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## **Fatigue Crack Propagation in Steel Prestressing Wires**

Propagation des fissures dues à la fatigue dans les fils de précontrainte

Ermüdungsrisssfortpflanzung in Vorspanndrähten

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## **SUMMARY**

This paper presents the results of research carried out to determine fatigue crack growth in steel prestressing wires. Measurements were made by using the compliance method and recording continuously the compliance of the sample by means of a dynamic extensometer. Tests performed at different frequencies, waveforms, stress ratios and load amplitudes show that fatigue crack growth can be described by a Paris law.

## **RESUME**

Cette communication présente les résultats d'une recherche réalisée pour déterminer la vitesse de propagation des fissures de fatigue dans les fils de précontrainte. Les mesures ont été faites au moyen de la méthode de la flexibilité avec un extensomètre dynamique. Les essais qui ont été réalisés à différentes fréquences, formes d'onde, relations de contraintes et amplitude de charge, ont montré que la propagation des fissures dues à la fatigue peut être décrite par une expression de Paris.

## **ZUSAMMENFASSUNG**

Der Beitrag stellt die Ergebnisse einer Untersuchung über das Ermüdungsrissswachstum in Vorspanndrähten vor. Die Messungen wurden mit einem dynamischen Extensometer durchgeführt. Die Versuche, die mit verschiedenen Frequenzen, Wellenformen, Spannungsverhältnissen und Belastungsamplituden durchgeführt wurden, zeigen, dass das Ermüdungsrissswachstum mit einer Gleichung von Paris beschrieben werden kann.





## 1. INTRODUCTION

In the prestressed concrete industry, steel tendons may be subjected to tensile axial fatigue. But despite the importance of this loading the very few published axial fatigue tests have been made without adequately distinguishing between crack initiation and crack propagation in accordance with most prevailing standards. Recent requirements, in particular the Model-Code [1] issued by CEB (Comite Euro-International du Beton), still specifies for this material an endurance limit from classical Wohler-type or S-N tests. The purpose of this paper is to complement classical approach by applying fracture mechanics techniques to techniques to tendons subjected to axial fatigue. Crack propagation has been measured and is the subject of this paper; work on crack initiation is in progress and will be published later.

## 2. MATERIALS TESTED

Three steels have been tested; all are eutectoid cold drawn steels, currently used in prestressed concrete structures. Steels A and B were stress-relieved while steel C was stabilized.

Table 1 shows the mechanical properties of all three steels tested. As the material is produced in the shape of wires, the fracture toughness of these steels can not be obtained with compact specimens. Therefore  $K_{IC}$  values listed have been determined by testing precracked samples, using single edge notches as starters of surface precracks produced in 3-point bending or in tension.

## 3. TESTS PERFORMED

Axial loading fatigue tests have been performed in an Instron dynamic testing machine. The observation of crack growth was made by the method of the compliance of the samples. First of all, the compliance of all steels tested has been determined as a function of crack depth by testing in tension several precracked samples with different crack depths. A 12.5 mm. gauge length extensometer provides enough accuracy to achieve a calibration curve compliance vs crack depth.

In fatigue tests, a 12.5 mm. gauge length dynamic extensometer attached to the precracked sample gives a continuous record of the compliance of the sample. A Hewlett Packard data acquisition system has been used to collect and compute compliance data provided by the extensometer and from the calibration curve, to plot the curves of crack depth vs. number of cycles.

All tests have been performed in air at  $20 \pm 1^\circ\text{C}$  of temperature and a relative humidity of  $50 \pm 5\%$ . Load amplitude and frequency have been held constant automatically during the test. Three waveforms have been studied: sinusoidal, triangular and square, and also five frequencies 0.2, 1, 4, 8, 10 and 20 Hz. The stress range was varied from 250 to 610 MPa and R ratio from values below 0.20 to 0.67.

## 4. EXPERIMENTAL RESULTS

In order to predict fatigue crack growth in prestressing steels it is worthwhile to ascertain whether the experimental results are based upon some general



fatigue crack growth law. Among them, the simplest one is the Paris law [2]:

$$\frac{da}{dN} = C(\Delta K)^n \quad (1)$$

where  $da/dN$  is the crack growth rate,  $\Delta K$  the stress intensity factor range and  $C$  and  $n$  constants.

