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Fracture Mechanics Predictive Technique Applied to Fatigue

Méthode prévisionnelle de la mécanique de la rupture appliquée à la fatigue

Bruchmechanische Methode zur Vorhersage der Ermüdungsfestigkeit

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SUMMARY
This paper presents a fracture mechanics technique for predicting the fatigue life of steel reinforcing bars in structural concrete elements. A comparison of the experimental results of five different laboratories and the results predicted by the theory shows that the technique is capable of estimating a reasonable lower bound for the experimental fatigue data. The use of the technique in studies of the influence of individual factors on the fatigue performance of reinforced concrete elements is demonstrated. The technique will be most useful for design purposes and in the development of new reinforcing bars of higher tensile strength.

RESUME
Cet article présente une méthode de la mécanique de la rupture pour la prévision de la résistance à la fatigue d'aciers d'armature pour des éléments de structure en béton. La comparaison entre les résultats expérimentaux de cinq différents laboratoires et les résultats théoriques montre que cette méthode permet de faire une estimation raisonnable de la borne inférieure des caractéristiques mesurées de fatigue. On montre l'application de cette méthode à l'étude de l'influence des différents paramètres sur la résistance à la fatigue. Cette méthode pourra être très utile pour le dimensionnement à la fatigue et pour le développement de nouveaux aciers d'armature à haute résistance.

ZUSAMMENFASSUNG
Der Beitrag stellt eine bruchmechanische Methode zur Vorhersage der Ermüdungsfestigkeit von Bewehrungseinlagen in Stahlbetonbauteilen vor. Ein Vergleich der Versuchsergebnisse aus fünf verschiedenen Laboratorien mit den theoretisch prophezeiten Resultaten zeigt, dass mit der Methode ein vernünftiger unterer Grenzwert der Ermüdungsfestigkeit bestimmt werden kann. Die Anwendung der Methode für das Stadium des Einflusses von Einzelfaktoren auf das Ermüdungsverhalten wird gezeigt. Das Verfahren kann für Bemessungszwecke und für die Entwicklung neuer Bewehrungsstähle mit höherer Festigkeit nützlich sein.

1. INTRODUCTION

Correct evaluation of fatigue behaviour of reinforced concrete elements has always required analysis of a large amount of experimental data (1,2). The data, though valuable, have been shown to be costly and time consuming to produce. Isolation of factors which influence the fatigue data has also been found difficult often impeding the development of conclusive findings.

This paper describes a theoretical approach based on the concepts of fracture mechanics which may be used to predict the fatigue life of steel reinforcing bars in concrete elements under constant amplitude cyclic stresses. Experimental data of some 100 tests were used for the development and verification of the theory. The influence of individual factors on fatigue performance of reinforced concrete beams was investigated.

2. THEORY OF FATIGUE LIFE PREDICTION

The fatigue life N is considered as the sum of the fatigue crack initiation life N_i and the crack propagation life N_p. The propagation life is assumed to consist of three components, namely slow propagation life N_{p1}, intermediate propagation life N_{p2} and fast propagation life N_{p3}. The fast propagation life is usually a small proportion of the total fatigue life and it is ignored, thus

$$N = N_i + N_{p1} + N_{p2}$$

Fatigue cracks initiate at stress concentration sites. Experimental work on formation of fatigue cracks in torbar (3) has shown that fatigue cracks normally initiate at the root of the transverse ribs on the surface of the bar. At the peak of the first load cycle a plastic zone may be formed at the root of the rib. The maximum depth of this zone in the bar a_p depends on the peak nominal stress σ_{max}, the stress concentration factor k_t, the radius of the rib root ρ and the yield stress of the steel σ_y. Assuming a Neuber stress distribution ahead of the rib the value of a_p can be estimated by the following expression:

$$a_p = \frac{\rho}{4}\left[\left(\frac{k_t \ \sigma_{max}}{\sigma_y}\right)^2 - 1\right]$$

