

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 37 (1982)

**Artikel:** Fatigue properties of concrete members subjected to torsion  
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**DOI:** <https://doi.org/10.5169/seals-28938>

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## Fatigue Properties of Concrete Members Subjected to Torsion

Propriétés de fatigue des éléments en béton soumis à la torsion

Ermüdungseigenschaften von Betonbauteilen unter Torsionsbeanspruchung

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### SUMMARY

Experimental investigation was undertaken to study fatigue properties of concrete members under pure torsion. The experiments included 148 tests consisting of plain, reinforced, prestressed and steel fibre reinforced concrete beam specimens. From results of the fatigue tests, S-N lines were obtained by statistical procedures. Fatigue strength at 10 millions cycles of loading was 53 to 56 % of the static ultimate strength of plain and prestressed concrete, while that of reinforced and steel fibre concrete was 38 to 53 %.

### RESUME

Des recherches expérimentales ont été entreprises pour étudier les propriétés de fatigue des éléments en béton sous l'effet de la torsion pure. Les expériences comprenaient 148 essais de poutres en béton seul, armé, précontraint et armé de fibres d'acier. Des résultats des essais de fatigue, les droites S-N ont été obtenues à l'aide de procédés statistiques. La résistance à la fatigue après dix millions de cycles de charges valait 53 à 56 % de la résistance ultime statique du béton seul et précontraint, tandis que celle du béton armé et de fibres d'acier valait 38 à 53 %.

### ZUSAMMENFASSUNG

Experimentelle Untersuchungen für das Studium der Ermüdungseigenschaften von Betonbauteilen unter reiner Torsionsbeanspruchung wurden durchgeführt, im ganzen 148 Versuche an unbewehrten, bewehrten, vorgespannten und stahlfaserbewehrten Balken. Aus den Ergebnissen wurden mit Hilfe statistischer Auswerteverfahren S-N-Kurven erstellt. Für vorgespannten sowie unbewehrten Beton betrug die Ermüdungsfestigkeit bei 10 Mio Lastwechseln 53 bis 56 % der statischen Bruchfestigkeit, während sie für Stahl- und Stahlfaserbeton bei 38 bis 53 % lag.



## 1. INTRODUCTION

The first research work on fatigue of plain concrete members was conducted by Van Ornum[1] in 1903, and since then many research works concerning fatigue properties of a wide range of plain concrete, reinforced concrete, reinforcing steel, prestressing tendon and prestressed concrete have been carried out. In recent years, design procedures of concrete structures have been transformed into ultimate strength design or limit state design, resulting in giving more concerns about fatigue properties of concrete members.

Though the case rarely occurs in which repeated torsion should be considered in designing concrete structures, it is a matter of interest for plain concrete as one of the basic properties of concrete under repeated diagonal tension due to torsional shear stress, and for prestressed concrete as fatigue problem under combined shear and prestress, i.e. under combined tensile-compressive state of stress. On the other hand, there have been many reports on deterioration due to cracking of reinforced concrete slab decks of highway bridges in Japan, and the cracks and their development are pointed out to be caused by repetition of moving wheel loads characterised in highway traffic[2]. This type of progressive failure of concrete slab may be considered due to fatigue under repeated torsion and shear.

Main objective of this study is to obtain basic data on fatigue properties of concrete under pure torsion. Torsional fatigue tests were carried out on plain, prestressed and reinforced concrete small specimens.

## 2. OUTLINE OF EXPERIMENT

The test program consisted of two series; series A on plain and prestressed concrete, and series B on plain, reinforced and steel fibre concrete. The mix proportions of concrete are listed in Table 1. Normal Portland cement, Yasu river sand for fine aggregate and Takatsuki crushed gravel for coarse aggregate were used in these mixes.

