Nonlinear Computations From The User's Point of View

Calcul non linéaire des structures: point de vue d'utilisateur

Nichtlineare Berechnungen vom Standpunkt des Anwenders

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Miloš Marinček, born 1918, obtained his civil engineering degree in Ljubljana and Ph. D. in Vienna. He founded the university institute for metal structures and was director until 1961. He was later involved in theoretical research in inelastic behaviour and safety of structures.

SUMMARY
Some suggestions are given regarding the more useful application of nonlinear computations of structures using dimensionless load-displacement diagrams for structures with standardised dimensionless inelastic material and cross-sectional properties.

RESUME
L'article fait état de quelques suggestions propres à améliorer l'application du calcul non linéaire des structures en recourant à des diagrammes charge-déplacement sans dimension, ainsi qu'à des lois de matériaux inélastiques et des propriétés de sections standardisées.

ZUSAMMENFASSUNG
Um eine bessere Anwendung von nichtlinearen Computer-Berechnungen zu erreichen, werden einige Vorschläge gegeben, indem dimensionslose Belastungs-Verschiebungs-Diagramme mit standardisierten dimensionslosen Material- und Querschnittseigenschaften benützt werden.

1. INTRODUCTION

The nonlinear computations of structures have among many possible aspect
of the informatics in the field of structural engineering an important
role in the further development and successful practical use of the limit
states design of structures. So much better assessment of the safety of
structures and therefore more rational use of the material in structures
is possible.

Owing to his responsibility the design engineer is much interested to
know the safety resp. reliability of structures: relating the appearance
of undesirable phenomena at the working state, regarding the beginning of
unacceptable damage due to the inelastic displacements at overloading,
and concerning the ultimate carrying capacity, which is connected with
the inelastic behaviour of material too.

The structural behaviour is best represented with the characteristic
load-displacement and /or time-displacement relationship. Normalising the
load-displacement relationship using the elastic limit state as the norm,
a very useful generalisation of the nonlinear behaviour of structures is
possible.

"Exact" nonlinear computations of structures represent the simulation of
the real structural behaviour and replace more and more the experimental
work. However the so called deterministic tests remain very important in
order to confirm the theory.

Using computers the consideration of the influence of different paramete-
rs is very simple (differnt material behaviour, mechanical and geometri-
cal imperfections). "Exact" nonlinear computations are also very useful
for the assessment of various approximate methods. They serve, further,
for the elaboration of differnt design aids regarding nonlinear behaviour
of structures, which simplify the every day work of the design engineer.
For this purpose a choice of appropriate representative normalised (dime-
nsionless) shapes of the material laws, geometries and loadings is necce-
ssary.

Reliability of nonlinear computations is very important. It is the condi-
tion for an understanding between suppliers of computer programs and
users. The activity of a user is not computer programing, but to solve
technical problems, to deside what to do with stresses, strains and dis-
placements obtained. He must rely that the stress equilibrium, strainm
compatibility and material low are adequately applied in the structural
computer programs. On the other hand he is much more interested in a clear
and simple enough input and output.

A complete confidence in nonlinear computations (regarding accuracy and
costs) can be obtained with systematic computer solutions of basic exam-
ples of structural mechanics as benchmarking cases, using representative
shapes of the material laws and geometries.

The paper deals with metal structures. The structures made of other ma-
terials can be treated similarly.

2. REPRESENTATIVE LOAD-DISPLACEMENT DIAGRAM OF THE STRUCTURE

It is easy to asses the nonlinear behaviour of a structure tracing the
representative load-displacement diagram for a given loading path, obta-
ined either experimentally or with the computer simulation. Hovever, the
great advantage of the computer simulation is a very simple separation
of influences of different parameters. So it is easy to divide geometri-
cal nonlinearities from material nonlinearities, the primary behaviour
from the additional one. It is simple to judge the influence of different
material laws, the influence of different geometrical and/or material

imperfections etc. In this way, with the deterministic knowledge regarding the influence of various parameters and with the knowledge relating their statistical distribution, the rational probabilistic predictiction of the real behaviour of structures is possible. In other words, the deterministic knowledge of the real behaviour of structures until the exhaustion is the condition for the successful probabilistic treatment of the safety resp.reliability of the structures.

