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**Autor:** Basu, S. / Chi, M.  
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## **Analytical Study for Fatigue of Bridge Cables**

Etude analytique de la fatigue des câbles de pont

Analytische Studie über die Ermüdung von freitragenden Brückenkabeln

### **S. BASU**

Senior Mechanical Eng.  
Chi Associates, Inc.  
Arlington, VA, USA

### **M. CHI**

Chief Engineer  
Chi Associates, Inc.  
Arlington, VA, USA

## **SUMMARY**

A comprehensive analytical formulation of the fatigue behaviour of highway bridge cables under wind loading is presented in this paper. The formulation includes the deflection and bending stress of highway bridge cables under wind-induced vortex shedding vibrations. The fatigue behaviour of bridge cables resulting from the stress reversals caused by vortex shedding is investigated analytically in this paper using the methodologies of linear elastic fracture mechanics. An attempt is made to compare the results of fatigue life obtained from the formulation with the available experimental data.

## **RESUME**

Une formulation analytique compréhensive du comportement à la fatigue des câbles de ponts autoroutiers soumis aux actions du vent est présentée dans cet article. La formulation inclut la déformation et la contrainte de flexion de ces câbles sous des vibrations dues à des tourbillons alternés de vent. Le comportement à la fatigue des câbles de ponts résultant d'inversions de contrainte produites par des tourbillons alternés est analysé dans cet article par la méthodologie de la mécanique de la rupture linéaire élastique. Un essai de comparaison des résultats de durée de vie obtenus par la formulation avec les données expérimentales disponibles, a été fait.

## **ZUSAMMENFASSUNG**

Der Beitrag behandelt die analytische Formulierung des Ermüdungsverhaltens freier Brückenkabel und -seile unter Windlast. Die Untersuchung berücksichtigt die Deformationen und Biegespannungen infolge der durch Wirbel hervorgerufenen Schwingungen. Das Ermüdungsverhalten unter Wechselbeanspruchung durch die Wirbelwirkung wird unter Anwendung der Methode der linear elastischen Bruchmechanik analytisch behandelt. Es wird der Versuch unternommen, vorhandene experimentelle Ergebnisse über die Lebensdauer unter Ermüdungsbeanspruchung mit der rechnerischen zu vergleichen.



## 1. INTRODUCTION

Although the concept that a bridge deck can be strengthened by stay cables was first conceived by Fantus Verantius [1] in Italy, during the early seventh century, the large scale application of this concept to major bridges was quite recent, especially in the U.S.A. The detailed behavior of cable-stayed bridges and the design and construction procedures have been well-established and are available in standard references [1, 2]. However, there are still gaps of knowledge at the present time; for example, there exists no guidelines for stay cable design for prevention of fatigue failure of stay cables due to wind-induced vibrations.

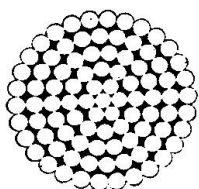
Stay-cables are susceptible to Strouhal vibrations. If the prevailing wind velocities in a geographic location frequently produce resonance conditions in the cable, it is conceivable, or even probable, that the individual wires in the cable may fail due to wind-induced fatigue loadings.

Although a number of fatigue tests of wires and cables are available in the literature, there is a paucity of analysis of wire fatigue through a rational procedure. Even more difficult is the problem to extrapolate the failure of individual wires to the probable failure of the cable as a whole.

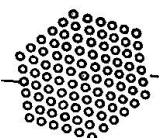
This paper entails an initial attempt to fill this gap of knowledge.

## 2. STRUCTURAL CHARACTERISTICS OF STAY CABLES

The fatigue behavior of stay cables depends, to a large extent, on its structural properties which, in turn, depend on the geometry of the cable configurations. Under the category of helical wire cables, single strand and multiple strand constructions of a wide variety of configurations are possible. Figure 1 shows the two types of cable configurations commonly used in bridge structures. These are: parallel wire cables and helical wire cables.



Cable with Helical Wires



Cable with Parallel Wires

Figure 1. Cable Configurations

Parallel wire cables are made of uncoated, stress-relieved wires which have ASTM designation A421-77BA. Helical wire structural strands are made of zinc-coated steel wires having ASTM designation A586-68, while multiple strand helical cables are manufactured according to ASTM specification A603-70. The mechanical properties of these materials are shown in Table 1.