Details of test specimens are shown in Fig.1. A square cross section of 15x15cm were used for all beam specimens in this experiment. In the plain and prestressed beam specimens a small amount of longitudinal reinforcement consisting of four corner bars of 6mm in diameter was provided to prevent a sudden and violent failure. Three kind of steel ratio of 1.0, 1.5 and 2.0 percent in volume were used in reinforced concrete beam specimens of series B. In the reinforced concrete beam specimens, closed stirrups of 12x12cm of deformed bar of 6mm in diameter (D6) were arranged within a torsional span, and the arrangement of stirrups and details of longitudinal reinforcement are listed in Table 2 and also shown in Fig.1.

Table 1 mix proportions of concrete

series	max.size of aggregate (mm)	slump (cm)	W/C (%)	S/a (%)	unit content (kg/m <sup>3</sup> )					W.R.A.*
					W	C	S	G	fibre	
A normal	20	10.0	41	41	196	478	671	1003	--	--
B normal	20	7.5	50	43	170	340	754	1038	--	used
B fibre	15	7.5	50	60	210	420	943	653	79	used

\*W.R.A.: water reducing agent

In all beam specimens, some additional stirrups were provided near the both ends of the specimen to prevent failure at the support of a torsion arm(see Fig.1). Strengths of steel bars used are listed in Table 3. Steel fibre concrete was

used in beams with steel ratio of 1.5 percent. All specimens were stripped one day after casting of concrete and then stored in a curing room of about 20°C and relative humidity of 90±5 percent during about four months in series A and during about one month in series B. And the age when fatigue test was performed was 122 to 181 days for plain concrete in series A, 144 to 151 days for pre-stressed concrete, 112 to 146 days for plain concrete in B series, 112 to 137 days for reinforced concrete and 152 to 157 days for steel fibre concrete.

In the prestressed concrete specimens, effective prestress of 6.91 MPa was introduced one or two weeks before starting fatigue test and re-introduced immediately before testing.

### 3. TESTING PROCEDURE

Schematic view of the fatigue test arrangement is shown in Fig.2. All the beam specimens were loaded with a hydraulic pulsator. Before fatigue test, static tests were carried out. The maximum loads in the cycle were selected to 87 - 65 percent of the ultimate load for most of beams, but to 85 - 59 percent for plain concrete beams of series A, and to 75 - 55 percent for steel fibre concrete beams of series B. The minimum load in the cycle was kept at 9.81kN which gave the torque of 490Nm.

Table 2 details of reinforcement

P	P <sub>1</sub> (%)	P <sub>v</sub> (%)	s(cm)
1.0	1.12(8D6)	1.13	6.0
1.5	1.83(4D10, 4D6)	1.50	4.5
2.0	2.54(8D10)	2.25	3.0

