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**Fatigue of Plain Concrete in Uniaxial Compression**

Fatigue du béton non armé en compression uniaxiale

Ermüdungsverhalten von unbewehrtem Beton unter zentrischer Druckschwellbelastung

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**SUMMARY**

The paper describes uniaxial compression tests on plain concrete subjected to either constant amplitude or random loading.

**RESUME**

L'article décrit des essais de compression uniaxiale sur du béton non armé soumis à des charges d'amplitude constante et aléatoires.

**ZUSAMMENFASSUNG**

Der Beitrag behandelt einaxiale Druckschwellversuche an unbewehrten Betonkörpern mit zufallsbedingter sowie konstanter Amplitude.



## 1. INTRODUCTION

The load variations to which concrete structures are subjected often display a random behaviour, which means that they are stochastic or unpredictable in magnitude and erratic in relation to time. Loads due to wind, wave motion, currents, earthquakes and traffic are of this kind. Random stress variations may in course of time cause fatigue failure of the concrete of a structure. However, most of the research activities that have been carried out in the past to study the fatigue behaviour of concrete, have been restricted to determining the influence of constant amplitude loading by means of the well-known Wöhler test (see fig. 1).

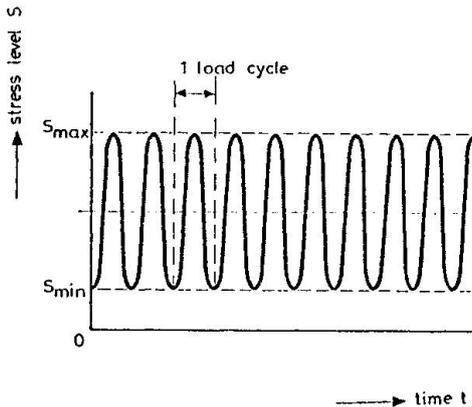


Fig. 1 Wöhler test

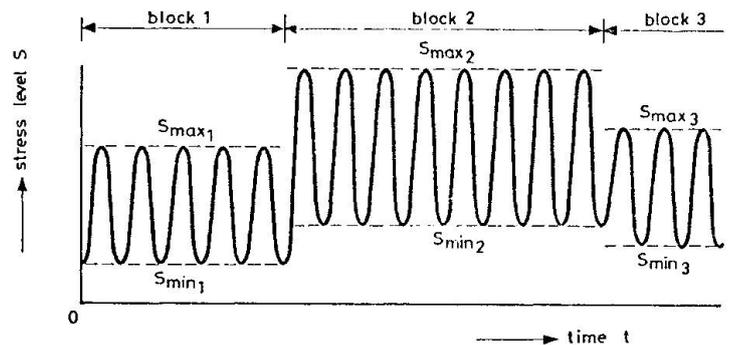


Fig. 2 Program loading test

The Wöhler test can be adopted as a method for comparing the influence of different factors on the fatigue behaviour. The test in itself is not sufficient to serve as a basis for predicting the lifetime of a structure that is subjected to random loads. To overcome this lack, tests have been done with non-constant amplitude loadings. In first instance these were program-loading tests (see fig. 2) with just two stress blocks [1, 2]. In the last few years a number of tests have been carried out on concrete with program loadings with more than two blocks [3, 4, 5, 6]. Recently tests have been carried out with variable amplitude loadings (see fig. 3) [6, 7]. In these tests the amplitude of the stress cycles changes in each cycle or half cycle. The changes can be made in respect to the mean stress (see fig. 3a.), the minimum stress (see fig. 3b) or the maximum stress (see fig. 3c).

To extend the present knowledge about the effects of non-constant amplitude loadings on concrete, experiments were set up. The specimens were loaded in uniaxial compression with various types of random loads.

## 2. RANDOM LOADINGS

Random vibrational loadings have to be defined in a statistical sense. In this respect two statistical functions are very important, i.e. the probability density function and the energy spectral density function. The probability density function  $f_R(\rho)$  gives the probability that the random loading  $R(t)$  has a certain value at time  $t$  (see fig. 4).