The stress intensity factor for an elliptical crack in the surface of a cylindrical bar in tension is given by the following expression, obtained from a finite element calculation [3]:

$$K = \sigma \sqrt{\pi a} \left\{ 0.473 - 3.286 \left( \frac{a}{D} \right) + 14.797 \left( \frac{a}{D} \right)^2 \right\}^{1/2} \left\{ \left( \frac{a}{D} \right) - \left( \frac{a}{D} \right)^2 \right\}^{-1/4} \quad (2)$$

where  $a$  is the crack depth,  $D$  the diameter of the wire and  $\sigma$  the remote applied stress.

The stress intensity factor defined by equation (2) has been shown to be a suitable fracture parameter. For each tested material the critical value of this parameter was found to be independent on flaw size. This critical value appears in Table 1 for the three materials as  $K_C$ .

From the curves crack depth vs. number of cycles plotted by the system, a numerical differentiation method has been applied to obtain the crack growth rates  $da/dN$  at different crack depths. Since the stress range  $\Delta\sigma$  was held constant for each test, the stress intensity factor range  $\Delta K$  can be calculated for these crack depths from equation (2). Then it is possible to plot  $\log. (da/dN)$  vs.  $\log. (\Delta K)$  and check if there exists a linear relationship as equation (1) predicts.

Steel A has been fully tested to crack the influence of load amplitude, waveform, stress ratio and frequency on the results.

Figure 1 shows the experimental results of crack growth rate vs. stress intensity range in a log. log. plot. for steel A and different load amplitudes. This figure shows that a relationship of the Paris form is able to describe crack growth in prestressing cold drawn steel wires, independently of the stress amplitude applied.

Figure 2 includes also the results obtained at different frequencies, with different waveforms and with different stress ratios. As can be seen, fatigue crack growth in prestressing steel wires is independent of waveform and frequency, as it has been observed for many other alloys [4]. The crack growth also appears independent of stress ratio  $R$ . Finally the figure shows the best fit by a straight line in the log. log. plot leading to the following crack growth law:

$$\frac{da}{dN} = 11.08 \times 10^{-12} (\Delta K)^{2.3}, \text{ where } a \text{ is in m. and } K \text{ in MPa m}^{1/2}. \quad (3)$$

Figures 3 and 4 show the experimental results obtained with steels B and C and the corresponding straight lines fitted. Values of the constants as is shown in those figures are  $C = 13.96 \times 10^{-12}$ ,  $n = 2.3$  for steel B and  $C = 16.05 \times 10^{-12}$ ,  $n = 2.3$  for steel C. In all cases the correlation coefficient is 0.97.

Values obtained for the constants  $C$  and  $n$  are very close for the three steels tested within the usual experimental scattering and are in agreement with the experimental results by Ritchie and Knott [5] for material with  $K_C$  values similar to those of steels tested.



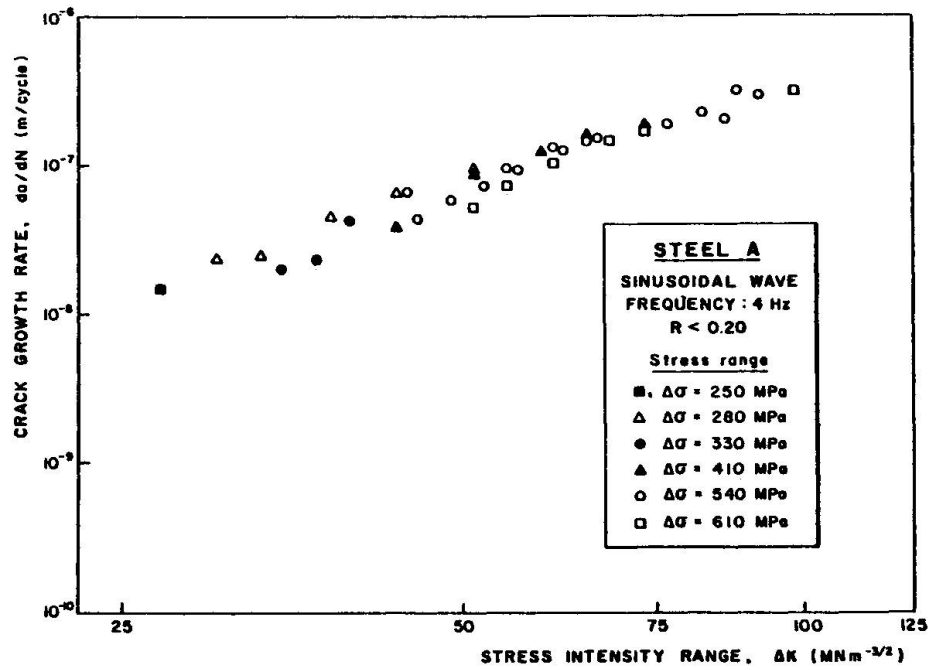


Figure 1. Crack growth rate versus stress intensity range for various stress ranges in steel A.