P<sub>1</sub>, P<sub>v</sub>: longitudinal and lateral steel ratio respectively

s: spacing of stirrup

Table 3 strength of reinforcing bar

	yield strength	ultimate strength
φ6	--	488
D6	347	534
D10	373	548

unit: MPa

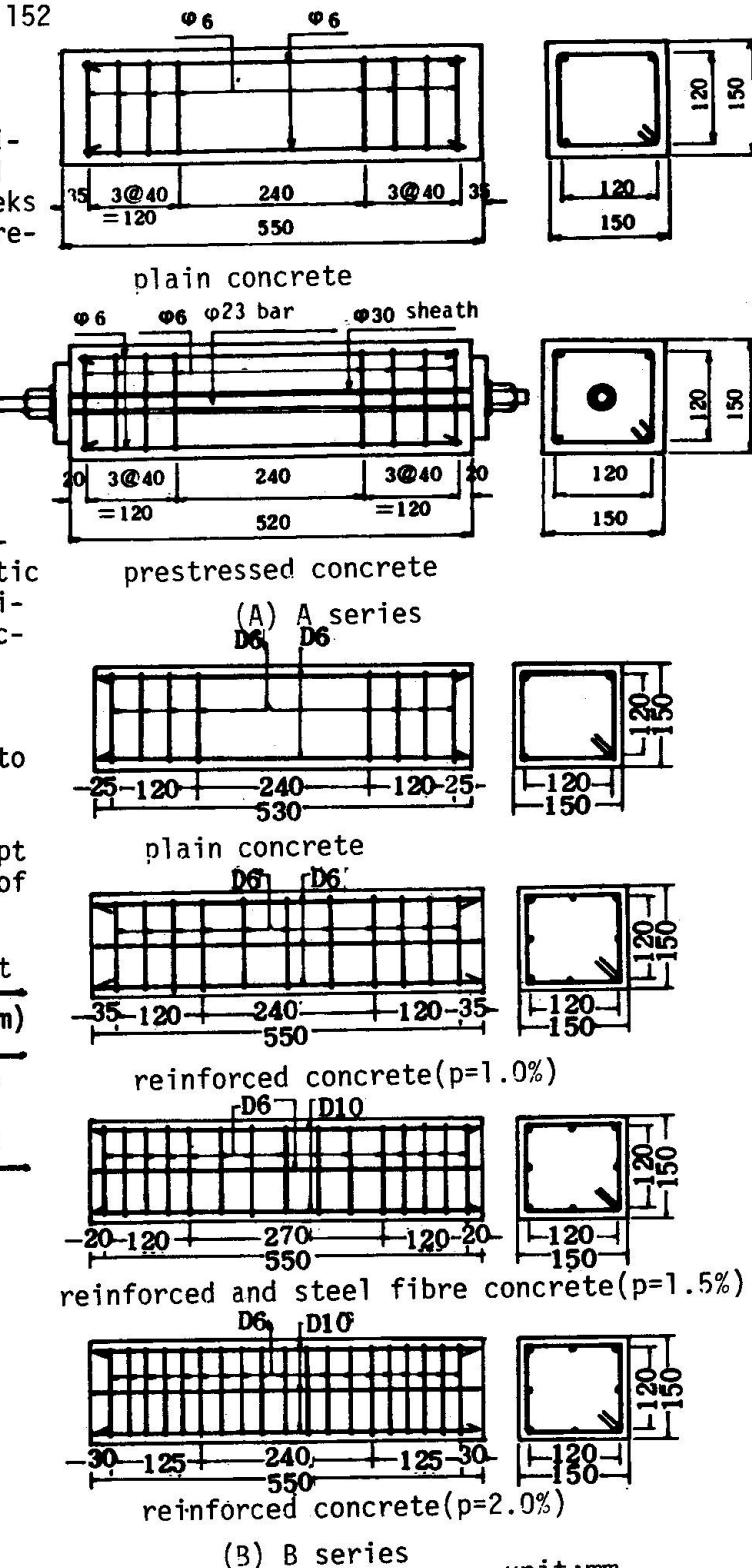


Fig.1 test specimen

The cyclic load was of a constant amplitude sinusoidal wave and its frequency was 7.5Hz for plain and pre-stressed concrete beams and 5.0Hz for reinforced and steel fibre concrete beams. The maximum and minimum loads were controlled and verified periodically by observing the output on a syncroscope of the load cell attached to the test arrangement (see Fig.2).

#### 4. TEST RESULTS AND DISCUSSIONS

##### 4.1 Static Strengths

Strength and elastic moduli of concrete are summarized in Table 4. Results of the static torsional tests are listed in Table 5. In series A the difference in tensile strength between two ages, by which the ultimate torque of the plain concrete was directly affected, was small,

but the ultimate torque at the age of 180 days increased by more than ten percent compared with that at the age of 121 days. Such an increase in the ultimate torque may be considered due to difference in moisture contents rather than in ages at testing. The test at the age of 121

days was performed two days after the specimens

were taken out from curing room, while the test at the age of 180 days was carried out after the specimens were fully dried in the testing room, and hence the ultimate strength at the age of 121 days seemed to decrease by the effect of drying shrinkage. The maximum

load ratio in the fatigue test of plain concrete in series A, therefore, were determined on the basis of the value at the age of 180 days.