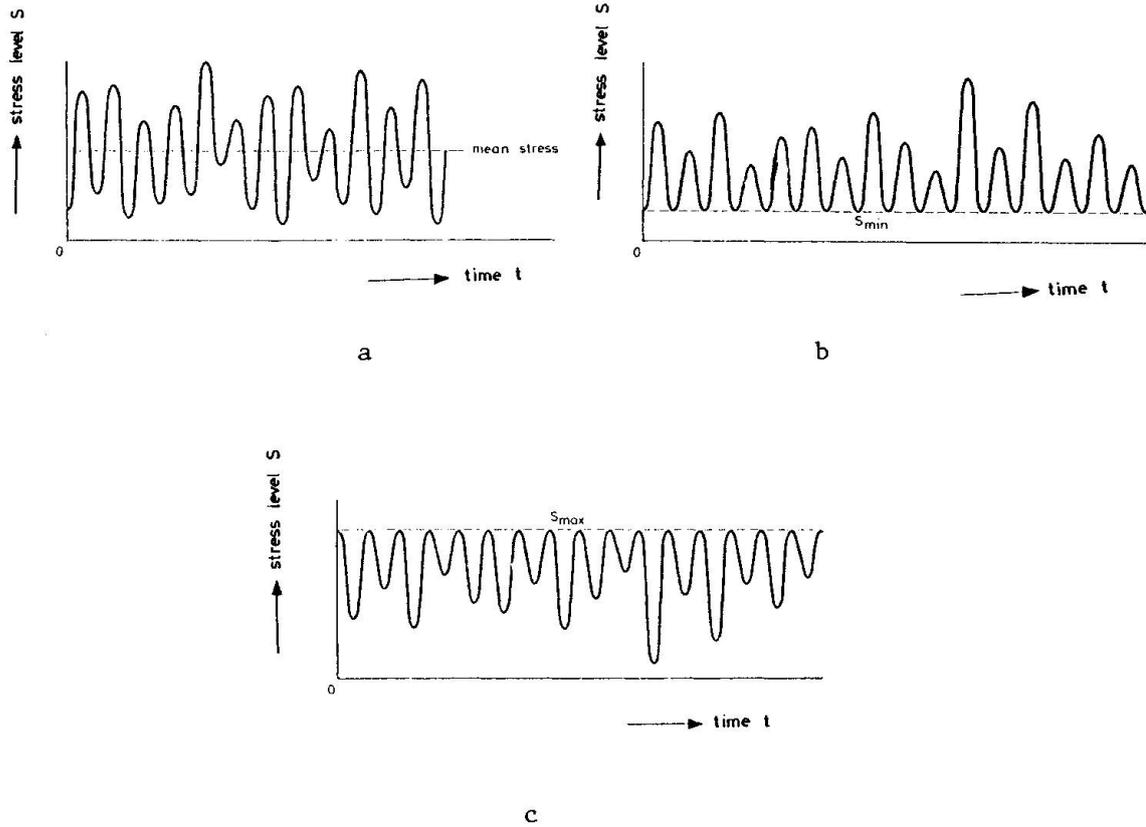


Fig. 3 Variable amplitude loading test

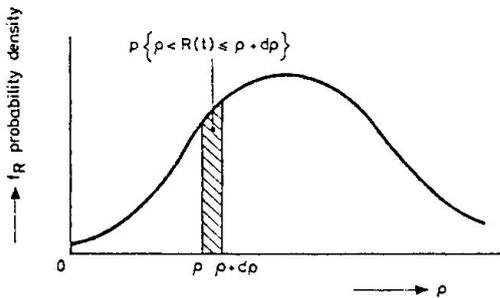


Fig. 4 Probability density function

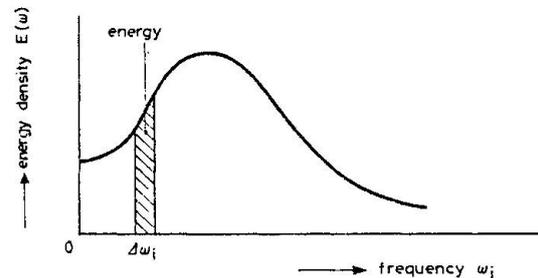


Fig. 5 Energy spectral density function

From the definition of the probability density function it follows that the probability  $P$  that  $R(t)$  will fall within the range  $[\rho, \rho + d\rho]$  is equal to the value  $f_R(\rho)$  multiplied with the width of that range. As a formula:

$$P\{\rho < R(t) \leq \rho + d\rho\} = f_R(\rho) \cdot d\rho \quad \dots(1)$$

The energy spectral density function  $E(\omega)$  gives the amount of power or energy that is involved with a certain random vibration (see fig. 5) and roughly how the energy is distributed over the various frequencies. The energy in a frequency range with a width  $\Delta(\omega)$  equals  $E(\omega) \cdot \Delta\omega$ .