Surface irregularities of different depths exist at the root of the rib. The maximum depth of these irregularities a_r depends on the surface condition of the bar. The critical fatigue crack is assumed to form at the deepest of these irregularities. Therefore, the fatigue crack initiation life is defined here as the number of cycles required to sharpen the trough of the deepest surface irregularity in the vicinity of the rib and/or damage the plastic zone. Thus at the end of the initiation period a crack of size a_o is formed in the bar and generally the initial crack size a_o is given by

$$a_o = a_r + a_p$$

The fatigue crack initiation life N_i is assumed to be related to the range of the stress intensity factor ΔK_o at the tip of the initial crack by the following equation

$$N_i = B_1 (\Delta K_o)^{m_1}$$

in which B_1 and m_1 are material constants which have been evaluated (4) for torbar steel and found to be: $B_1 = 1.448 \times 10^8$ and $m_1 = - 2.701$ in MN and m units. The above relation is analogous to those proposed by Jack and Price (5) for mild steel and Barnby and Holder (6) for cast steels.

The results of fatigue tests (3) on cracked specimens taken from torbar have shown that the rate of fatigue crack propagation da/dN may be described by a bi-linear relationship of the following form,

$$\frac{da}{dN} = C_1 (\Delta K)^{n_1} \qquad \text{for } \Delta K < 9 \ \text{MN.m}^{-3/2}$$

and

$$\frac{da}{dN} = C_2 (\Delta K)^{n_2} \qquad \text{for } \Delta K > 9 \ \text{MN.m}^{-3/2}$$

in which ΔK is the range of the stress intensity factor, $C_1 = 3.83 \times 10^{-29}$, $n_1 = 20.862$, $C_2 = 3.16 \times 10^{-12}$ and $n_2 = 3.143$.

Based on finite element analyses (7) of edge cracked round bar the following expression for K was derived (4):

$$K = 0.886\, k_t . \alpha(\tfrac{a}{D}) . f_b(\tfrac{a}{D}) . \sigma_t \sqrt{\Pi a} \left[f_t(\tfrac{a}{D}) + 0.76 \frac{D}{2d(1 - \frac{x}{d})} \right]$$

This expression allows for the following effects,
- the curved shape of the crack front,
- the stress concentration effect of the rib,
- embedment in concrete and debonding between the concrete and the bar,
- strain gradient in the bar because of embedment in the tension side of concrete flexural elements.

The correction functions $\alpha(\frac{a}{D})$, $f_b(\frac{a}{D})$ and $f_t(\frac{a}{D})$ account for the effects of the decay of the rib effect, the debonding and the curved shape of an edge crack in circular bar under uniform tension respectively. The last term in the K expression represents the strain gradient effect in which D, d and x are the bar diameter, the effective depth of the concrete section and the depth of the compression zone respectively.

The integration of the two fatigue crack propagation equations yields values of N_{p1} and N_{p2}, i.e.

$$N_{p1} + N_{p2} = \frac{1}{C_1} \int_{a_o}^{a_t} \frac{da}{(\Delta K)^{n_1}} + \frac{1}{C_2} \int_{a_t}^{a_{cr}} \frac{da}{(\Delta K)^{n_2}}$$

in which a_t is the crack depth at transition from slow growth to intermediate growth and a_{cr} is the final crack depth which may be taken equal to half the bar diameter.

3. VERIFICATION OF THE THEORY

The above theory was used to generate theoretical stress range - life data for concrete beams reinforced with torbar having dimensions and loading conditions similar to those tested experimentally in five different laboratories (4, 8, 9, 10, 11). For all cases, the measured values of $\rho = 0.3$ mm, $a_r = 0.071$ mm and $k_t = 1.85$ for 32 mm torbar were taken as typical critical values. In addition the values of B_1, m_1, C_1, n_1, C_2, n_2 and ΔK_t given in section 2 were also assumed typical.