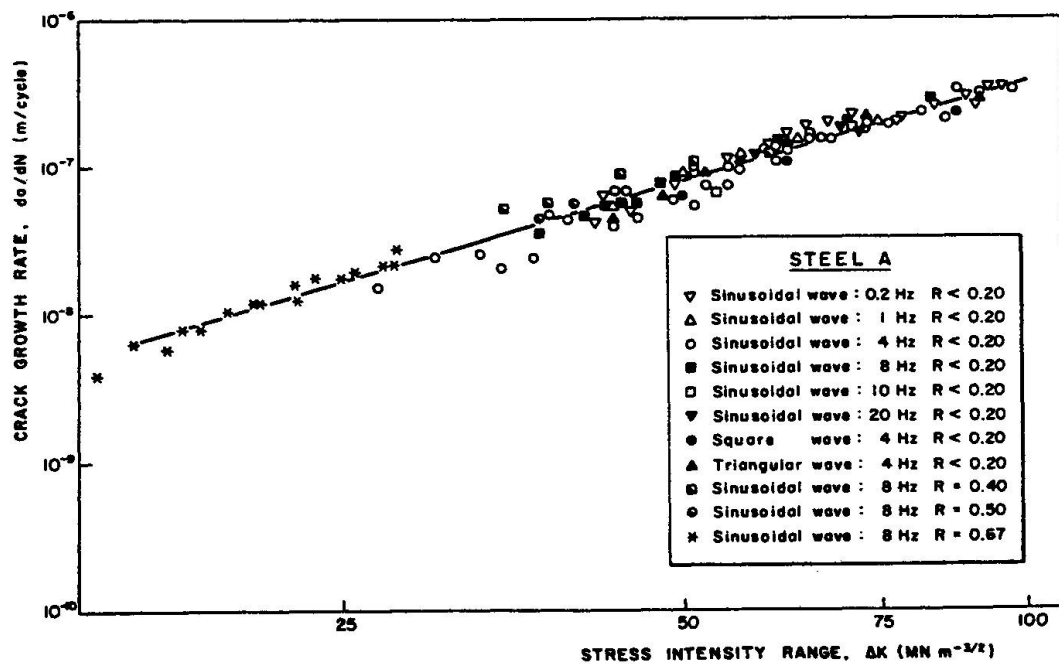


Figure 2. Crack growth rate versus stress intensity range in steel A.



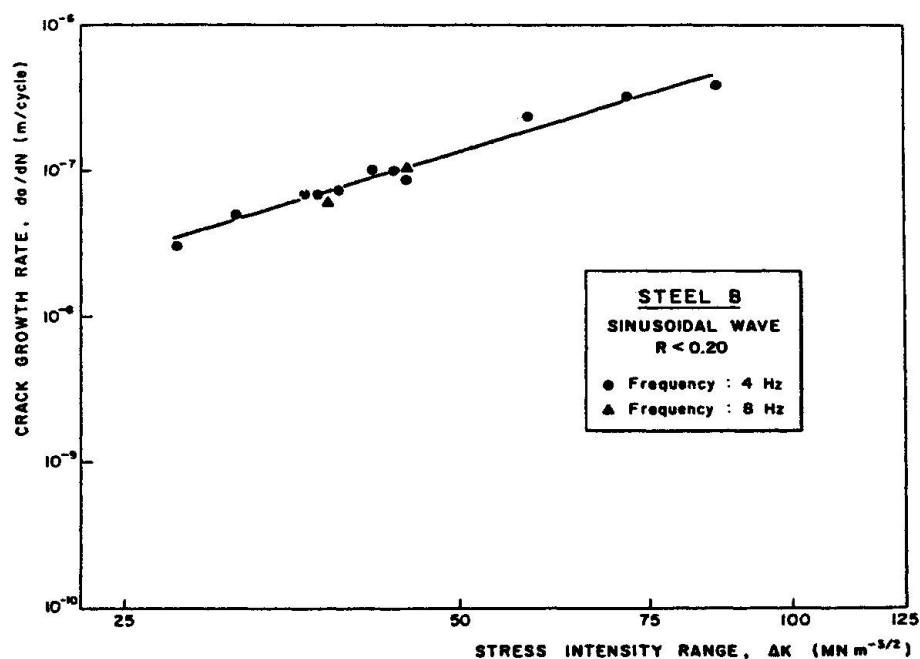


Figure 3. Crack growth rate versus stress intensity range in steel B.