The energy spectral density function is often used for a quick characterisation of the type of random loading. In figure 6 some examples are given.

As loadings on structures come from different sources like wind, waves, currents, earthquakes and traffic, it is obvious that various types of probability density functions and energy spectral density functions are



needed to describe loadings.

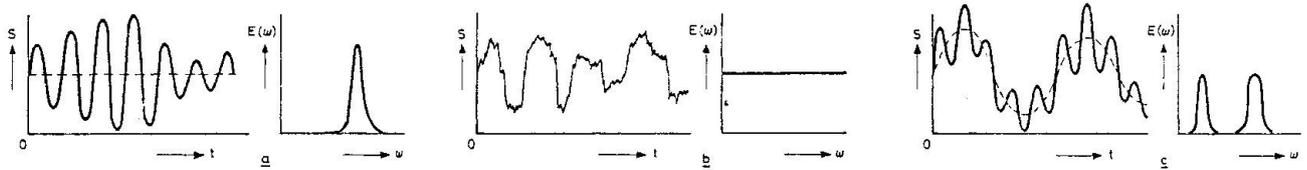


Fig. 6 Various types of random stresses. Single peak or small band (a), broad band (b), double peak (c).

Due to the structural response, given by the transfer function  $H(\omega)$ , stresses  $S(\omega)$  can have a totally different character compared to the original loading  $R(\omega)$  :

$$S(\omega) = H(\omega) \cdot R(\omega)$$

In figure 7 an example is given. This example is taken from [8]. The structure is schematized to a single degree of freedom system. The loading is broad banded. The response is however single peaked. In practice the foregoing means, that random stresses that can lead to fatigue failure, have to be described by a great variety of probability and energy spectral density functions.

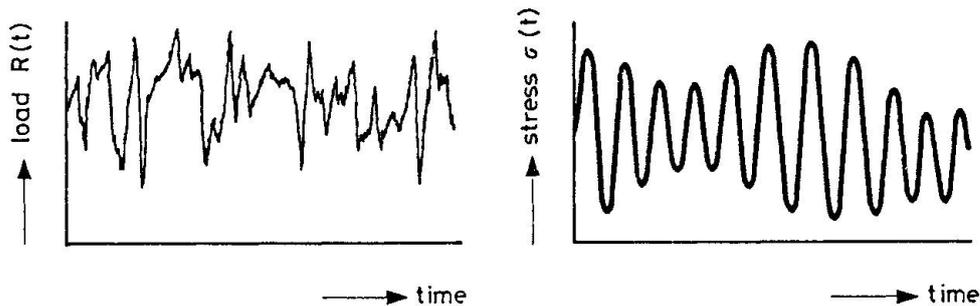


Fig. 7 Example of structural response

### 3. EXPERIMENTS

#### 3.1 Specimens and testing conditions

Constant amplitude tests and random loading tests were carried out on cylinders with a diameter of 150 mm and a height of 450 mm. The cylinders were made of a high-strength concrete with a mean cube strength at 28 days of about  $45 \text{ N/mm}^2$  (coefficient of variation 5%). The composition of the concrete is indicated in tabel 1.

Table 1 Composition of the high-strenght concrete.

ordinary portland cement	360 kg
water	162 kg
water cement ratio	0.45
gravel (maximum particle 32 mm)	1860 kg
fineness modulus	5.40

The testing specimens were cured and tested in fresh water. The testing took place in the fifth week after the casting of the specimens. The plain concrete cylinders were tested in uniaxial compression.

The tests were controlled by a servo-hydraulic system. The constant amplitude loading were generated by means of a standard MTS digital sine wave generator. The random loadings were generated by means of a micro-processor controlled random generator that was developed by TNO-IBBC.

### 3.2 Digital random generator

The random generator [9] consists of two parts that cooperate of line (see fig. 8).

