Fatigue Behaviour of Reinforced Concrete Beams under Shear Force

Comportement à la fatigue de poutres en béton armé sollicitées par un effort tranchant

Ermüdungsverhalten von Stahlbetonbalken unter Ermüdungsbeanspruchung

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SUMMARY
This report summarises a series of investigations on the shear behaviour of reinforced concrete beams under static and repeated loading. Procedures are given for calculating the average strain of stirrups in a beam under varied loading with variable maximum and minimum shear levels. The assumptions on which these procedures are based and the fatigue life of beams, with or without stirrups, are also presented.

RESUME
Ce rapport énumère une série de recherches sur le comportement à l'effort tranchant des poutres en béton armé soumises à des charges statiques et répétées. On donne des procédés pour calculer la déformation relative moyenne des étriers dans une poutre soumise à une charge variable oscillant entre les niveaux d'effort tranchant maximum et minimum. Les hypothèses sur lesquelles sont basées ces procédés et la durée de vie des poutres, avec ou sans étriers, sont également présentées.

ZUSAMMENFASSUNG
Der Beitrag beschreibt zusammenfassend eine Reihe Untersuchungen über das Schubverhalten von Stahlbetonbalken unter statischer und wiederholter Belastung. Das Vorgehen für die Berechnung der mittleren Bügeldehnungen in Stahlbetonträgern unter variabler maximaler und minimaler Schubbeanspruchung wird beschrieben. Die Annahmen, auf welche sich die Berechnungsmethoden abstützen, sowie die Lebensdauer von Schubträgern mit und ohne Bügelbewehrung werden besprochen.

1. INTRODUCTION

Recent shear design tends to request less web reinforcement than in the previous codes. This tendency demands the further study on fatigue. Concerning the fatigue behavior of beams with stirrups it was pointed out that (1) stirrup strains increased during fatigue loading [1][2][3], (2) fatigue fracture of stirrup was found [2][4][5] and (3) fatigue strength of stirrup was smaller than that of bar itself [2][5].

However no report, which deals with the phenomena systematically and rationally, is available. In order to get the fundamental knowledge of fatigue behavior, a series of research works have been carried out. This paper presents the summary of these investigations and the details are found in the oridinal papers [6][7][8].

2. BEHAVIOR OF BEAMS WITH STIRRUPS UNDER FATIGUE LOADING

The general shear behavior of beams with stirrups under fatigue loading can be summarized as follows [6][7].

(1) Under the effect of repeated loading the inclined cracks increase gradually in width and extend upward or downward, but new diagonal cracks are seldom formed. With increase of the inclined cracks in width and in length, strains of stirrups intersected by the cracks increase. The rate of increase is not always the same among the stirrups, but the total strain or the average strain in a beam continues to increase at an almost constant rate against the logarithms of loading cycles.

(2) When the applied maximum shear level is lower than the shear capacity of concrete, average strain of stirrups hardly increases during the early stage of loading, and it begins to increase noticeably after some cycles of loading when two or three inclined cracks appear.

(3) Due to repeated loading fracture of stirrup generally occurs. The first fractured stirrup is usually the one that has developed the highest strain in the beam. When fracture of one leg occurs, the strain of another leg or the leg of the adjacent stirrup increases exceedingly, and the next fracture usually occurs in either of the legs after some thousand cycles. The beam will fail if it becomes unable to sustain the applied maximum shear force with the remaining legs.

(4) Fracturing of longitudinal bar is sometimes found. The fracture usually occurs just beside the first fractured stirrup.

3. FATIGUE STRENGTH OF BEAMS WITHOUT WEB REINFORCEMENT

Knowledge of fatigue strength of beams without web reinforcement is significant for investigating how stirrup strains increase under fatigue loading and when stirrup strains begin to increase in the cace that the applied maximum shear level is lower than the shear capacity of concrete.

The experimental researches on fatigue strength of beams without web reinforcement had made clear the followings:

(1) Fatigue strength at 10^6 cycles is about 60% of the static strength [9][10][13][14].