Table 6 shows the theoretical values of the static ultimate torque. The values obtained from Hsu's equation [3], non-linear analysis [4] and Equation (1) gave good estimations for the ultimate strength of plain concrete.

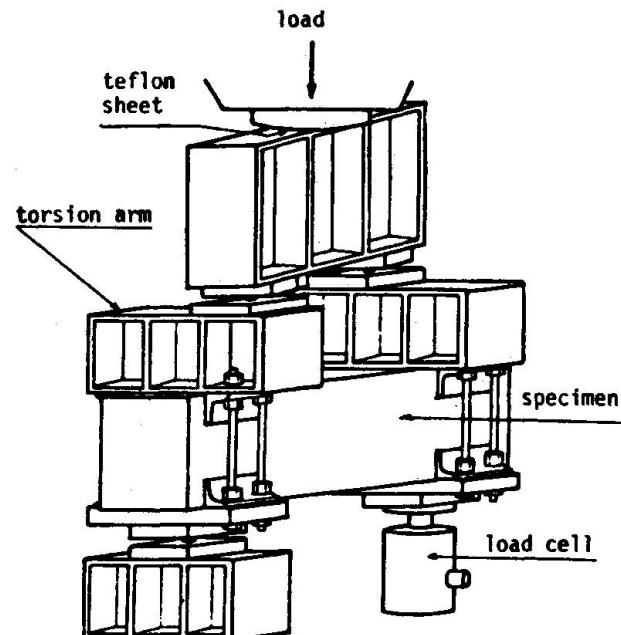


Fig.2 test arrangement

Table 4 strengths and elastic moduli of concrete

series		A series normal		B series normal fibre	
age	(day)	118	182	87	89
compressive strength $f_{cu}$	(MPa)	44.4	48.3	45.7	46.9
tensile strength $f_{tu}$	(MPa)	3.40	3.45	3.45	4.82
modulus of rupture $f_{bu}$	(MPa)	5.34	6.35	5.14	8.65
Young's modulus $E_c$	(GPa)	36.3	36.2	30.3	29.4
Poisson's ratio		0.214	0.204	0.188	0.195
shear modulus $G_c$	(GPa)	14.9	15.0	12.7	12.3

Table 5 torsional strength by static tests

series	ultimate torque	age (day)	series & p(%)	cracking torque	ultimate torque	age (day)
plain concrete			reinforced concrete			
A	3190	121	B, 1.0	3600	5720	89
A	3520	180	B, 1.5	3840	7000	90
B	2840	87	B, 2.0	4090	9340	91
prestressed concrete			steel fibre concrete			
A	6730	155	B, 1.5	4170	9910	92

unit:Nm

Table 6 shows the theoretical values of the static ultimate torque. The values obtained from Hsu's equation [3], non-linear analysis [4] and Equation (1) gave good estimations for the ultimate strength of plain concrete.

Table 6 theoretical strengths (unit: Nm)

	Hsu[3]	Kojima[4]	Eq(1)
plain (A series)	3430*	3610*	3390*
concrete (B series)	3370	3320	3050
prestressed concrete	5940	6255	5870
reinforced concrete	cracking Hsu[3]	ultimate Lampart[5]	Hsu[3]
p=1.0%	3670	4510	4420
p=1.5%	3820	6430	5850
p=2.0%	4020	9410	7540

\* : the values were calculated from the strength at 182days.

Equation (1) was derived as the approximate equation from the results of the non-linear analysis [4], that is

$$T_{up} = T_e + \frac{1}{4}(f_{bu}/f_{tu}-1)(5-f_{bu}/f_{tu})(T_p-T_e) \quad (1)$$

where  $T_e = 1/(3+1.8b/h) \cdot b^2h$  f<sub>tu</sub>, and  $T_p = 1/2 \cdot (1-b/3h)b^2h$  f<sub>tu</sub>. b and h are shorter and longer sides of rectangular section respectively. f<sub>tu</sub> and f<sub>bu</sub> are tensile and modulus of rupture of concrete respectively.