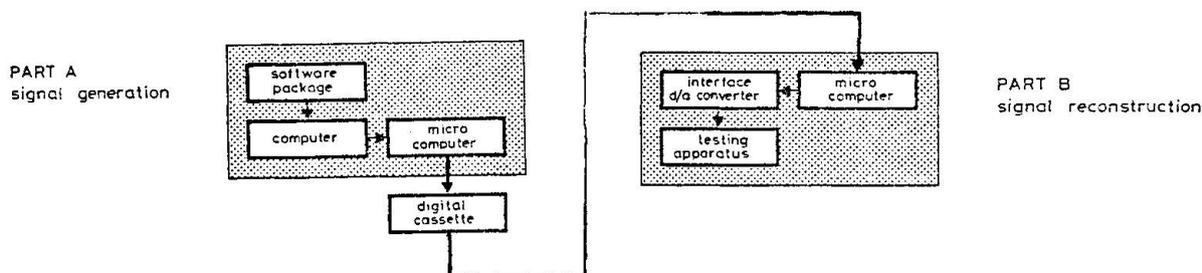


Fig. 8 Digital random generator

In the first part A a computer program generates a basic random signal (Gaussian white noise, i.e. random with a normal probability density function and a broad banded energy spectral density function). On this signal two digital filters are applied. The parameters of these filters are calculated from the probability density function and the energy spectral density function of the random signal to be generated. From the output signal of the software generator. Only the coordinates of the extreme values are stored on a digital compact cassette.

The second part B of the generator is linked to the testing apparatus and serves as a control unit. It consists of a micro-computer with a digital cassette unit and an interface. In this part the information of the compact cassette is read and the original signal is reconstructed by fitting cosine functions between successive extreme values. This signal is sent to the testing bench by means of a digital/analog-converter.

The number of extreme values to the experiment, are continually counted. The instrument is connected to the test bench driving unit in such a way that the generator stops when the specimen to be tested collapses. The total number of extreme values of the signal is registered.

This digital random generator was developed by TNO-IBBC because of its unique properties. Any type of random signal can be generated, departing from the desired statistic characteristics. As the signal is generated digitally it is fully reproducible and also easy accessible for computer calculations. This is important when after the testing the random signal has to be processed with counting methods and damage rules.

### 3.3 Constant amplitude tests

The constant amplitude tests serve as a basis for future processing the test results of the random loading tests with damage rules. As most of these damage rules are based on sine waves with a certain maximum stress level  $S'_{\max i}$ , a certain stress ratio  $R = S'_{\min i} / S'_{\max i}$  and a certain frequency, the constant amplitude tests were performed at various combinations of  $S'_{\max i}$ ,  $R$  and frequency. The testing program is given in table 2.

As indicated the maximum stress is relative to the mean static compressive strength  $m(f'_{bu})$ . Each test was repeated at least six times. A total number of 109 cylinders were tested. From the test results  $N_i$  - i.e. the number of cycles leading to fatigue failure - the mean values



$m(\log N_i)$  and the standard deviations  $s(\log N_i)$  were calculated.

Table 2 Constant amplitude tests - testing program and results

Test	Frequency in Hz	R	$S'_{\max i} / m(f'_{bu})$	$m(\log N_i)$	$s(\log N_i)$
1	6	0,1	0,8	2,980	0,154
2	6	0,1	0,7	3,780	0,121
3	6	0,1	0,6	4,695	0,176
4	6	0,4	0,8	3,386	0,174
5	6	0,4	0,7	4,507	0,349
6	6	0,7	0,8	4,264	0,299
7	0,6	0,1	0,8	2,474	0,099
8	0,6	0,1	0,7	3,226	0,080
9	0,6	0,1	0,6	4,062	0,144
10	0,6	0,4	0,8	2,879	0,149
11	0,6	0,4	0,7	3,554	0,569
12	0,6	0,7	0,8	3,353	0,259
13	0,06	0,1	0,8	1,949	0,070
14	0,06	0,1	0,7	2,644	0,092
15	0,06	0,1	0,6	3,426	0,062
16	0,06	0,4	0,8	2,146	0,160
17	0,06	0,4	0,7	3,165	0,203

These values are also given in table 2. It should be pointed out that the relatively high standard deviation for test number 11 is caused by only one extremely low test result. The scatter of the other test has the same magnitude as in foregoing investigations [4, 6, 7]. So it may be concluded that the dispersion in  $N_i$  is not larger than is to be expected on the basis of the dispersion in the static compressive strength. Probability aspects of fatigue itself have apparently no significant influence on the magnitude of the dispersion. This contrasts with results of constant amplitude tests in uniaxial tension and in alternating tension-compression loading [10].

