paper presents a model to estimate the probability of ship collision with bridge piers constructed over a strait or bay. The model includes, at the operational variables, the design variables such as the span length and the pier diameter, the traffic volume and the fairway width. Some numerical examples of the collision probability for these variables are also presented. In computation, statistical data collected in Japan are used.

RÉSUMÉ

L'article présente un modèle pour estimer la probabilité de collision d'un navire contre les piles d'un pont construit au-dessus d'un détroit ou d'une baie. Les variables de conception telles que la portée du pont, le diamètre des piles, le volume du trafic et la largeur de la voie maritime sont pris en considération. Quelques résultats numériques de la probabilité de collision sont calculés, sur la base de données statistiques au Japon.

ZUSAMMENFASSUNG

Ein Modell wird vorgeschlagen, um die Wahrscheinlichkeit einer Schiffskollision mit Brückenpfeilen in einer Meerenge oder Bucht abzuschätzen. Die Bauparameter einer Brücke, wie zum Beispiel die Spannweite und der Pfeilerdurchmesser, und der Verkehrsumfang von Schiffen werden hier als die operativen Variablen behandelt. Ferner werden einige numerische Ergebnisse der Kollisionswahrscheinlichkeit für diese Variablen anhand der statistischen Daten in Japan gezeigt.

1. INTRODUCTION

Ships carrying hazardous material such as oil and LNG tend to increase in number and size. On the other hand, a number of maritime structures such as oil-platforms and bridges over a bay or strait recently become to be constructed. Therefore, once a ship happens to collide with those structures, there will be tremendous losses and damages.

Taking these situations into account, not only the structural safety but also the ships' navigational safety should be considered in planning and design of those structures.

This paper discusses a model to estimate the collision probability of ships with a bridge pier. The model has been developed based on the model proposed previously by the authors ([1], [2]). In order to give useful information for planning and design of bridges constructed over a bay or strait, this model includes the span length, the pier diameter, the fairway width and the marine traffic characteristics as the operational variables. Numerical examples by the proposed model are also presented and discussed.

2. FACTORS INFLUENCING SHIP COLLISIONS

The authors ([1]) classified the factors which influence the collision of ships with obstacles in or near the fairway such as the bridge piers and other offshore structures as shown in Table 1. These factors can be devided into two groups : operational factors and non-operational ones.

In the operational factors, fairway width, curvature and obstacles are related to location and design of the structure. In case of bridges, the fairway width and obstacles are represented by the span length and the pier cross-sectional diameter, respectively, and the curvature partly depends on the location of the bridge. Since this paper aims to obtain the probability of ship collision with a bridge pier, the factors of fairway length, fairway crossing and fairway side shape are not considered explicitly. Curvature of the fairway is assumed to be infinite. That is, the fairway is assumed to be straight under given bridge location. However, it should be notified that these fairway characteristics can not be omitted in case of the probability of collisions between ships.

In the non-operational factors, navigator's and natural conditions are implicitly taken into consideration as the random variation of the distance where the ships start their give-way motions to avoid the collision with the bridge pier.

Table 1 Factors Influencing Collisions

Operational		Non-Operational		
Channel Characteristics	Traffic characterstics	Navigators Characteristics	Ship Characteristics	Natural Conditions
1. Fairway Width	1. Ship Size Distribution	1. Quality	1. Ship Size	1. Tidal Stream
2. Fairway Length	2. Sailing Velocity Distribution	2. Illegal Sailing	2. Speed Performance	
3. Depth				2. Wave
4. Curvature	3. Total Traffic Volume	3. Bad Watching	3. Steering Performance	3. Sight Distance
5. Fairway Crossing	4. Traffic Volume Ratio in Different Directions	4. With or Without Pilot	4. Stopping Performance	4. Wind Direction
6. Navigation Mark	5. Crossing Traffic Volume			
7. Obstacles	6. Wake Position Distribution		5. Radar Equipment	5. Wind Force
8. Channel Side Shape				6. Weather
	7. Headway Distribution			7. Time