Table 1. Mechanical Properties of Cable Materials

Material	Zinc Coating Class	Nominal Diameter mm	Minimum Stress at 1% Extension MPa	Minimum Tensile Strength MPa
A 586-68	A	1.016-2.794	1477	1517
		2.820	1576	1517
	B	2.286	1477	1448
		2.286	1379	1379
A 603-70	A	1.016-2.794	1477	1517
		2.820	1576	1517
	B	1.016	1477	1448
		1.016	1379	1379
A 421-77 (Type BA)	C	4.978	1407	1655
		6.350	1407	1655
		7.010	1377	1620

When cables are used as structural members, consideration must be given to various factors which determine their load bearing capacities and performance characteristics. One of these factors is the end anchorage which connects a cable to other supporting structural members. End anchorages vary widely in their design and manufacturing techniques and their selection depends on the sizes and properties of the cables to which they are attached. An overview of different cable constructions with particular emphasis on various end anchorages may be found in references [1] and [2]. The reader is also referred to a recent paper on fatigue resistant tendons for cable stayed construction by Birkenmaier [3].

### 3. FATIGUE ANALYSIS OF STAY CABLES

#### 3.1 Wind-Induced Fatigue Loading

Stay cables used as structural components in bridges are frequently subjected to wind forces which result in vortex shedding. If the vortex shedding frequency is close to any of the natural frequencies of a stay cable, a nonlinear phenomenon known as synchronization or lock-in occurs, and in unfavorable conditions, the cable can undergo large amplitude vibrations.

Recently Basu and Chi [4] investigated the dynamic response of stay cables by considering a simplified vortex excitation model whereby the driving wind force was harmonic with magnitude proportional to the lift coefficient and with frequency equal to the Strouhal frequency. The analytical procedure was used to determine the deflections and bending stresses of stay cables.

Table 2 illustrates typical results of deflections and bending stresses for the main cables of the Intercity Bridge on Columbia River in Pasco-Kennewick, Washington, U.S.A. Properties of the cables are given below:

Length	154 m
Outer diameter of cable	15 cm
Wire diameter	6.35 mm BBR-type
Number of wires	283
Maximum moment of inertia	855 cm <sup>2</sup>
Axial stress	745 MPa
Young's modulus	200 GPa



Table 2. Computation of Deflections and Stresses

Mode	Natural Frequency rad/sec	Critical Wind Velocity km/hr	Maximum Deflection mm	Maximum Bending Stress MPa
1	6.5578	2.74	1.082	0.5647
3	19.0608	8.27	1.215	1.7149
5	43.7219	18.98	1.315	3.9064
10	64.5815	28.04	1.093	5.8299
15	96.2724	41.79	1.216	8.8690
29	190.1227	82.52	1.215	18.5110

It is seen that for commonly occurring wind velocities, the magnitude of the maximum bending stress is less than 25 MPa. Moreover, the maximum bending stress is expected to occur at end anchorages. It should be noted that this wind induced bending stress is reversible and cyclic in nature which may cause cables to fail in fatigue.

### 3.2 Fatigue Life of a Wire

Consider a single wire subjected to wind-induced fatigue loading. The fatigue initiation life of the wire,  $N_i$ , is given by the following relationship:

$$N_i = C_1 (\Delta\sigma)^{-\gamma} \quad (1)$$

where  $\Delta\sigma$  is the nominal stress range, and where  $C_1$  and  $\gamma$  are two constants which depend, in general, on the material properties. Basu and Chi [4] computed the values of  $C_1$  and  $\gamma$  by a statistical fit of various test data on the fatigue life of high strength steel wires, and found that for wires with average tensile strength of 1650 MPa:

$$C_1 = 1.57 \times 10^{26} \text{ and } \gamma = 7.5$$

The fatigue initiation life of a single wire can therefore be calculated from the knowledge of  $C_1$  and  $\gamma$ .

The fatigue propagation life of a single wire can be determined from the following relationship:

$$\frac{da}{dN} = C_2 (\Delta K)^{\mu} \quad (2)$$

where  $a$  is the crack size,  $\Delta K$  (or equivalently,  $\Delta K_I$ ) is the stress intensity factor range, and  $C_2$  and  $\mu$  are two parameters which depend, among other things, on material properties. Barsom [5] tested various martensitic steels for fatigue crack propagation and found that:

$$\mu = 2.25 \text{ and } 0.27 \times 10^{-8} \leq C_2 \leq 0.66 \times 10^{-8}$$

for steels having yield strength ranging from 550 MPa to 2000 MPa. We therefore assumed that  $\mu = 2.25$  and  $C_2 = 0.66 \times 10^{-8}$  for the fatigue propagation in a single wire.

The propagation life can be obtained by the direct integration of Equation (2). For a circumferential crack in a solid cylinder, the stress intensity factor,  $K_I$ , can be approximated as follows [4]:

$$K_I = \sigma \{1 + .054 \left(\frac{a}{R}\right)^2\} \sqrt{a} \quad (3)$$

where  $R$  is the radius of the wire. Equation (3) may be used to compute the critical crack size,  $a_0$ , if the fracture toughness,  $K_{IC}$ , and the maximum stress level,  $\sigma_{max}$ , are known. Figure 2 shows the fatigue propagation life of a single wire for various values of initial crack size,  $a_0$ , and for a fracture toughness of  $100 \text{ MPa } \sqrt{\text{m}}$ .