It must be emhasised that not only the strength but also the inelastic deformability plays an important role in the assessment of structures. Fig.1 shows different load-displacement diagrams, F-U, with the same ultimate strength. In Fig. 2 the working state (WS), the inelastic deformation limit state (IDLS) and the ultimate limit state (ULS) are presented. While ULS is always on the peak of the curve, the IDLS as the second criteria for the determination of the allovable working state never can be defined uniquely. It depends too much on specific requirements for the given function of individual structures.

It is suitable to treat the inelastic structural behaviour from the point of view of three characteristic plastic strain regions: compression instability with small strains, tension plastic instability with medium strains and fractures with large strains.

The load-displacement diagram of the primary behaviour of a stocky compression component has usually the shape shown in Fig. 3. Material and geometrical imperfections like residual stresses and initial bow can substantially reduce the ultimate limit load. Additional reduction is possible due to lateral or local buckling. The ductility requirements of the material for compression components are low. They are more technologically conditioned.

The load-displacement diagram of the primary behaviour of a tension component with the constant cross-section has a typical shape according to Fig. 4. It has the full similarity to the course of the stress-strain diagram obtained with the tensile test, until the tensile strength is reached at the finish of the uniform elongation. Therefore besides the elastic modulus and the yield stress the tensile strength and the uniform elongation are the leading material properties for tension components. Residual stresses, initial geometrical imperfections and lateral loads have no effect on the ultimate limit state of a tension component if there is no condition for an earlier fracture. Only in the region of small strains the inelastic deformability can be more expressed.

Ductile fracture behaviour with large strains can be observed on bending or torsion test besides usual tensile test at necking. The large plastic rupture strains have an immense influence at sharp strain concentrations like cracks.

In general the primary load-displacement diagram of a component or of a structure can be shortened (decreasing of the carrying capacity and of the ductility) due to the lateral or local buckling at compressed parts or due to sudden fracture or stable crack growth at tension parts. At the stable crack growth the decreasing of the load carrying capacity with the increasing of the crack area has to be considered as the decreasing of the safety with the time, Fig. 6.

Besides the influence of the triaxiallity of the stresses on the deformability and fracture the inelastic behaviour of the material depends also on the temperature and strain velocity (impact, creep). There is an infinite number of possible working diagrams of the materials, with further complications: the anisotropy, the physical nonhomogeneity (different working diagrams) and the geometrical nonhomogeneity (defects).

For the computer simulation of the inelastic structural behaviour the assumption of the linear distribution of strains through the thickness of bars, plates and shells enables a considerable simplification. However , the eventual plastic strain reversal , e.g. at strong seismic loading, together with the influence of Bauschinger-Effect, makes again the computer simulation very complex and costly, also for the research.

3. NORMALISED LOAD-DISPLACEMENT DIAGRAM AND THE USE OF REPRESENTATIVE DIMENSIONLESS PARAMETERS

To overcome the complexity of inelastic computer simulations of individual cases there are two measures at disposal.

First measure is the use of normalised (dimensionless) load-displacement relationship with the elastic limit state as the norm for the load and the corresponding displacement as the norm for the displacement. In this way every individual computation (or experimental test) is generalised, because it is valid for all geometrically similar structures with similar shape of the working diagram of the material.

The second much more effective measure is the choice of typical dimensionless stress-strain diagrams of the material, cross-sections, components and systems, when neccessary also representative dimensionless material and geometrical imperfections can be used. So a lot of generalised computations of typical examples can be made in advance, once for ever. Of course an international cooperation for this purpose is necessary.

Such generalised computations also enable a quantitative classification of structures regarding their inelastic behaviour (ductility, plastic reserve) and the creation of many design aids in the form of technical data sheets as an addition to the future international structural codes, which can be prepared very efficiently with the help of computers.

As an example Fig. 7 shows the dimensionless "horizontal force- horizontal displacement" for a cantilever column. The results of the parametric nonlinear computations of this simplest case are very useful in the judgment of the seismic behaviour of framed structures. In advance prepared moment-curvature relationships for a given type of the cross-section and material certainly considerably reduce the price of the computations.

4. GENERALISED REPRESENTATIVE STRESS-STRAIN DIAGRAMS OF THE MATERIAL

There are two types of the working diagrams for metals: with the continuous strain hardening after the linear behaviour (e.g. aluminium alloy, austenitic steel) and with the additional plastic plateau (ferritic steel). While the plastic plateau clearly represents the yield stress, there is a need of a redifinition of the yield stress for the continuously hardening materials. The yield stress defined with 2%o inelastic strain should be replaced with the new yield stress defined with the equality of elastic and plastic strain (Fig. 8). In this way the well known Ramberg-Osgood stress-strain relationship, normalised with this yield stress and the corresponding elastic strain, has a simple oneparametric expression

$$\bar{\varepsilon}' = \bar{\sigma}' \left(1 + \bar{\sigma}'^{N} \right)$$

and $\left(1 + \bar{\sigma}'^{N} \right)^{-1}$ represents the dimensionless secant modulus. The exponents N=4, 10 and 30 can be taken as representative.