(2) S-N curve of a beam with larger span-depth ratio is different from that with smaller span-depth ratio [14].
(3) Fatigue fracture of longitudinal bar tends to occur at the crossing point of diagonal crack when the span-depth ratio is small [13][14].
(4) The beam, which should fail in flexure under static loading, sometimes fails in shear under fatigue loading [10].

However, the influence of loading range on the fatigue strength and the size effect had not yet been cleared. There had not been any S-N curves which include these factors.

Therefore, fatigue tests of beams were carried out under the condition in which the loading range and effective depth were changed. As a result, the influence of the loading range was clarified and an equation of the fatigue strength of beam was introduced for larger span-depth ratio [8].

$$\log(V_{max}/V_u) = -0.036 \ (1-r|r|) \log N_f \tag{1}$$

Vu   calculared static shear strength of beam without web reinforcement [15]
Vmax   applied maximum shear level
Vmin   applied minimum shear level
r   Vmin/Vmax
Nf   failure life of a beam

The calculated values of Eq.(1) fit well the authors' and the previous tested values [9]-[14] as shown in Fig.1.
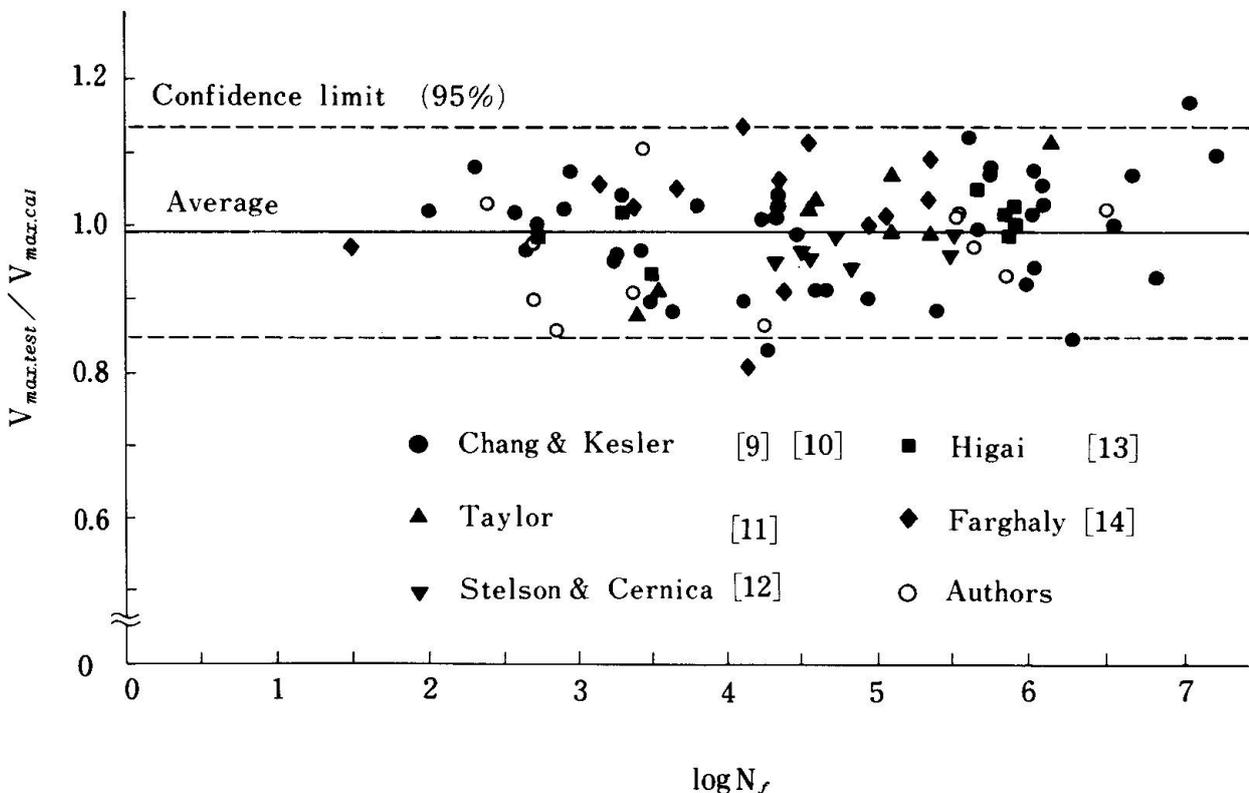