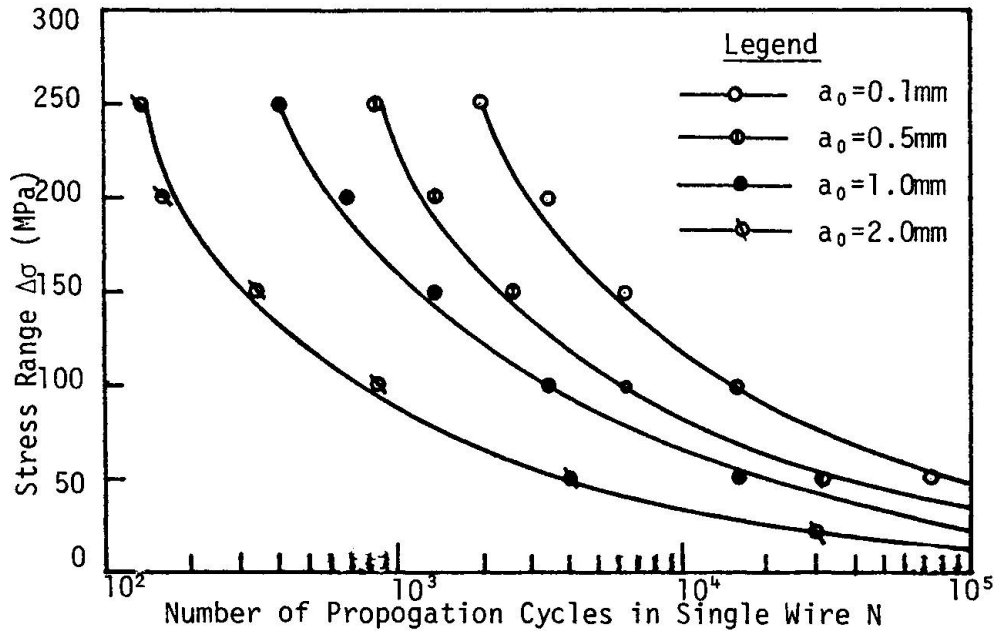


Figure 2. Fatigue Propagation Life of a Single Wire

### 3.3 Fatigue Life of Cable

Our model for the fatigue failure of a cable is based on the assumption that whenever a wire fails, its load is redistributed equally among the remaining wires. As the number of broken wires increases, the axial stress of unbroken wires would eventually reach the ultimate tensile strength of the wire material at which point the final failure of the cable occurs. Obviously, the number,  $m$ , of wires that fail by fatigue before the cable collapses is smaller than  $n$ , the number of wires in the cable.

Consider the fatigue life of a single wire is Gaussian distributed with mean life  $\bar{N}$  and a standard deviation  $s$ . Then the probability of a single wire having a fatigue life of  $N$  cycles is given by:

$$\phi(N) = \int_0^N \frac{1}{\sqrt{2\pi}s} e^{-\frac{(x-\bar{N})^2}{2s^2}} dx^2 \quad (4)$$

The problem of determining the fatigue life of a cable is now equivalent to computing the probability of failure of  $m$  wires out of a bundle of  $n$  wires which is given by:

$$P(m,n) = \binom{n}{m} \phi^m (1-\phi)^{n-m} \quad (5)$$

This type of probabilistic formulation has been developed elsewhere by Andrä and Saul [6] for parallel wire cables. Assuming now that the joint probability distribution,  $p(m,n)$  is also Gaussian, the average life of the cable may be calculated.



Figure 3 shows typical plots of the average fatigue life of a 7x1 wire cable in comparison to the fatigue life of a single wire.