For small strain problems the above Ramberg-Osgood equation is generally valid. For medium and large strain problems the elastic limit strain $\varepsilon_e = \sigma_e : E$ as an additional parameter has to be taken into account for the dimensional treatment. Fig. 9 taken from /1/ shows normalised natural $(\bar{\sigma}' - \bar{\Sigma}')$ and engineering $(\bar{\sigma} - \bar{\varepsilon})$ stress-strain diagrams for chosen re-

presentative N=4, 10, 30 and for the practical limits of ε_e from 0,001 to 0,01.

For small strain problems the additional plastic plateau (without supplemental difficulty with the upper yield stress) makes a lot of truble because of its extreme complexity due to the inhomogeneous spreading of plastic strains (Lüder's bands). Mathematically simple horizontal line is physically exceptionally complicated.

It is still open question how to choose typical dimensionless working diagrams for the materials with plastic plateau. In any case the Lüder's strain is an additional parameter for such materials.

5. APPROXIMATE NONLINEAR COMPUTATIONS

Besides the "exact" nonlinear computations there is a need for different approximate nonlinear computations of structures (geometrical and/or material). Of course, a simplified method should not be too uneconomical and should be on the safe side. It is appropriate to integrate the approximate and more accurate computations in one programing system.

Approximate computations can serve also for the preliminary design. However it is important to consider their assumptions and limitations. E.g. the plastic hinge theory can be useful for the predominant bending, but it is on the unsafe side for the predominant compression. For the predominant compression an inelastic bifurcation computation gives a physically clear upper limit of the carrying capacity if no geometrical imperfections are taken into account.

It has to be mentioned that the possibility of an automatic jump from linear to the nonlinear computation should be introduced im the successful structural programs, corresponding to the suitable criteria.

6. RELIABILITY OF NONLINEAR COMPUTATIONS

The best assurance of the reliability of nonlinear computations is a large public use of the corresponding software, selected on the base of a sound competition, taking into account exclusively the criteria important for the structural engineering profession as whole. Such programs are optimal regarding the portability, maintenance and updating. After a certain time their mistakeability is negligible, which is the most important property.

Computer graphics with possible interactive guidance of the nonlinear computation and tracing the corresponding load-displacement diagram, including an eventual additional information regarding the convergence resp. the change of the determinant, is the best tool for the reliable nonlinear computation.

As an example Fig. 10 shows the graphical result of the zero determinant search for an inelastic bifurcation computation for a truss. The decreasing of the inelastic bending rigidity of the cross-section with the increasing axial force is taken from ECCS-column buckling curve "c" (fictitous rigidity, which includes the effect of geometrical imperfections besides the residual stresses). The thin curve represents the verification with the elastic bifurcation calculation, using the inelastic rigidities at the inelastic bifurcation as constant elastic rigidities.

Fig. 11 gives three inelastic bending rigidity curves for axially loaded aluminium alloy cross-section according to ECCS (effective without the influence of geometrical imperfections, fictitious with this influence).

Finally, the confidence into the inelastic computer simulations can partacularly be obtained with the systematic computer solutions of the ba-

sic examples of the structural mechanics, using internationally recogni-
sed representative dimensionless stress-strain curves of the material.
Such examples are: tension of the round bar with the necking, torsion of
the round and of the rectangular bar, bending of the bar, tension of a
strip with the hole, shear and tension of the fillet weld, Brinell hard-
ness test, Griffith's crack, penny shaped crack, single and multiple po-
res, buckling of the bar with the Shanley-Effect etc. Besides the norma-
lised load-displacement diagrams the change of typical stresses and stra-
ins has to be given, if the fracture criteria have to be considered.

Nonlinear computations of this kind, verified by parallel computing with
differnt programs and eventually tested with deterministic experimental
tests, would have a historical value for structural mechanics and engi-
neering. For the beginning naturally a homogeneous and isotropic materi-
al has to be used. Later representative cases of the nonhomogeneity and
anisotropy will have to be considered. Such activity should awake the
interest for the cooperation between many international assotiations
(IABSE, IUTAM, RILEM and the corresponding assotiations for mechanical
engineering and metallurgy).