3. PROBABILITY MODEL OF SHIP COLLISION

3.1 Modeling Process

As shown in Fig. 1, the basic consideration starts with modeling the give-way motion of a ship of particular size, B_k, sailing at the position X_k. The collision of this ship with the bridge pier is defined as the event of failure of the give-way. This is given by the function of B_k, X_k, the pier diameter, D, fairway width,W, span length,L, and the distance l_k,between the ship and the center of the pier section when the ship starts give-way motion. Hereafter, the distance,l_k, is called as the give-way starting distance (GWS-distance). Since the GWS-distance can be regarded as a random variable, the event of failure of give-way

Fig. 1 Modeling Process of Collision Probability

becomes a random event. The probability of occurrence of this event is defined as the " failure probability of give-way", and denoted by P_f. On the other hand, the sailing position,X_k, can be considered as a random variable whose probability density function (p.d.f.), $\phi_x(X_k/W,Q)$, is specified by the fairway width and the traffic volume per hour, Q. Thus the expected failure probability of give-way, P_{ef}, is given as a function of B_k, D, W, and Q. Based on this probability, probability of ship collision, P_c, is obtained.
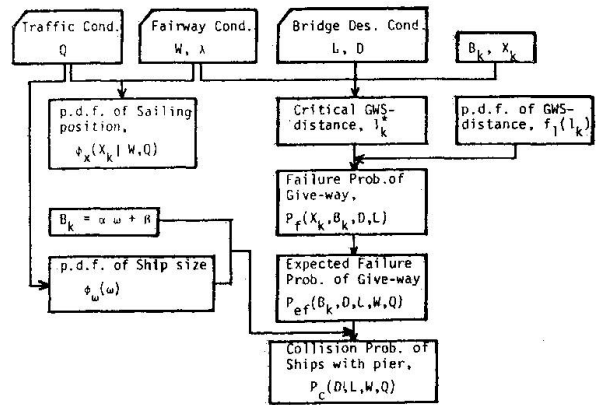
3.2 Failure Probability of Give-way

As discussed in the previous section, modeling the give-way motion is the basis of a mathematical treatment of ship's collision with a bridge pier.   Suppose a ship of particular size, B_k, sailing at the position, X_k, with the velocity,V_k, takes a give-way motion (Fig. 2a). In general the give-way motion includes altering course by steering, speed-down , anchoring, and so forth. However, the present model consideres only the steering motion because speed-down, anchoring and other motions are quite rare comparing with the steering motion. Let l_k be the distance between the ship and the pier in the y-coordinate when the ship starts the give-way by the angle , θ , of altering course (see Fig. 2b). The distance , d(t), between the center of the ship and the pier when time t is passed after starting the give-way motion is given by

(a) Give-way motion

(b) Critical GWS-distance

$$d(t) = [\{X_k - V_k t \sin\theta - \frac{L}{2}\}^2 + \{l_k - V_k t \cos\theta\}^2]^{1/2} \quad (1)$$