The theoretical values of the ultimate torque of prestressed concrete were obtained as those of plain concrete increased by the factor  $\sqrt{1+f_p/f_{tu}}$  due to pre-stress, where f<sub>p</sub> was effective pre-stress, and they were slightly less than the measurement.

The cracking strengths of reinforced concrete beams increased with steel ratio, and Hsu's equation [3] gave good estimations. As to the ultimate torque of reinforced concrete, the theoretical values by Lampart [5] were agreed with the test results in the range of high steel ratio, but the values by Hsu [3] were about 20 percent lower than the test results. The ultimate strength of steel fibre concrete beam increased by 42 percent compared with that of reinforced concrete beam having the same steel ratio of 1.5 percent.

#### 4.2 Fatigue Properties

In the fatigue tests five specimens for each maximum load ratio were tested for plain concrete in series A, three specimens for prestressed concrete, and six specimens for all beams in series B. Relations between fatigue life and maximum load ratio are shown in Fig.4. An average fatigue life for each maximum load ratio was determined by the statistical procedure as used in analysis of fatigue data of concrete under compression [6]. The relation between probability of survival obtained from fatigue test for the same maximum load and the cycles at fatigue failure indicated an almost linear line when it was plotted on a logarithmic normal probability paper as shown in Fig.3 which is an example of the reinforced concrete with steel ratio of 2.0 percent. And in this study the average fatigue life for each maximum load was determined as the cycles at fifty percent probability of survival given by the regression line obtained from the method of least square. These values are listed in Table 7. In this study S-N curve was assumed to be linear and was determined as a regression line by the method of least square using the value in Table 7. Equations of S-N lines and fatigue strength at two million and ten million cycles of loading obtained by the S-N lines are summarized in Table 8, and S-N lines are also shown in Fig.4.

Fatigue strength of plain concrete at ten million cycles was 56.2 percent in

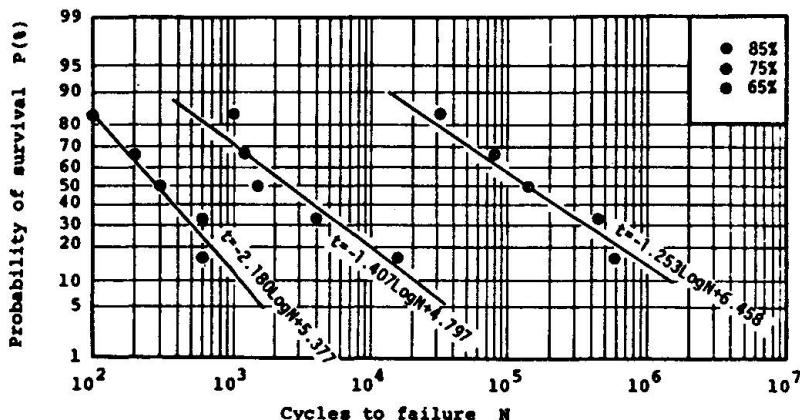


Fig.3 an example of relations between probability of survival and cycles to failure (reinforced concrete:p=2.0%)

Table 7 average fatigue life

	s	$\bar{n}$	s	$\bar{n}$	s	$\bar{n}$	s	$\bar{n}$
plain (A series)	85%	9200	77%	119100	68%	565700	59%	4627900
concrete (B series)	85	72700	75	- -	65	1944800		
prestressed concrete	85	900	75	34200	65	211000		
reinforced concrete	p=1.0%	85	676	75	881	65	25600	
	p=1.5%	85	1210	75	1210	65	364000	
	p=2.0%	85	293	75	2570	65	142800	
steel fibre concrete				75	2070	65	8180	55 407100

s:maximum load ratio,  $\bar{n}$ :average fatigue life(probability of survival:50%)