The need for an international cooperation also regarding the practical
use of the results of nonlinear computations can be illustrated with the
Fig. 12, taken from /2/. For a typical I-profile (IPB$_v$340 and geometri-
cally similar) and for the exponent N=4 of the Ramberg-Osgood material
the ductility functions K_φ (in this case dependet of normalised bending
moment $\bar{M}=M:M_e$ only) have been determined (see Fig.13 for axial force
$\bar{N}=N:N_e=0$). K_φ is the ratio between the total bending curvature and the
corresponding elastic curvature. It represents the reciprocal value of
the normalised secant modulus. Then for the beam with the uniformly di-
stributed load, clamped on both ends, the ductility functions K_S of
the load-displacement relationship have been computed in a semi analy-
tical way

$$K_S = \frac{6}{\bar{M}_1 + \bar{M}_2} \int_{\bar{M}_2}^{\bar{M}_1} \bar{M} \cdot K_\varphi \cdot d\bar{M}$$

With the condition

$$\int_0^{\bar{M}_2} \frac{\bar{M} \cdot K_\varphi}{\sqrt{1 - \frac{\bar{M}}{\bar{M}_2}}} \cdot d\bar{M} = \int_0^{\bar{M}_1} \frac{\bar{M} \cdot K_\varphi}{\sqrt{1 + \frac{\bar{M}}{\bar{M}_2}}} \cdot d\bar{M}$$

for the relation between the negative moment $\bar{M}_1$ and positive moment $\bar{M}_2$,
fulfiled with the iteration. In the normalised load-displacement
diagram in Fig. 12 many different limit states are presented. They are
defined either with different procentage of unrecoverable displacements
(1-7), actual (8,9,10) or linear stress limits (11,12) and also accor-
ding the criteria of the plastic hinge theory (13-16). The index 0,2
belongs to the 0,2 yield stress and the index2to the yield stress with
the equality of elastic and inelastic strain. It is clear that the pro-
blems are not so much in the correct use of the nonlinear computations
but in the establishment of the generalised criteria for the practical
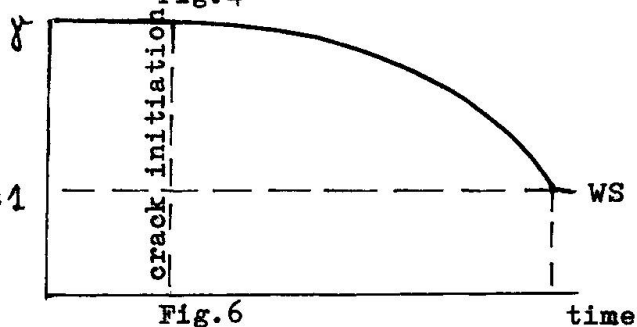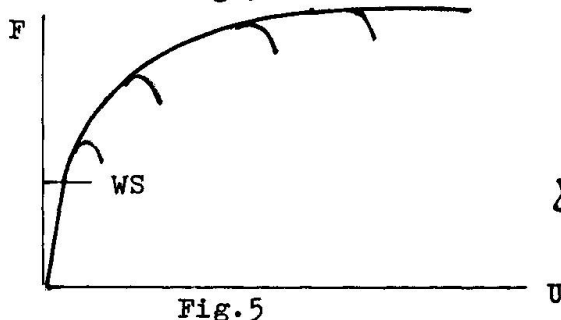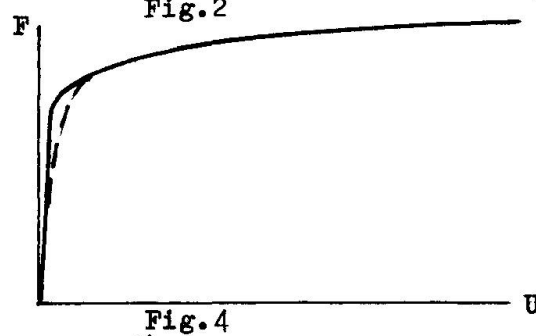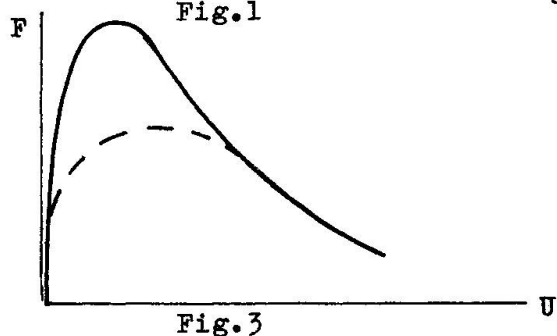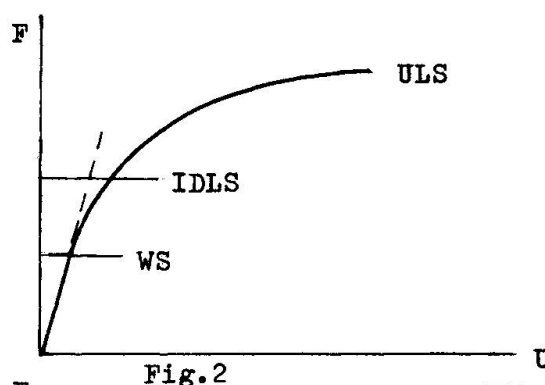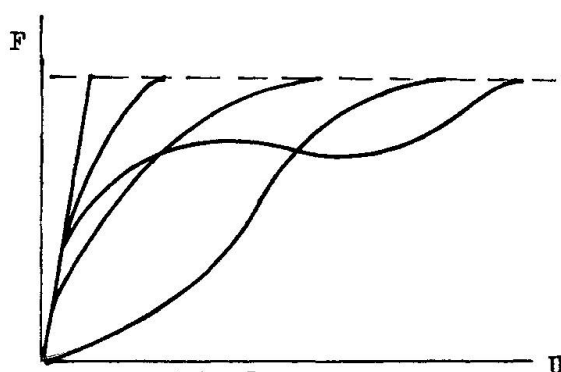application of results.