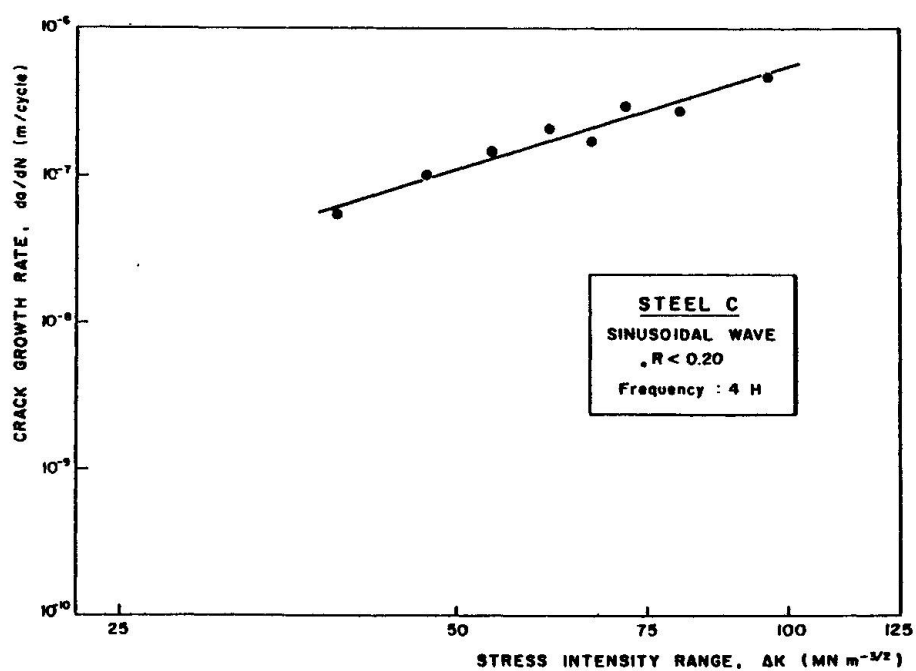


Figure 4. Crack growth rate versus stress intensity range in steel C.





Table 1. Mechanical Properties

	Steel A	Steel B	Steel C
Young's modulus (MPa)	$2.01 \times 10^5$	$2.05 \times 10^5$	$1.81 \times 10^5$
0.2% Proof stress (MPa)	1400	1520	1560
Tensile Strength (MPa)	1620	1740	1800
Elongation under max. load (%)	6.5	5.5	6.5
Reduction in area (%)	40	30	35
Fracture Toughness $K_{IC}$ (MPam <sup>1/2</sup> )	108.8	112.3	101.7

## 5. APPLICATION FOR DESIGN

Two valuable results have been obtained which permit to propose a simple method to predict the life of a prestressing steel wire subjected to cyclic stresses. The first result is that the fatigue crack propagation for these steels follows the Paris law and is independent on stress ratio, frequency and waveform within the abovementioned range. The second one is that the parameters C and n of the Paris law have similar values for all these steels.

If the nominal stress in the wire has a range of variation  $\Delta\sigma$ , the variation of the stress intensity factor for a crack depth a is given by:

$$\Delta K = \Delta\sigma \sqrt{\pi a} M \quad \text{where} \quad M = \left\{ 0.473 - 3.286 \left( \frac{a}{D} \right) + 14.797 \left( \frac{a}{D} \right)^2 \right\}^{1/2} \left\{ \left( \frac{a}{D} \right) - \left( \frac{a}{D} \right)^2 \right\}^{-1/4} \quad (4)$$

$$M = \left\{ 0.473 - 3.286 \left( \frac{a}{D} \right) + 14.797 \left( \frac{a}{D} \right)^2 \right\}^{1/2} \left\{ \left( \frac{a}{D} \right) - \left( \frac{a}{D} \right)^2 \right\}^{-1/4} \quad (5)$$

By substituting equation (4) in the Paris law (1) and integrating, the following expression is obtained:

$$N_R = \frac{(\Delta\sigma \sqrt{\pi D})^{-n}}{C} \left\{ F \left( \frac{a_R}{D} \right) - F \left( \frac{a_0}{D} \right) \right\} \quad (6)$$

in which the subscript R indicates values at fracture,  $a_0$  is the depth of preexisting flaws in the wire and F the non dimensional function:

$$F(x) = \int_0^x \left[ \sqrt{x} M(x) \right]^{-n} dx \quad (7)$$

which has been calculated for  $n = 2.3$  and is plotted in figure 5.