Table 8 S-N line and fatigue strength

	S-N line	fatigue strength at 2x10 <sup>6</sup> cycles	10 <sup>7</sup> cycles
plain (A series)	$S = -9.81 \log N + 125$	63.0%	56.2%
concrete (B series)	$S = -13.7 \log N + 153$	65.8	56.1
prestressed concrete	$S = -8.07 \log N + 110$	58.8	53.1
reinforced concrete	$S = -10.2 \log N + 110$	45.4	38.2
p=1.5%	$S = -7.78 \log N + 108$	58.5	53.1
p=2.0%	$S = -7.23 \log N + 102$	56.0	51.0
steel fibre concrete	$S = -8.10 \log N + 99.7$	48.6	43.0

series A and 56.1 percent in series B, and it was almost same in spite of difference in mix. That of prestressed concrete was 53.1 percent. It is said that fatigue strength of concrete at ten million cycles is about 55 percent in compression and flexure [7],[8]. It suggests that fatigue strength of concrete under torsional shear stress is not much different from that under compressive or tensile stress.

In reinforced concrete, the static strength under torsion is determined by yielding of reinforcing steel in the range of under-reinforcement. In this experiment, the beams of p=1.0 and 1.5percent were under reinforced and the beam of 2.0 percent was slightly over-reinforced and therefore the fatigue strength of under-reinforced beam under torsion may be affected by fatigue properties of steel bars. By visual observation of the failed specimens, however, steel bars which failed due to fatigue could not be found at all in the beams the maximum load ratio of which was relatively low. It is indicates that the mechanism of fatigue failure of reinforced concrete under torsion should be considered for two cases; one for failure at low cycles by higher maximum load ratio and one for failure at high cycles by lower maximum load ratio. In the former case failure mechanism is similar to that in static failure, but in the latter one the beam usually passes through from uncracked state and gradually to cracked state up to failure. In the