Fig.1 Ratio of Tested Value of (Vmax/Vu) to Calculated One by Eq.(1)

4. STRAIN OF STIRRUP UNDER REPEATED LOADING WITH CONSTANT MAXIMUM AND MINIMUM SHEAR LEVELS

4.1 Strain of Stirrup at the Maximum Shear Level

Although behavior of each stirrup under fatigue loading is fairly complicated, average strains apparently increase approximately in proportion to the logarithm of loading cycles as shown in Fig.2. And the rate of increase is not dependent on the strain at the first cycle, but is almost the same in spite of differences in the applied maximum shear level. This seems strange but can be explained by the following idea.

Since it may be reasonably considered from Reference [16] that the total shear force is carried by Vs and Vc, where Vs is shear force carried by assumed truss with 45 degree's diagonals and Vc is shear force carried by other means than the assumed truss, Eq.(2) is obtained under the maximum shear level in the fatigue test. The shear force Vc can be obtained by extending the shear − stirrup stress line toward the shear axis. Under fatigue loading the value of Vc is not considered as constant but should be decreased with the increase in number of loading cycles. When the decrease of Vc is assumed to be the same as that of
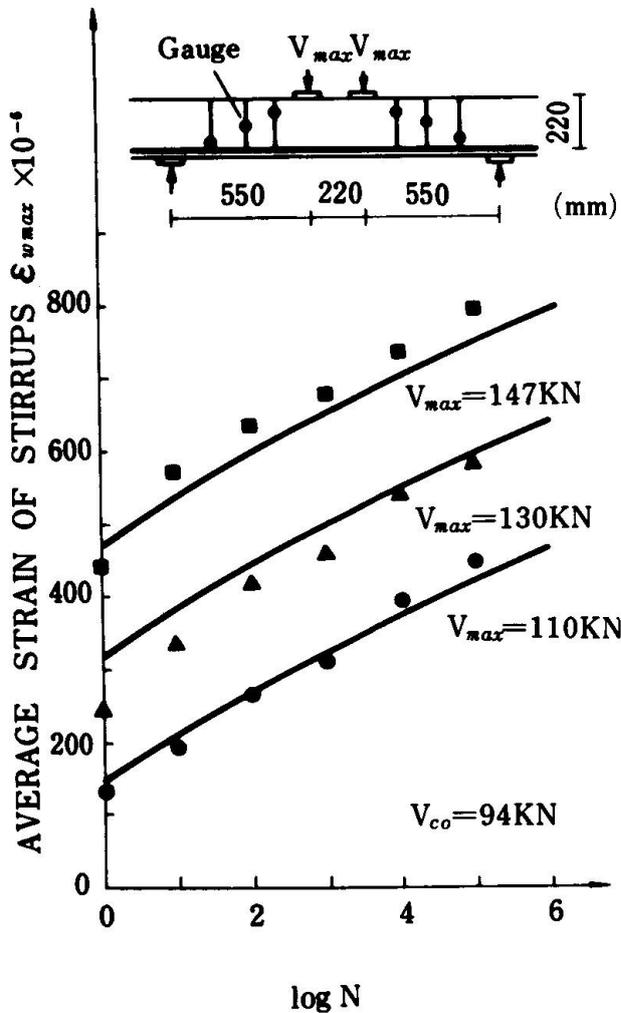