The theoretical stress range-life curves and the corresponding experimental data are compared in Figures 1.a to 1.e. It can be seen that the theory represents a reasonable lower bound for all the experimental data. Only 13 beams out of a

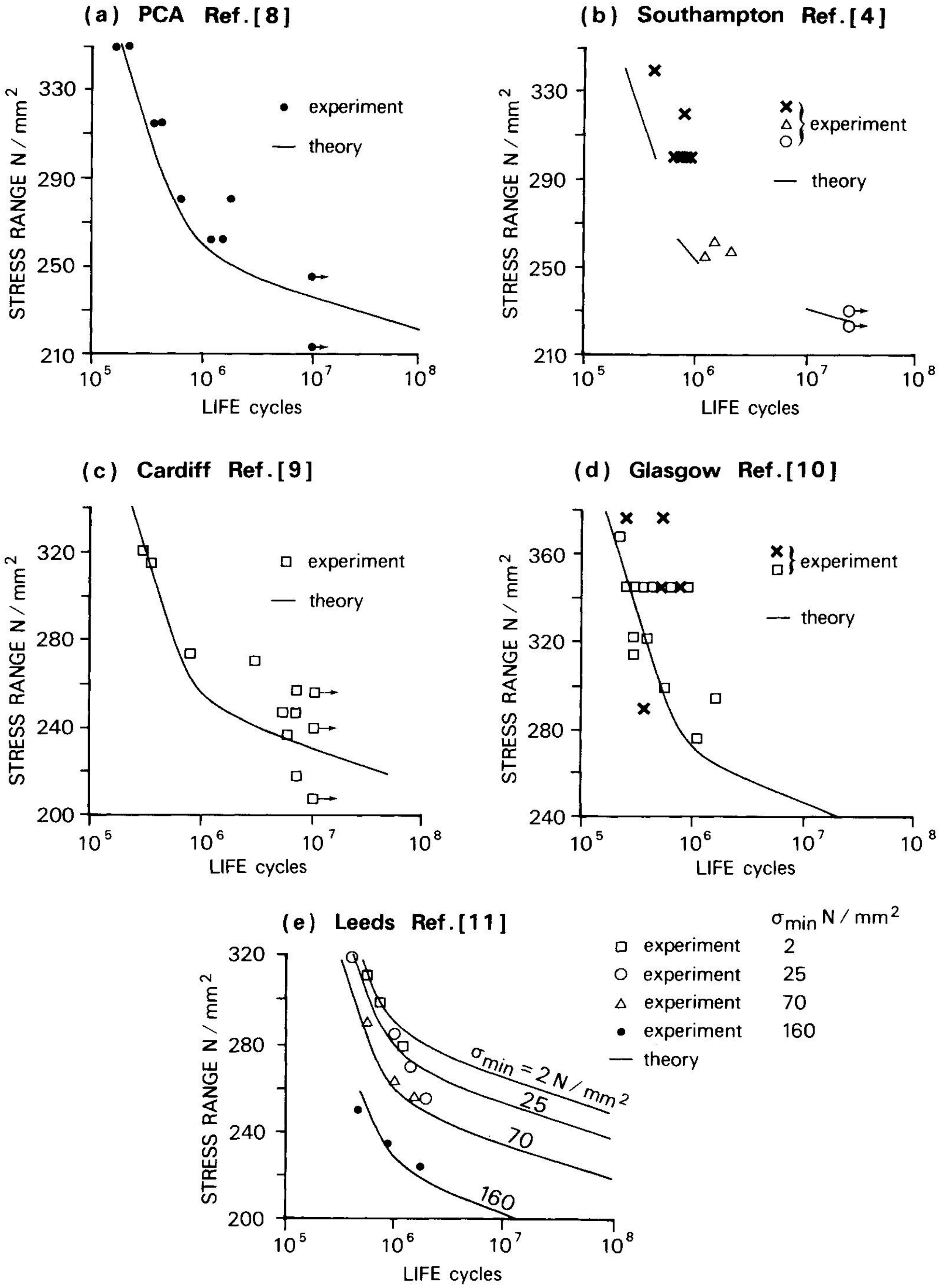

Figure 1 **Comparisons between the theory and experimental data from different laboratories**