in which

$$L = W + (2\lambda + 1)D \quad (2)$$

This is reduced on the assumption that the ship can be approximated by the circle of diameter, B_k, which denotes the
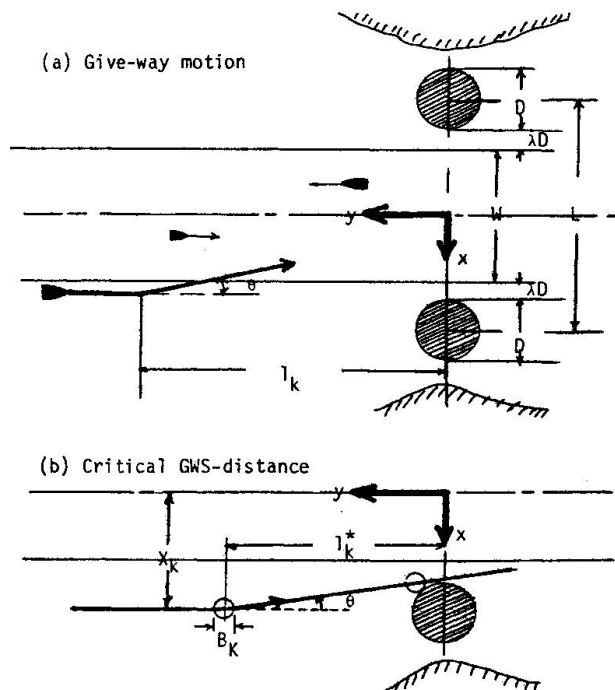
Fig. 2 Give-way Motion and Critical GWS-Distance

width of the ship. Eq.(1) gives the minimum distance, d^*, as

$$d^* = l_k \sin\theta + (X_k - \frac{L}{2})\cos\theta \tag{3}$$

Then the collision of the ship with the pier is defined as the event that the distance,d^*, is less than equal to the "collision diameter,D_{ck}",which is defined by

$$D_{ck} = (B_k + D)/2 \tag{4}$$

that is,

$$d^* \leq D_{ck} \tag{5}$$

Applying Eqs.(3) and (4) to Eq.(5), the critical distance , l_k^*, is given by the following equation :

$$l_k^* = D_{ck}\operatorname{cosec}\theta + (X_k - \frac{L}{2})\cot\theta \tag{6}$$

The critical GWS-distance ,l_k^*, means that the ship will collide with the pier if she starts the give-way motion at the distance to the pier less than l_k^*. In the practical situation, ships start the give-way motions at the various distance depending on the conditions of their own instruments and navigators and others.Based on the authors' observational data at Obatake in Japan, the GWS-distance, l_k, follows lognormal distribution as shown in Fig. 3. Thus the p.d.f. of l_k is approximated by

$$f_1(l_k) = \frac{1}{\sqrt{2\pi}l_k\sigma_1}\exp[- (\frac{\log l_k - \mu_1}{\sqrt{2}\sigma_1})^2] \tag{7}$$

In the above equation, the mean μ_1 and the standard deviation σ_1 could be a function of the conditions discussed previously. For instance, the observational data by the Ministry of Transpotation of Japan ([3]), the GWS-distance under head on situation between two ships is the function of their velocities and sizes. However, enough data of GWS-distances are not collected to identify the function statistically. Therefore, in this paper the GWS-distance l_k is assumed to follow a lognormal distribution with constant mean and standard deviation.

Taking into account that l_k follows the lognormal distribution,   the failure probability of give-way, P_f, is calculated as follows :

$$P_f(X_k,B_k,D,L) = \text{Prob.}[l_k \leq l_k^*] = \begin{cases} \frac{1}{2}[1+\text{ERF}(\frac{I_{ck}}{\sqrt{2}})] & \text{for } X_k \leq X_k^* \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

wher ERF($\cdot$) is the error function, I_{ck} and X_k^* are given by

$$I_{ck} = \frac{\log l_k^* - \mu_1}{\sigma_1} \tag{9}$$

and

$$X_k^* = \frac{W + 2\lambda D - B_k}{2} \tag{10}$$

3.3 Collision Probability

Ships can be expected to take their sailing position on the fiarway at their will. However, their positioning could be affected by the fairway conditions and the traffic condition.   Inoue ([4]) reported that the sailing position is affected by the fairway width and the traffic volume per hour modified by the ship length, and that the sailing position follows the normal distribution as shown in Fig. 4.
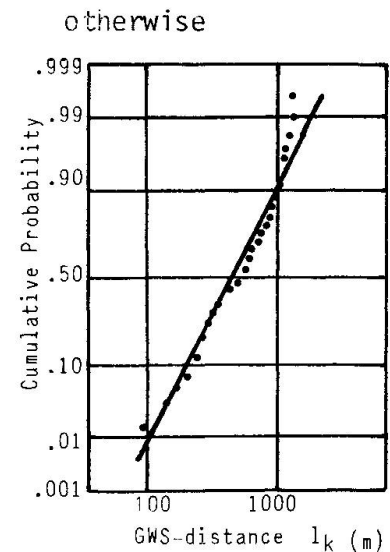

Fig. 3 Prob. Distribution of GWS-distance,l_k.