Fig.2 Average Strain of Stirrups
(Vmax>Vco)
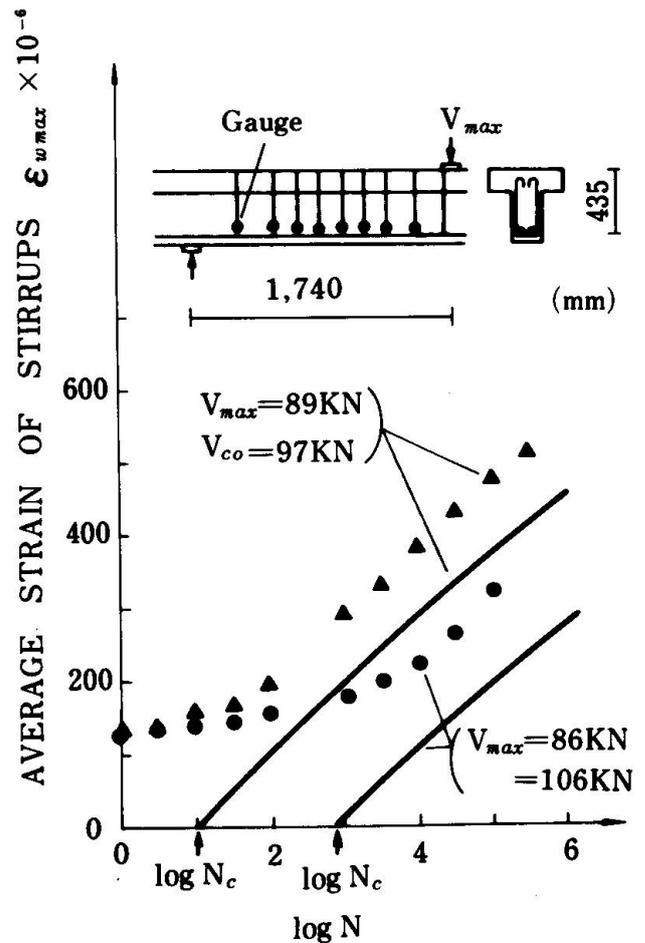
Fig.3 Average Strain of Stirrups
(Vmax<Vco)

fatigue strength of beam without web reinforcement, Eq.(3) is obtained. When the maximum shear level Vmax is constant, the value of Vs should increase with decrease in the value of Vc as indicated in Eq.(2). Eq.(2) is, however, for the part of a beam where the influence of the support or loadng point is negligibly small. For the part where this influence does exist, the shear force Vs is lightened by multiplying coefficient $\beta x (<1)$ [16]. Finally, the strain of the stirrup at the maximum shear level is expressed by Eq.(4). The values calculated by Eq.(4) are also shown in Fig.2.

$$Vs = Vmax - Vc \qquad\qquad (2)$$

$$Vc = Vco\ 10^{-0.036(1-r|r|)logN} \qquad\qquad (3)$$

$$\varepsilon wmax = \beta x \{Vmax-Vco\ 10^{-0.036(1-r|r|)logN}\}/(Aw\ Ew\ z/s) \qquad (4)$$

Vco   Vc at the initial loading
N   cycles of loading
εwmax   strain in stirrup at the applied maximum shear level
βx   coefficient covering the influence of support or loading point [16]
Aw   cross sectional area of stirrups within distance s
Ew   Young's modulus of stirrup
z   arm length of truss
s   spacing of stirrups

When the applied maximum shear level is lower than the shear capacity of concrete, the increase in strains of stirrups is very small until a certain number of loading cycles. The idea on which Eq.(4) is based can be extended to estimate not only the number of cycles when stirrup strains begin to increase but also the increase of the strains thereafter.

$$logNc = - log(Vmax/Vco)\ /\ \{0.036(1-r|r|)\} \qquad\qquad (5)$$

Nc   loading cycles when average strain of stirrups begins to increase

After the loading cycles exceed the value of Nc, the stirrup strain at the applied maximum shear level can be calculated by Eq.(4), substituting for N the total loading cycles from the start of fatigue loading. The calculated values compared with experimental ones are shown in Fig.3.