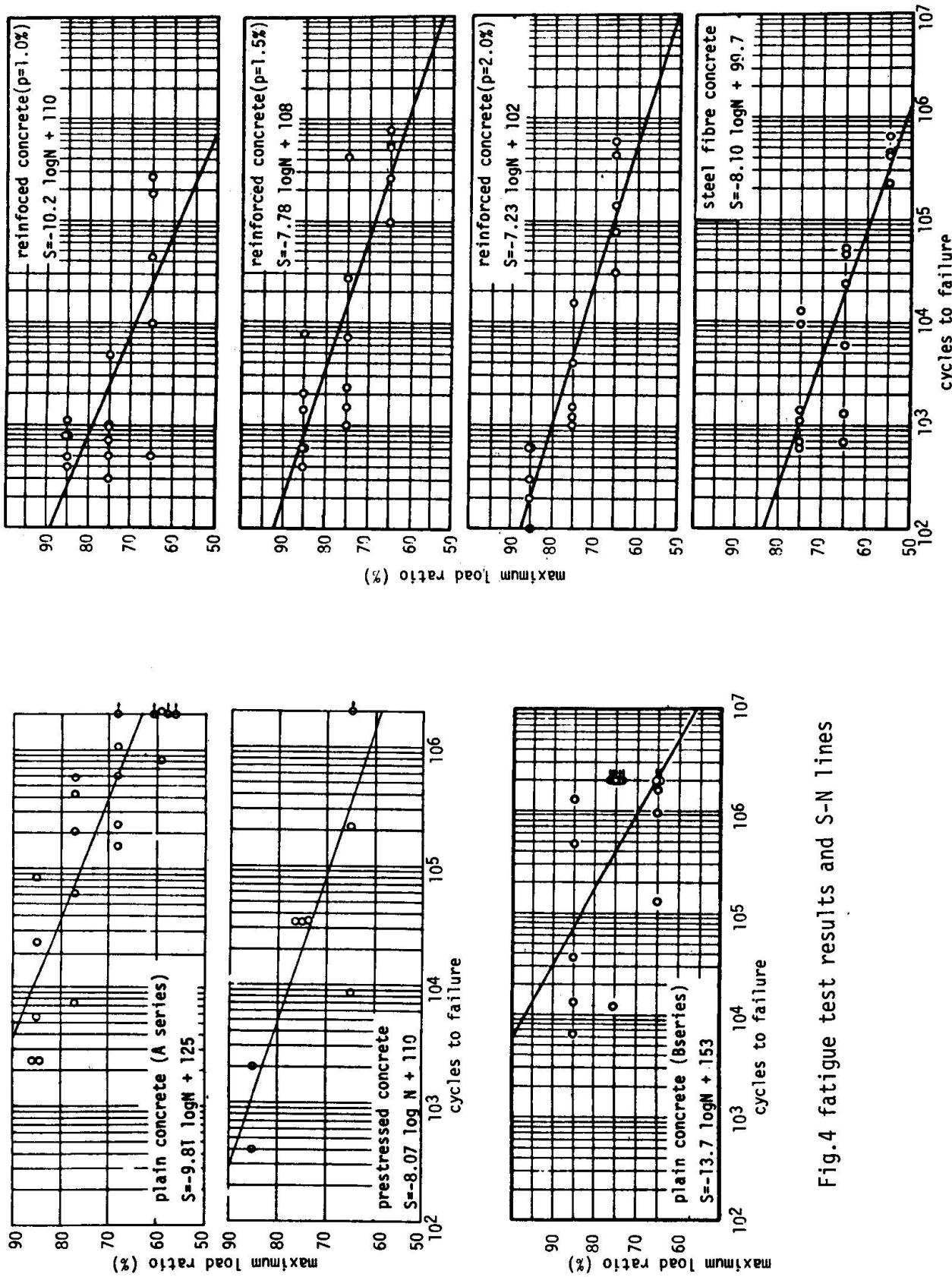


Fig.4 fatigue test results and S-N lines



uncracked state almost no stress occurs in reinforcing steel, while, once cracks occur, torsional shear stress is re-distributed gradually to the bars and torsional stiffness is reduced at the same time. In addition, in concrete near the reinforcing steel across the diagonal cracks, dowel force will act. Therefore failure of reinforced concrete might be caused by fatigue failure of concrete due to dowel force of the reinforcing steel.

Fatigue strength of the reinforced concrete beam of  $p=1.0$  percent was somewhat lower than that of the other beams, probably because that its smaller after-cracking-torsional stiffness accelerated failure of concrete due to dowel force.

Fatigue strength of steel fibre concrete was ten percent lower at ten million cycles compared with that of reinforced concrete with the same reinforcement ratio. The effect of steel fibre concrete on fatigue resistance was smaller than that on the static strength, but when the maximum load ratio is calculated on the basis of the static strength of reinforced concrete beam ( $p=1.5$  percent), its fatigue strength becomes 60.9 percent at ten million cycles, and as the effect of fibre concrete on fatigue resistance may be noticeable.

## 5. CONCLUSIONS

From the experimental investigation of fatigue of concrete members under pure torsion, the following conclusions may be drawn.

- (1) Fatigue properties of plain and prestressed concrete under torsion were almost same as those of concrete under compression or flexure, and fatigue strength was 53 to 56 percent of the static strength at ten million cycles of loading.
- (2) Fatigue failure of reinforced concrete did not occur due to failure of reinforcing steel but did due to failure of concrete near diagonal crack, though the static strength was determined by yielding of the bars.
- (3) Fatigue strength of reinforced concrete decreased when steel ratio was low. The effect of torsional stiffness after cracking may be responsible for this fact, but additional research is still needed.
- (4) Fatigue strength of steel fibre concrete was lower than that of reinforced concrete, but when compared with the basis of the static strength of reinforced concrete with the same reinforcement, the effect of fibre concrete was noticeable.

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