In figures 9 to 11 the results of the constant amplitude tests are represented. Curves are given for  $m(\log N_i)$ . Comparison of these figures shows a reduction in fatigue strength with a decreasing rate of loading. This effect has been observed before [7].

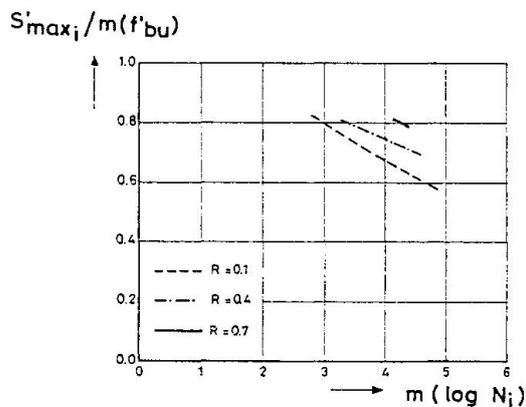


Fig. 9 Results of the Wöhler tests at 6 Hz.

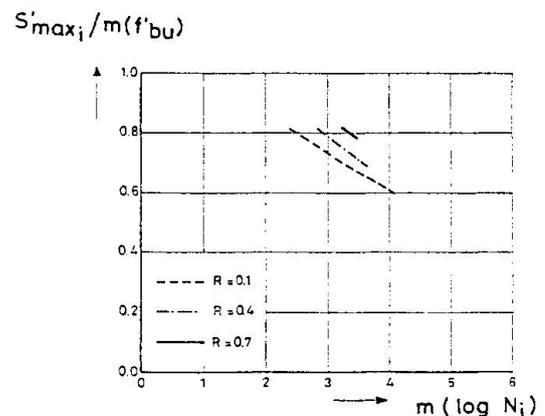


Fig. 10 Results of the Wöhler tests at 0,6 Hz.

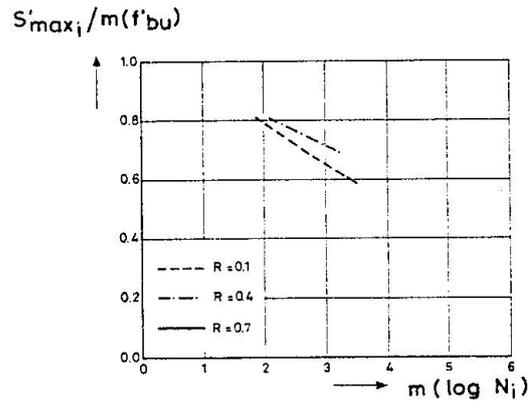


Fig. 11 Results of the Wöhler tests at 0,06 Hz

### 3.4 Random load tests

The random load tests served in first instance to find out if in the case of random loading the lifetime of a specimen is a reproducible quantity. Second, the test results will serve to study counting methods and damage rules. This latter part is still under way. To reach the goals of the investigation it was thought necessary to do testing with a large variety of random loadings, i.e. with different combinations of various probability and energy spectral density functions. Arbitrarily the combinations mentioned in table 3 are chosen. The types of random loadings are not characterized by a formula but by the names of their probability density functions in combination with their statistical parameters (i.e. the mean value  $m$  and the standard deviation  $s$ ) and by the names of their energy spectral density functions with their parameters (the central frequency in the case of a single peak signal, the two central frequencies in the case of a double peak signal and the band limits in case of a broad band signal). The tests 25 and 26 have the same characteristics as test 20. But they are new realisations.

The probability density functions of all tests were truncated at the value  $0,10 \cdot m(f'_{bu})$  and at the value  $0,80 \cdot m(f'_{bu})$ , the minimum stress amounted to  $0,10 \cdot m(f'_{bu})$  and the maximum stress to  $0,80 \cdot m(f'_{bu})$ .

Truncation is necessary to avoid extreme stresses. Without truncation tensile stresses are often theoretical possible and also compressive stresses above the static compressive strength. The values of the truncation barriers  $0,10 \cdot m(f'_{bu})$  and  $0,80 \cdot m(f'_{bu})$  were chosen in such a way that the tests could be expected to last about 1.000 to 100.000 half cycles. The test results are given in tabel 3 by means of the mean value and the standard deviation of  $\log C$ , where  $C$  is the number of half cycles causing fatigue failure (the number of half cycles is equal to the number of extremes of the loading signal). The means and standard deviations are calculated on basis of at least seven test results.