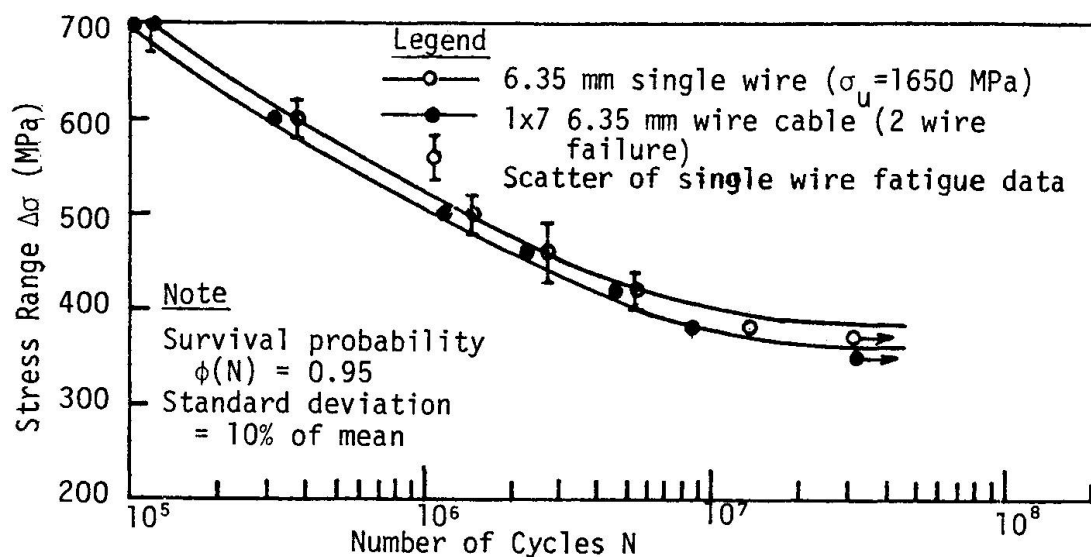


Figure 3. Fatigue Life of Wire and Cable

#### 4. COMPARISON WITH EXPERIMENTAL RESULTS

Fatigue testing of high tension wires and cables have been reported by a number of various authors but the variables in these tests are so different from one another that a direct comparison between the test results and the analytical results are not possible. Only an order of magnitude comparison can be made in a meaningful manner.

Birkenmaier [3] reported test results of dynamic tensile fatigue of 7 mm diameter single wires and wire bundles subjected to 2 million fatigue cycles at different stress ranges. The results are shown in Figure 4. The fatigue limits have also been computed for single wires using the above analytical formulation, and the results are also plotted in Figure 4. The comparison between analytical and experimental results shows a good agreement for single wires.

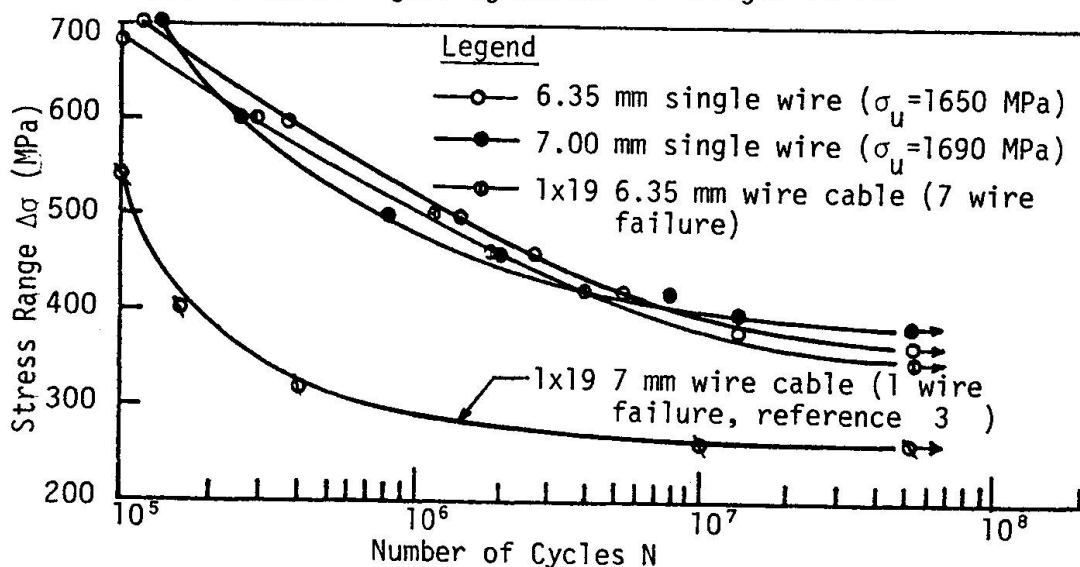


Figure 4. Comparison of Wire and Cable Fatigue Data [3]



## 5. CONCLUDING REMARKS

The analysis shows that there is a correlation between the fatigue life of a single wire and that of a cable or a wire bundle. In principle, it is now possible to determine the fatigue behavior of stay cables on basis of the structural characteristics and fatigue behavior of single wires. Evidently, a complete understanding and analysis of the same requires an extensive amount of additional research and the present paper may be considered as an initial effort in this direction.

Conversion of units:

$$\begin{aligned} 1 \text{ m} &= 39.37 \text{ in.} & 1 \text{ MPa} &= 1 \text{ N/m}^2 = 145 \text{ psi} & 1 \text{ GPa} &= 1 \text{ KN/m}^2 = 145 \text{ ksi} \\ 1 \text{ MPa } \sqrt{\text{m}} &= 0.91 \text{ ksi } \sqrt{\text{in.}} \end{aligned}$$

## ACKNOWLEDGEMENTS

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