The critical crack depth  $a_R$  has to be obtained from equation (2) by equating the stress intensity factor with its critical value  $K_{IC}$ , the stress  $\sigma$  being the maximum value of the stress applied.

The method proposed can be successfully applied to wires with surface flaws of similar size to those used in this work. On the other hand it would be worthwhile to ascertain whether the method could be applied for the life prediction of prestressing wires without any defect except those produced during the steel processing. Therefore the method proposed has been applied to the fatigue tests results carried out on smooth samples by Elices and Sánchez-Gálvez [6].



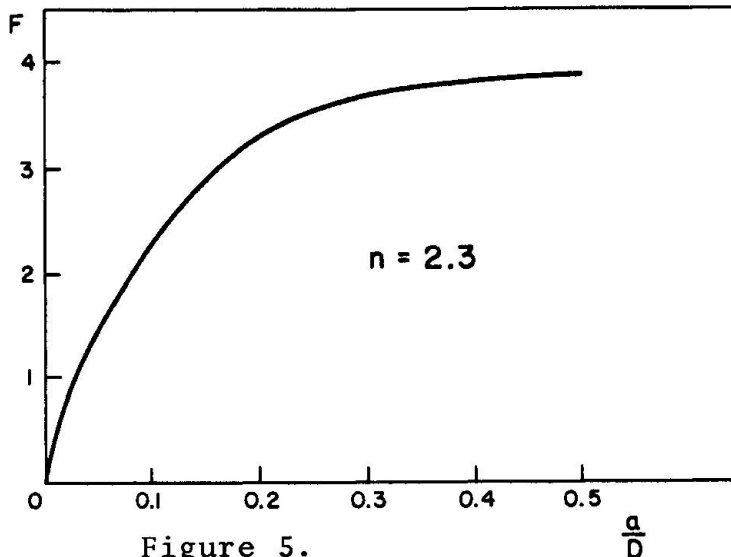


Figure 5.

From all tests results only those tests with stress ranges high enough to ensure that initiation times are negligible as compared with propagation times have been selected.

Table 2 shows for both steels the results of the test (each result is the mean value of four tests) and the predictions obtained from equation (6). To apply this equation a value for  $a_0$  has to be assumed.

However figure 8 shows that  $F(a_0/D)$  is much smaller than  $F(a_R/D)$  for usual values of  $a_0$  (below 50  $\mu\text{m}$ ) and  $a_R$  (which appears in table 2). Therefore  $f(a_0/D)$  has been neglected in the calculation. The parameters  $n$  and  $C$  chosen have been those from the steel A of this work,  $n = 2.3$  and  $C = 11 \cdot 10^{-12}$ .

Table 2 shows that the values of  $N_R$  obtained in the tests are in good agreement with the predicted values from the equation (6). Although Linear Elastic Fracture Mechanics (LEFM) has been questioned for such high  $\sigma_{\text{max}}$  values, it is hoped that for lower values the agreement should be even better. In consequence this method could be a valuable aid for the designer.

Table 2. Tests results and Predictions

Data of the steels				Steel 1	Steel 2
Tensile strength (MPa)				1630	1720
Fracture Toughness $K_{IC}$ (MPa $\text{m}^{1/2}$ )				93.5	82.0
Diameter (mm)				7	7
Tests results and Predictions					
	$\Delta\sigma$ (MPa)	$\sigma_{\text{max.}}$ (MPa)	$a_R$ (mm)	$N_R$ (measured) $10^5$ cycles	$N_R$ Prediction) $10^5$ cycles
Steel 1	330	1300	1.50	3.1	2.8
	410	1380	1.41	1.3	1.7
	330	1460	1.33	2.3	2.7
Steel 2	340	1370	1.23	2.1	2.4
	340	1540	1.03	2.0	2.2

## 6. CONCLUSIONS

For the first time the crack growth rate in axial loading fatigue for prestressing steel wires has been determined based upon Fracture Mechanics concepts.

Test results show that fatigue crack growth for cold drawn steels can be





described by Paris law. The results are independent of waveform, frequency and stress ratio within the intervals used.

From the tests results it is difficult to ascertain the influence of the heat treatment of the steel on the fatigue crack propagation since the behaviour of all three steels tested has been very similar taking into account the usual scattering of this kind of measurements.

Finally an application to design has been made by predicting the life of wires subjected to axial fatigue. The agreement found with theoretical predictions seems to validate this approach, based on LEFM, although some plasticity should be developed. Starting from known values of  $K_C$ , fatigue life under constant amplitude load can be predicted.

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