According to his conclusions, the mean sailing position , μ_X , in a two-way traffic fairway of width , W , can be approximated by

$$\mu_X = a W \qquad (11)$$

where a is a constant defined for the given fairway conditions, that is,

a = 0.2 with centerline mark

a = 0.1 without centerline mark

and the standard deviation of sailing position ,σ_X, in a certain direction on the two-way traffic fairway is

$$\sigma_X = -7.170+0.105W+2.168Q_L^* \qquad (12)$$

(a) North Bisan Strait   (b) Irago Channel

Fig. 4 Distribution of Sailing Position

in which W is measured in meter and Q_L^* is the traffic volume per hour modified by the ship length when the standard ship length , L_S^* , is employed as L_S^* = 35 m. Eq.(12) means that sailing position in x-direction (see Fig. 2b) tends to spread outer side of the fairway as the width and the traffic volume increase.

The modified traffic volume Q_L^* is calculated by the traffic volume , Q', per hour in a certain direction and its ship size distribution.Based on the data presented by Fujii ([5]), the ship length in the traffic volume Q' follows the lognormal distribution. Namely, denoting ω as the natural logarithm of ship length L_S,

$$\omega = \log_{10} L_S \qquad (13)$$

the p.d.f. of ω is given by

$$\phi_\omega(\omega) = \exp[- \frac{1}{2}(\frac{\omega - \mu_\omega}{\sigma_\omega})^2]/\sqrt{2\pi} \, \sigma_\omega \qquad (14)$$

Using Eq.(14), the modified traffic volume Q_L^* in Eq.(12) can be calculated by

$$Q_L^* = Q'\int_0^\infty 10^\omega \, \phi_\omega(\omega) d\omega \qquad (15)$$

From Eqs.(11) and (12), the p.d.f. of X_k is given by

$$\phi_X(X_k \mid W,Q') = \exp[- (\frac{X_k - \mu_X}{\sqrt{2} \, \sigma_X})^2]/\sqrt{2\pi} \, \sigma_X \qquad (16)$$

Applying Eq.(16) to Eq.(8), the expected failure probability of give-way, P_{ef}, is

$$P_{ef}(B_k,D,L,W,Q') = \int_{X_k^*}^\infty \frac{1}{2}[1 + ERF(\frac{I_{ck}}{2})] \, \phi_X(X_k \mid W,Q')dX_k \qquad (17)$$

This probability is the elementary probability in the sense that any one ship of size B_k is expected to have the probability P_{ef} to collide with the pier under the hourly traffic volume ,Q'. Therefore, when Q traffic volume per year is expected and Q_k ships of size B_k exist in Q, the Probability, P_{sk}, that any ship of Q_k does not collide with the pier is

$$P_{sk}(D,L,W,Q',Q) = (1 - P_{ef})^{Q_k} = 1 - P_{ef} Q \, \phi_\omega(\omega_k)d\omega_k \qquad (18)$$