total of 78 beams fractured by fatigue of the bar had lives shorter than those predicted by the theory. The theoretically predicted fatigue limit for the 12 mm torbar tested by Bannister is 229.5 N/mm^2 at 10^7 cycles which compares very well with the experimental value of 230 N/mm^2 which he has reported (9). Similarly, Bennett and Joynes (11) have reported a safe limiting stress range of 240 N/mm^2 for 10 mm torbar loaded at a minimum stress of up to 100 N/mm^2. The corrresponding theoretical value for this case is 237.9 N/mm^2 at 2 x 10^6 cycles. Also, the theory predicts a fatigue limit of 250 N/mm^2 at 2 x 10^6 cycles for the 25 mm torbar tested by Hanson et al (8) which compares well with the experimental value of 258.6 N/mm^2 (37.5 ksi).

4. EFFECT OF INDIVIDUAL FACTORS

In order to demonstrate the use of the theory to investigate the effect of individual factors on fatigue performance of reinforced concrete beams, the case of a 32 mm bar in concrete beam is considered. Theoretical stress range - life data were produced for this case in different conditions in which each of the influencing factors was varied individually from 0.8 to 1.2 of the initial values given below:

Factor	initial value
minimum stress σ_{min}	75 N/mm^2
yield strength	425 N/mm^2
stress concentration k_t	1.85
surface roughness a_r	0.071 mm
strain gradient D/2d(1-x/d)	0.0631 mm
bar diameter D	32 mm

Figures 2.a to 2.f show the effect of $\pm$ 20% variation of an individual factor while all other factors are kept constant. These figures show that the stress concentration at the root of the rib is the most effective factor in explaining the variation in performance. This confirms the significant effect of the rib geometery observed in previous experimental studies (8, 12). The second most effective factor is the yield strength of the bar, indicating that provided the initiation and propagation properties of the steel are constant, improvement in the yield strength can improve the fatigue performance of the bar. Surface roughness as measured by the maximum depth of surface irregularities and the minimum stress have approximately identical effects. Figure 2.e and 2.f indicate minimal effects due to the change in either the strain gradient or bar size.

The relative effect of each of the above factors as measured by the change in fatigue limit at 10^7 cycles due to 1% increase in a factor is given below:

Factor	average change in fatigue limit at 10^7 cycles due to 1% increase in factor	
	N/mm^2	%
stress concentration	- 3.05	- 1.40
yield strength	+ 1.10	+ 0.51
surface roughness	- 0.35	- 0.16
minimum stress	- 0.30	- 0.14
strain gradient	- 0.06	- 0.03
bar diameter	- 0.02	- 0.01