7. CONCLUSION

Nonlinear computations have to be added to the usual linear structural
software systems using contemporary possibilities of pre- and postpro-
cessing for CAD, CAM and public computer network including computers
from micros to supers, with the suitable data bases /3/.
A suitable standardisation on an international level can help essentially
to the further successful development of the structural engineering pro-
fession in the arising informatic society /4/.

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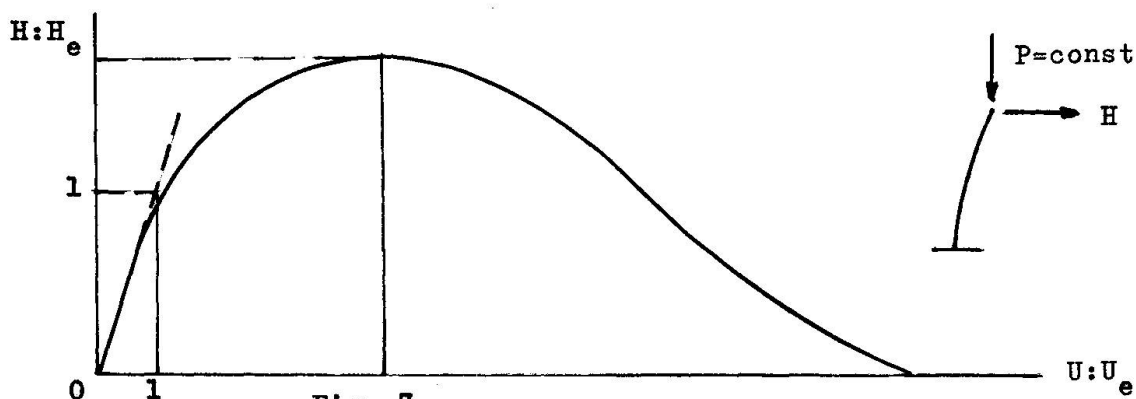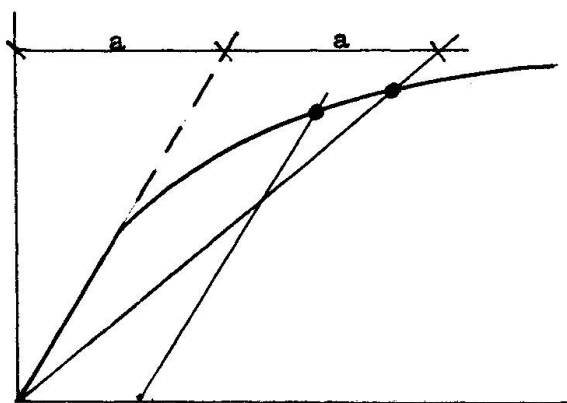/5/ J.J.Servan-Schreiber: Le defi mondial, 1980
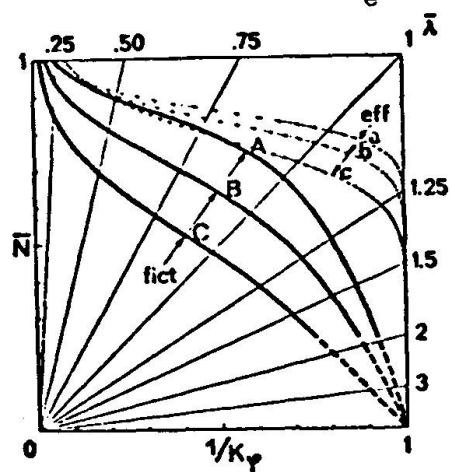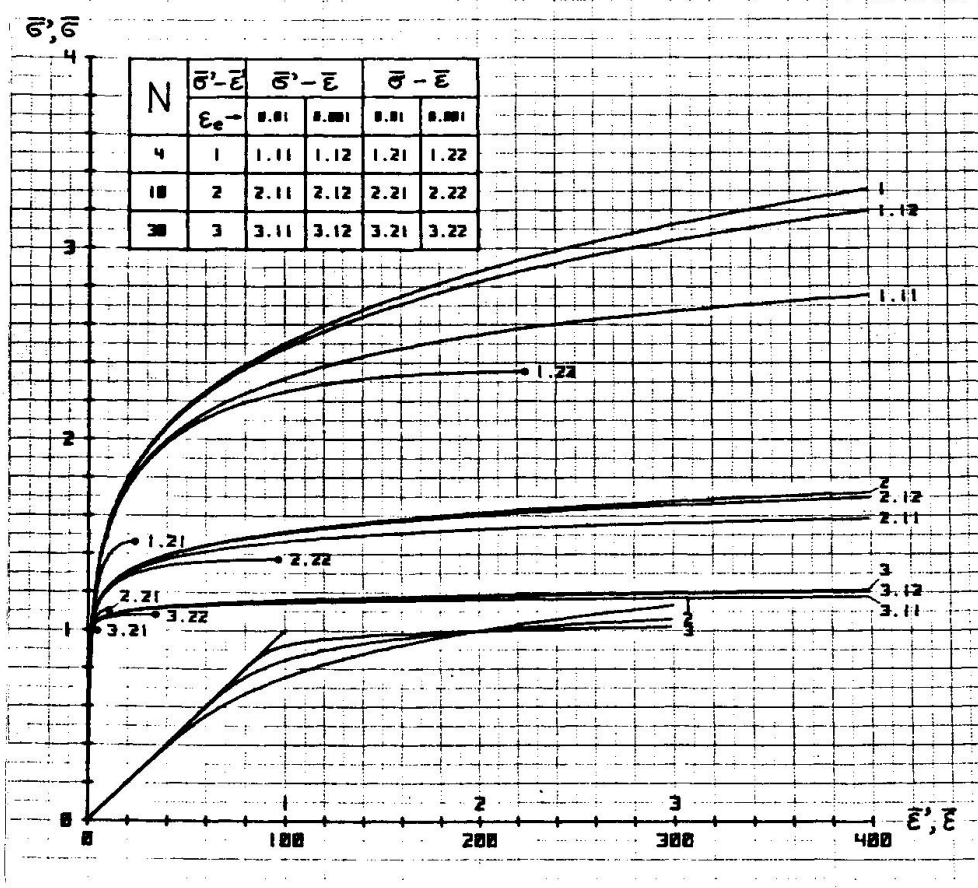
FIGURES

Fig.1

Fig.2

Fig.3

Fig.4

Fig.5

Fig.6

Fig. 7

Fig. 8

Fig. 11

N	$\bar{\sigma}'-\bar{\epsilon}$	$\bar{\sigma}'-\bar{\epsilon}$		$\bar{\sigma}-\bar{\epsilon}$	
	$\epsilon_e \rightarrow$	0.01	0.001	0.01	0.001
4	1	1.11	1.12	1.21	1.22
10	2	2.11	2.12	2.21	2.22
30	3	3.11	3.12	3.21	3.22

Fig. 9

Fig. 10

Fig. 12

$\overline{M}$

Fig. 13