4.2 Strain of Stirrup at the Applied Minimum Shear Level or Strain Range

Fig.4 shows the relationship between average strain of stirrups and applied shear force. The relation is considered as a straight line, and the inclination becomes larger with increase in number of cycles of fatigue loading, so that the strain range becomes larger. And a significant increase of the residual strain is observed.

The observed relationship can be explained if it is assumed that the curve is on the line between the point representing the strain at the applied maximum shear level and the specific point (-Vco, 0) as shown in Fig.4. Then, the strain range is calculated by Eq.(6).

$$\varepsilon wr = \{(Vmax-Vmin)/(Vmax+Vco)\}\ \varepsilon wmax \qquad\qquad (6)$$

εwr   strain range in stirrup

ε_w, $\times 10^{-6}$

—— Calculated lines

—●— Tested values

at 10^4 cyc.

1,000

800

600

at 1 cyc.

200

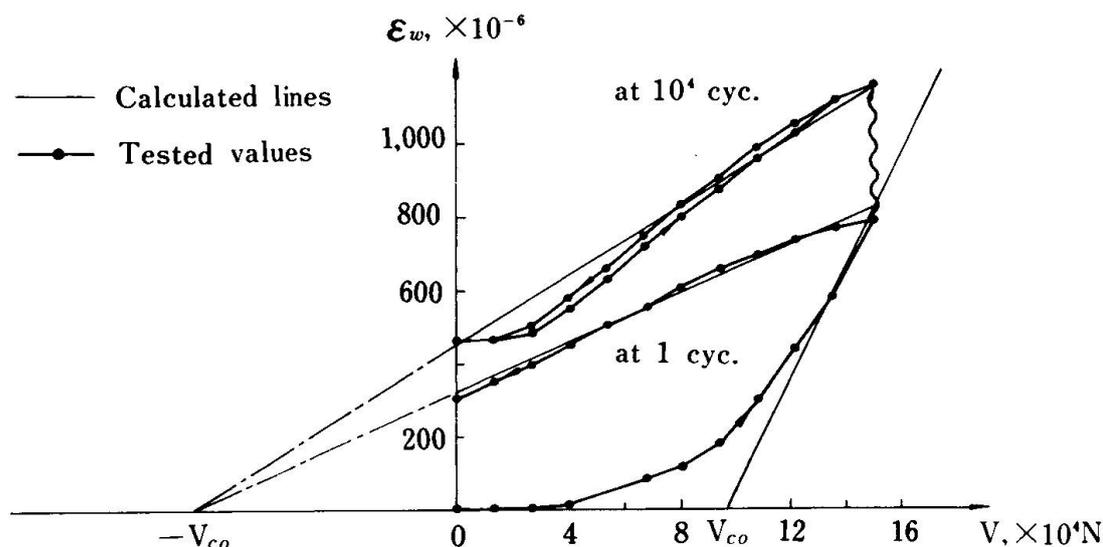$-V_{co}$ 0   4   8 V_{co} 12   16   V, $\times 10^4$N

Fig.4 Observed and Assumed Relationships between Average
Strain of Stirrups and Applied Shear Force

5. STRAIN OF STIRRUP UNDER FATIGUE LOADING WITH VARIED LOADING RANGE

Since actual structures are not subjected to fatigue loading with constant load range but generally subjected to varied loadings, it is necessary to know the behavior in this case.

General fatigue loading can be devided to some sets of repeated loading with constant maximum and minimum shear levels, and each set is named the first repeated loading, the second and so on according to the sequence of loading. The stirrup strain after subjected to the first repeated loading can be calculated as in Section 4. The strain during the second repeated loading can be calculated if the loading history of the first loading is expressed as equivalent cycles of loading whose maximum and minimum shear levels are equal to those of the second one. All the changes of stirrup strain under loading with varied loading range thus can be calculated by the same way.

In order to calculate the equivalent cycles, the following assumption is made. If stirrup strains produced by the shear force applied are same in beams subjected to different loading history, the changes of the strains during subsequent loading are essentially same in spite of the difference of the previous loading history. In other words, the behavior of a stirrup after subjected to a certain loading, static or fatigue or sustained loading, is only dependent on the strain corresponding to the shear force applied. Consequently any loading history can be substituted by an equivalent fatigue loading with the constant maximum and minimum shear levels as shown in Fig.5, and the stirrup strain can be calculated.