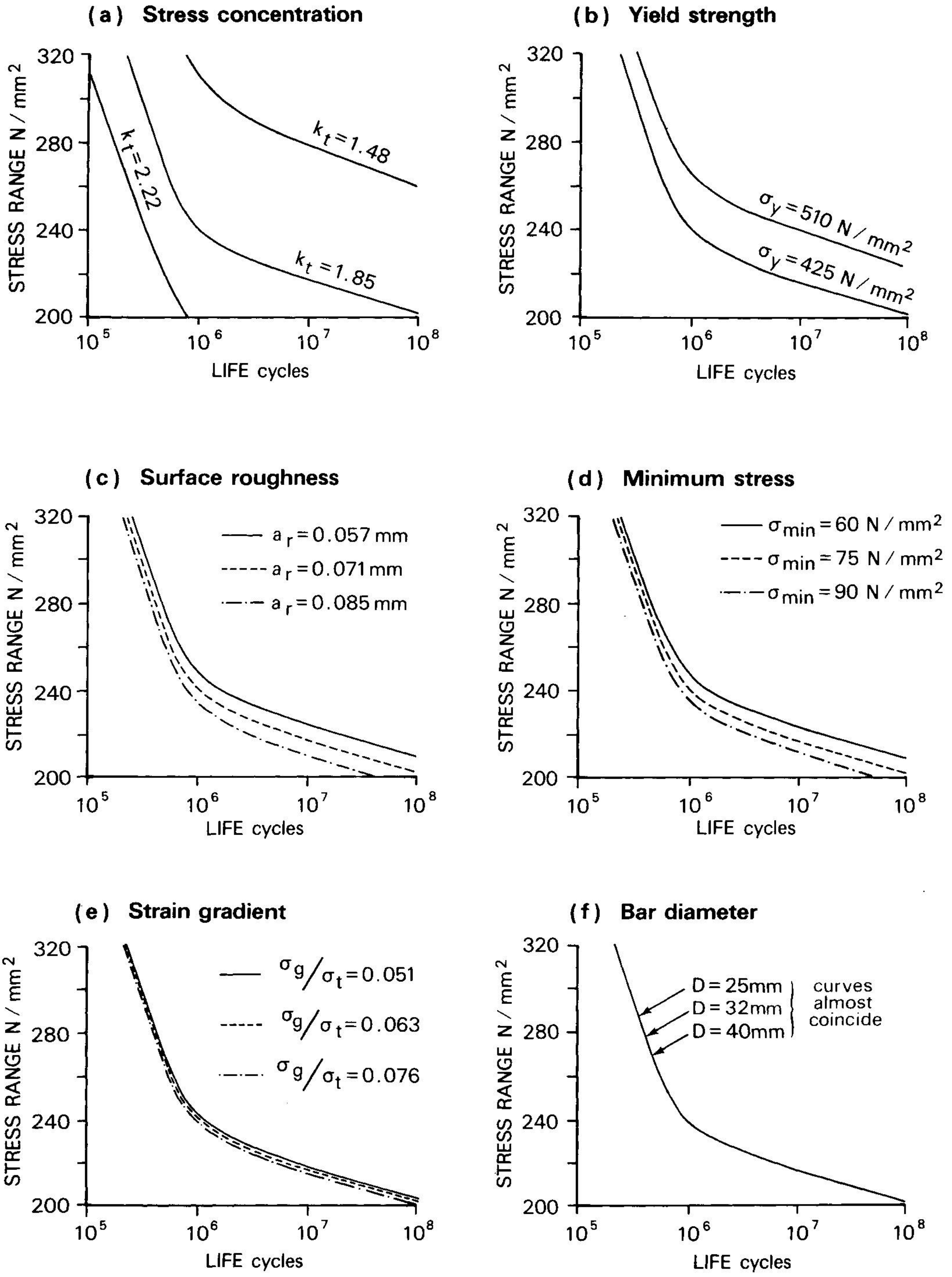

Figure 2 Effect of ± 20% change in individual factors on the fatigue performance of reinforcing bar

5. CONCLUSIONS

A theory for fatigue of reinforcing bars in concrete elements based on the concepts of fracture mechanics is presented and used to predict stress range - life data. The predicted results were verified against experimedntal data for 86 concrete beams reinforced with torbar tested by the authors and other investigators in different laboratories. It can be concluded that the theory may be used to predict an acceptable lower bound of the fatigue life.

The application of the theory requires information about crack initiation and propagation properties of the reinforcing bar, rib geometery, surface roughness, stresses in the bar and yield strength of the steel.

The theory may also be used to study the effect of individual factors on fatigue performance of reinforced concrete structural elements. For the cases considered in this study it was found that 1% increase in the stress concentration factor at the rib results in 1.4% decrease in the fatigue limit at 10^7 cycles. A similar increase in either the minimum stress or the surface roughness of the bar results in about 0.15% decrease in the fatigue limit at 10^7 cycles.

The proposed theory will be most useful for design against fatigue and for development of higher strength bars. The extension of the theory for variable amplitude and random loading conditions would be of practical value.

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