Therefore, the probability that all of Q ships do not collide with the pier is

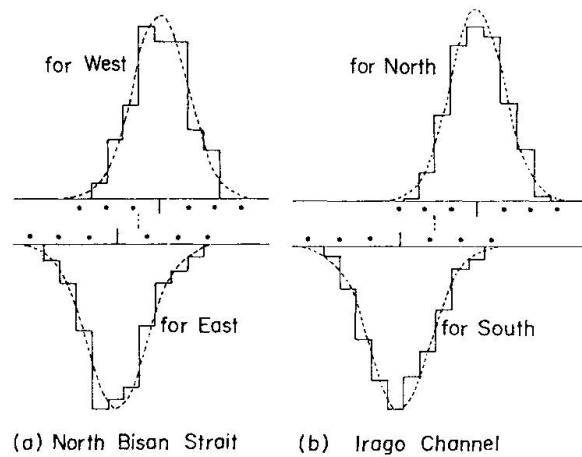$$P_s(D,L,W,Q',Q) = 1 - Q\int_0^\infty P_{ef} \, \phi_\omega(\omega_k)d\omega_k \qquad (19)$$

In the above integration it should be noticed that P_{ef} is the function of ω_k, because B_k has a unique relationship with the ship length L_k. Fujii ([5]) gives the relation as follows

$$B_k = \alpha \, \omega_k + \beta \, , \qquad \alpha = 0.88, \qquad \beta = -0.47 \tag{20}$$

Since P_s gives the probability that none of the ships of volume Q collide with the bridge pier, the probability that at least one ship collides with the pier is approximated by

$$P_c(D,L,W,Q',Q) = 1 - P_s = Q \int_0^\infty P_{ef} \, \phi_\omega(\omega_k) d\omega_k \tag{21}$$

where

$$Q' = Q / 8760$$

3.4 Average Number of Collision Ships

Since every ship of size B_k is expected to have the elementary probability of collision, the probability that N_k ships of Q_k will collide with the pier is given by the binomial distribution as

$$P_c(N_k) = \binom{Q_k}{N_k} (1 - P_{ef})^{Q_k-N_k} P_{ef}^{N_k} \tag{22}$$

This gives the average number , $\bar{N}_k$, of collision ships of size B_k as

$$\bar{N}_k = P_{ef} \, Q_k \tag{23}$$

Therefore, the average number, $\bar{N}_c$, of collision ships when total traffic volume Q per year is expected is given by

$$\bar{N}_c = Q \int_0^\infty P_{ef} \, \phi_\omega(\omega_k) d\omega_k \qquad \text{[ships/year]} \tag{24}$$

This has the same form as Eq.(21). However, Eq.(21) is the approximate form of the probability. Therefore, it does not exceed unity even if Q becomes very large number, while Eq. (24) gives the average number of collision ships if it goes over unity.

4. NUMERICAL EXAMPLES AND DISCUSSION

In computation of numerical examples, the values of the parameters in the model are used as shown in Table 2. The angle of altering course , θ , is based on the fact that the steering angle used by most of the ships in altering their courses is about 15 degree. The mean, μ_l, and the standard deviation, σ_l, of GWS-distance are from the data observed at Obatake in Japan (see Fig. 3)

Table 2 Values of Parameters Used in Examples

$\theta = 30°$	$\mu_l = 6.15$
$\alpha = 0.88$	$\sigma_l = 0.59$
$\beta = -0.47$	$\mu_\omega = 1.40$
$a = 0.2$	$\sigma_\omega = 0.15$

The statistical parameters , μ_ω , and , σ_ω , of the ship size distribution are assumed from those of the traffic in some straits in Japan (Fujii[5]).

Fig. 5 gives the relation between average number $\bar{N}_c$ of collision ships per year and the span length under a given fairway width and the traffic volume. From this figure, it can be understood that the number of collision ships will decreases as L increases. This is resultant from that the marginal space between the fairway edge and the pier becomes large as the span length increases. However, according to the authors' previous study ([2]), the average number of collision between ships per year does no change so long as the fairway width is constant. This is

shown by the dotted line in the figure. On the contrary, _in Fig. 6 is shown the relation between N_c and the fairway width under constant span length. In this case, the number of collision ships increases as the width increases.This might be felt strange. However, it should be notified that when the fairway width increases the marginal sea space decreases and in addition,ships tend to sail widely out of the fairway as shown in Eq.(12). While the number of collision between ships is reduced when the fairway width increases. This is also shown by the dotted line in the figure. The trade-off between them should be considered in planning and design of bridges over a strait or bay.