After a stirrup strain is subjected to N_2 cycles of the second repeated loading, stirrup strain at V_{max2} can be calculatd by using Eq.(4), substituting $N_{eq}+N_2$ for N (See Fig.5). Therefore the strains hardly increase if the figures of N_2 are smaller than that of N_{eq} (See Fig.6(a)). When the applied maximum shear level of the second repeated loading is above the point A in Fig.5, there is no

influence of the previous fatigue loading and hence Eq.(4) can be used without any modification for calculating the strain of stirrup (See Fig.6(b)).
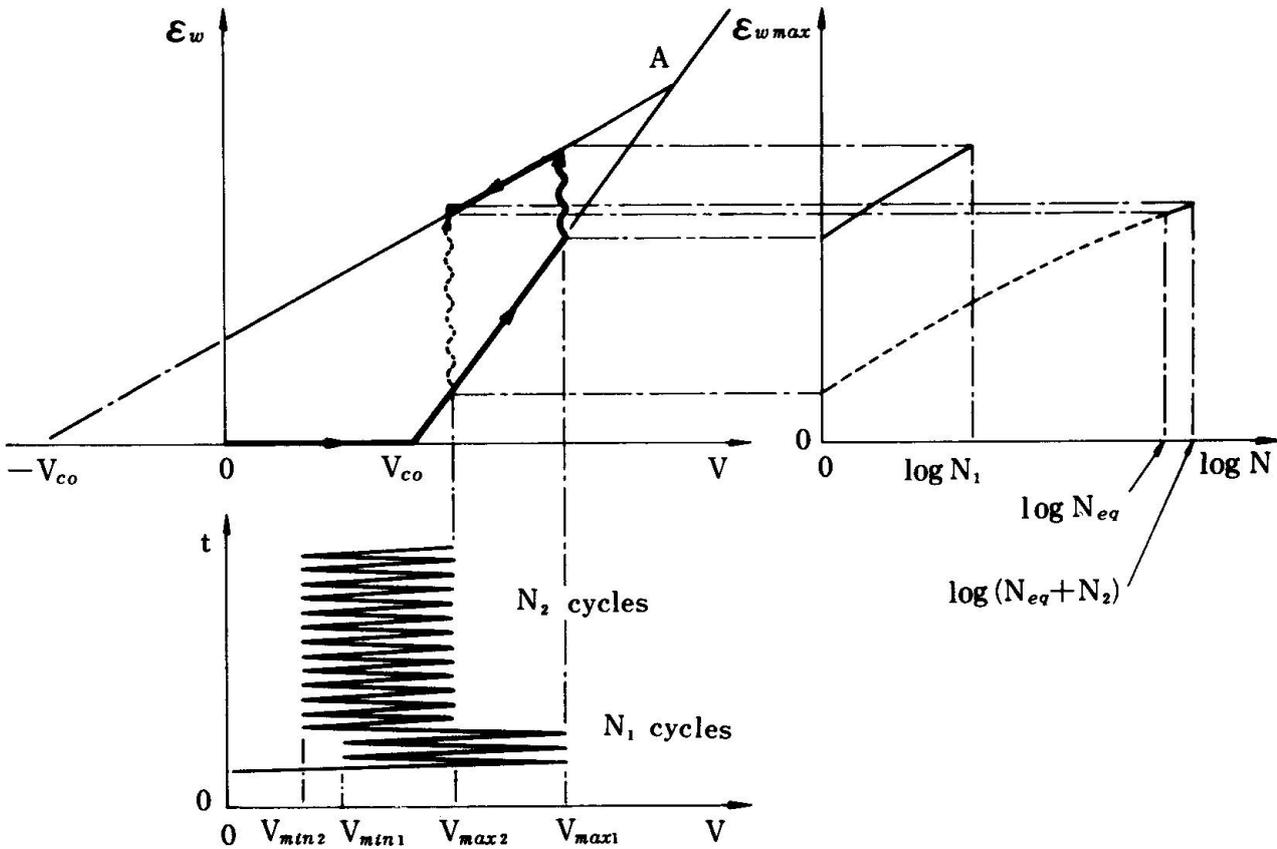

Fig.5 Assumption of an Equivalent Fatigue Loading (Neq is equivalent loading cycles with constant maximum and minimum shear levels.)