From the values of  $s(\log C)$  in tabel 3 it follows that the random loading tests are well reproducible, In general these standard deviations are smaller than with variable amplitude tests [6, 7]. So the randomness seems not to add to the dispersion of the test results.

Only in test 25 a rather high standard deviation was found. This is due to the fact that two from the eight test results were out of range. No explanation was found. It seems they are coincidental.

It also follows from the results of tests 20, 25 and 26 that the test results are well reproducible. As mentioned before these tests were



Table 3 Random loading tests - test program and test results

test	probability density function	m in % of $m(f'_{bu})$	s in % of $m(f'_{bu})$	energy spectral density function	$\omega$ in Hz	m(logC)	s(logC)
18	normal	0,45	0,175	single peak	0,16	3,433	0,118
19	normal	0,45	0,175	"	1	4,047	0,218
20	normal	0,45	0,175	"	6	4,298	0,171
21	normal	0,625	0,175	"	6	4,211	0,126
22	normal	0,275	0,175	"	6	4,464	0,203
23	normal	0,45	0,35	"	6	4,064	0,129
24	normal	0,45	0,2625	"	6	4,319	0,121
25	normal	0,45	0,175	"	6	4,422	0,546
26	normal	0,45	0,175	"	6	4,276	0,148
27	normal	0,45	0,175	broad band	0,16-6	4,562	0,179
28	normal	0,45	0,175	"	1 -6	4,622	0,212
29	normal	0,625	0,175	"	1 -6	4,392	0,121
30	normal	0,275	0,175	"	1 -6	4,841	0,159
31	normal	0,45	0,35	"	1 -6	4,458	0,187
32	normal	0,45	0,2625	"	1 -6	4,520	0,144
33	normal	0,45	0,175	double peak	0,16/6	4,686	0,088
34	normal	0,45	0,175	"	1/6	4,395	0,292
35	Rayleigh	-	0,14	single peak	6	>6,525	-
36	Rayleigh	-	0,21	"	6	5,021	0,194
37	Rayleigh	-	0,28	"	6	4,466	0,147

carried out with the same type of random loading, but with three different realisations of this loading. Despite of the different realisations the numbers of half cycles to failure in the three tests have the same magnitude.

The main conclusion from the test results is, that it is possible to determine the lifetime of concrete with sufficient accuracy by means of random loading tests with an arbitrarily chosen realisation.

#### 4. CONCLUSIONS

The following conclusions can be drawn from this investigation:

For constant amplitude loading, Wöhler curves could be determined for various combinations of minimum and maximum compressive stress-strength levels and loading frequencies.

With decreasing rate of loading a reduction of the fatigue strength (i.e. the number of cycles to failure) was observed. This is in accordance with an earlier investigation [4, 7].

The dispersion in the results of the constant amplitude tests is not larger than the dispersion in the static compression strength.

For various realisations of the same type of random loading the same lifetime was measured.

For random loading the lifetime of concrete is a reproducible quantity.

## 5. FUTURE RESEARCH

At the moment the investigation is continued in two different ways. First the results of the constant amplitude tests and the random loading tests will be combined in a study concerning counting methods and damage rules. In this respect attention will be paid to the Miner rule, which appeared [4, 7] to be satisfactory for variable amplitude stresses. Moreover experimental research is carried out to verify if the conclusions of this investigation are more general. In this respect random loading tests are carried out on various types of concrete, on concrete with various ages and various curing conditions.

## 6. NOTATION

- C = number of half cycles in a random loading test causing fatigue failure;
- $E(\omega)$  = energy spectral density function;
- $f_R(\rho)$  = probability density function of the random loading  $R(t)$ ;
- $H(\omega)$  = transfer function;
- $m(x)$  = mean value of  $x$ ;
- $m(f'_{bu})$  = mean static cylinder compressive strength;
- $N_i$  = number of cycles in a constant amplitude test causing fatigue failure;
- P = probability
- R = stress ratio  $S'_{\min i} / S'_{\max i}$  ;
- $R(t)$  = random loading;
- $S'_{\max i}$  = maximum compressive stress;
- $S'_{\min i}$  = minimum compressive stress.

## 7. ACKNOWLEDGEMENT

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