Fig. 7 gives the relation between $\bar{N}_c$ and the traffic volume under a constant L and W. This can be intuitively understood.

5. CONCLUDING REMARKS

From these numerical results, it can be expected that under the traffic volume Q = 900 ships per day and the span length L = 1 km, and the fairway width W = 800m, ship collision with the pier is expected once in every two years.

The proposed model has many assumptions and simplifications. However, this model is expected to give useful information for planning and design of maritime structures such as bridges over sea,oil-platforms in the sea and so forth. Further developed model is under studied by the authors.

6. ACKNOWLEDGEMENT

Fig.5 $\bar{N}_c \sim L$ Relation

Fig.6 $\bar{N}_c \sim W$ Relation

Fig.7 $\bar{N}_c \sim Q$ Relation

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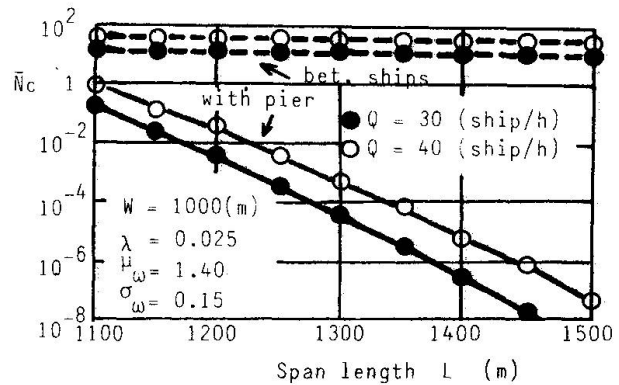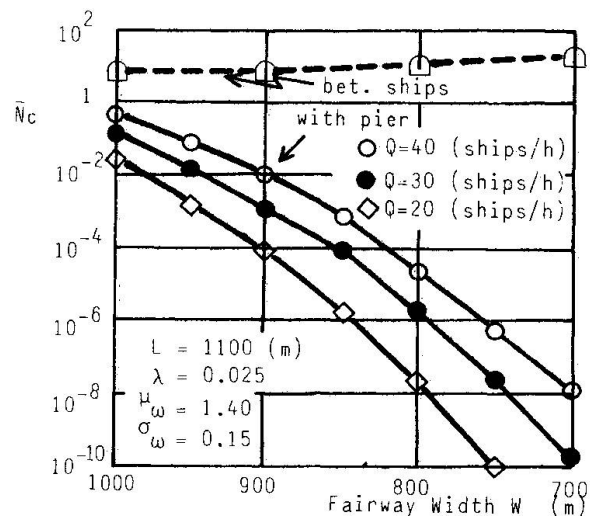4. INOUE K., On the Separation of Traffic at Straight Waterway by Distribution
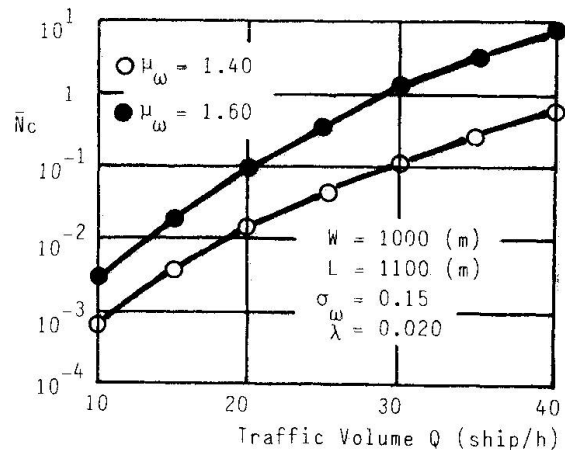
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