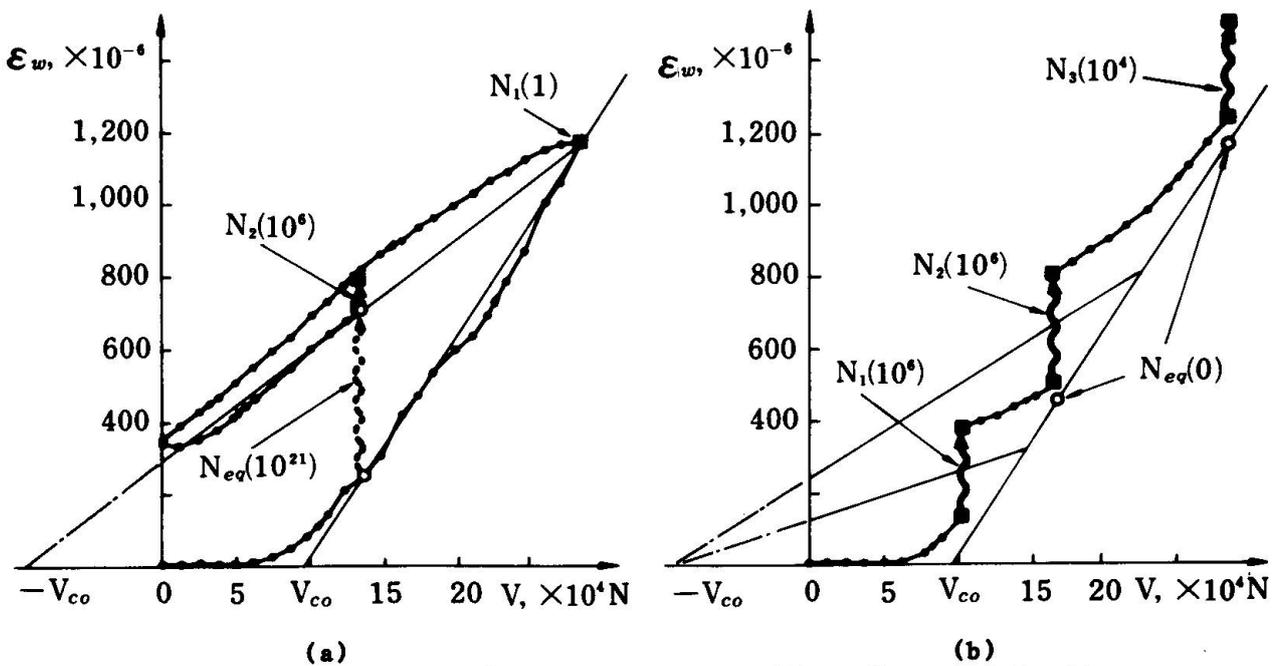(a)   (b)

Fig.6 Average Strain of Stirrups after the First Repeated Loading

6. DESIGN CONSIDERATIONS

The fatigue fracture of stirrup occurrs not only at lower bent portion where stirrup is bent around longitudinal bars but also at middle straight portion and upper hook portion. The portion of fracture is generally along the main diagonal crack making the beam to fail. The fatigue strength of stirrup is not cleared at present but supposed to lie between the fatigue strength of the straight bar and that of the bent bar, which is about 50% of the fatigue strength of the straight bar [2][14][17]. It is suggested that the fatigue life of a beam failing in shear due to stirrup fracture has the mutual correlation with the fatigue strength of stirrup [7].

ACKNOWLEDGEMENT

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