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Prediction of High Cycle Fatigue under Multiaxial Stress Conditions

Détermination de la résistance à la fatigue sous état de contraintes multiaxiales

Bestimmung der Ermüdungsfestigkeit bei mehraxialen Spannungszuständen

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SUMMARY
Four criteria for prediction of high cycle fatigue under multiaxial stress conditions are described, i.e. the octahedral stress criterion, the shear stress criterion, the internal friction criterion and the shear stress intensity criterion. From comparison between predictions and experimental results from the literature it is found that the two latter criteria give the best predictions.

RESUME
Quatres critères pour la détermination de la résistance à la fatigue sous conditions de contraintes multiaxiales sont décrits, c'est-à-dire le critère de contrainte octahèdral, le critère de contrainte de cisaillement, le critère de friction interne et le critère d'intensité de contrainte de cisaillement. La comparaison entre les résultats calculés et expérimentaux, tirés de la littérature, montre que les deux derniers critères donnent de meilleures prédictions.

ZUSAMMENFASSUNG
Es werden vier Berechnungskriterien für die Bestimmung der Ermüdungsfestigkeit bei mehraxialen Spannungszuständen beschrieben. Es sind dies die Kriterien der Oktaederspannung, der Schubspannung, der inneren Reibung und das Kriterium der Schubspannungsintensität. Ein Vergleich von Berechnung und Versuchsergebnissen aus der Literatur zeigt, dass mit den letzteren zwei Kriterien die besten Vorhersagen erzielt werden können.

1. INTRODUCTION

By means of modern fatigue analysis, for example by the finite element method, the stress state in any point of a component may be determined. In the general case, it will be given in terms of the stress matrix

$$T = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{bmatrix} \qquad (1)$$

In the case of fatigue loading, each of the six independent components of the matrix may in principle vary arbitrarily. Fatigue data available to the designer, however, are usually given in terms of SN curves measured with test specimens having uniaxial stress. The designer then needs some criterion for converting the multiaxial load variation to some equivalent, one dimensional stress variation, before the fatigue evaluation can be performed.
In the following, four criteria for such conversion are described and evaluated. It is assumed that all the stress components vary sinusoidally and with the same frequency, i.e.

$$\begin{aligned} \sigma_x &= \sigma_{xs} + \sigma_{xa} \cdot \sin(\omega t - \phi_x) \\ \sigma_y &= \sigma_{ys} + \sigma_{ya} \cdot \sin(\omega t - \phi_y) \\ \sigma_z &= \sigma_{zs} + \sigma_{za} \cdot \sin(\omega t - \phi_z) \\ \tau_{xy} &= \tau_{xyz} + \tau_{xya} \cdot \sin(\omega t - \phi_{xy}) \\ \tau_{xz} &= \tau_{xzs} + \tau_{xza} \cdot \sin(\omega t - \phi_{xz}) \\ \tau_{yz} &= \tau_{yzs} + \tau_{yza} \cdot \sin(\omega t - \phi_{yz}) \end{aligned} \qquad (2)$$

2. METHODS

2.1 The octahedral stress criterion

With this criterion, the variation of the shear stress in the octahedral plane is analysed.
In this plane, the shear stress is expressed as a vector with length proportional to the von Mises equivalent stress. If the stress components vary proportionally (in phase), the direction of the vector is fixed throughout the cycle, whereas the lenght of the vector varies. Hence a line segment in the plane defines the points that have been coverered by the vector during the cycle. The length of this segment is defined as the octahedral shear stress range. The equivalent stress variation is defined to be proportional to the octahedral shear stress range. With proportional loading, therefore, the octahedral stress criterion is identical to the von Mises criterion.
In the case of non proportional loading, the direction of the vector changes with time. During the cycle, the end point of the vector describes a closed curve in the plane. In this case, the octahedral shear stress range is defined as the greatest distance obtainable between two points of this curve.

2.2 The shear stress criterion

The shear stress (or Tresca) criterion could be applied directely to fatigue analysis in cases of proportional loading. For cases of non proportional laoding, an extended version of the criterion is described in ASME's Boiler and Pressure Vessel Code. The principle there is that the greatest shear stress range obtainable in any plane in space is used as the characteristic stress parameter. The stress range in a plane is defined in the same manner as described above for the octahedral plane. The equivalvent stress variation is defined to be proportional to this range. In the Code, a special, analytical procedure is applied to arrive at this result.

2.3 The internal friction criterion

This criterion appears as an extension of the shear stress criterion. Here, not only the shear stress range in the plane $\Delta\tau$ is considered, but also the normal stress range on the plane $\Delta\sigma$. They are combined linearly to $\Delta\tau_c$

$$\Delta\tau_c = \Delta\tau + \alpha \cdot \Delta\sigma \tag{3}$$

The equivalent stress variation is defined to be proportional to $\Delta\tau_c$. The plane of analysis is the plane where $\Delta\tau$ is the greatest. In simple cases, this plane can be determined analytically, whereas for complicated load cycles a numerical approach offers the simplest solution.
The constant α can be found if the material's fatigue strength is known in tension as well as torsion (denoted σ_o and τ_o respectively). The ratio between them (σ_o/τ_o) is by many ivestigators regarded as a material parameter. For ductile steels, σ_o/τ_o - ratios in the range 1.45 to 1.75 have been measured.
In lack of measured data, the value $\sigma_o/\tau_o = \sqrt{3}$ is recommended. This corresponds to α = 0.16 in eq. (3).

2.4 The shear stress intensity method

This method is based on an idea put forward by Novozhilov in (2) and modified for application to fatigue by Simbürger in (3). The idea is the following: Through a point in space, an infinite number of planes can be laid. A stress state in the point is reflected as a shear stress τ_p in each of these planes. The root-mean-square value of all these shear stresses is defined as the shear stress intensity, τ_i. Regarding the planes as tangential planes to a sphere with surface A, we get

$$\tau_i = \sqrt{\int_A \tau_p^2 \cdot dA} \tag{4}$$

It can be shown that τ_i is proportional to the von Mises equivalent stress.
For fatigue evaluation, the shear stress in a plane τ_p is exchanged by the shear stress range in the plane, $\Delta\tau_p$. Hence a $\Delta\tau_i$ is determined, and the equivalent stress variation is defined to be proportional to $\Delta\tau_i$. In cases of proportional loading, thus, this criterion is identical to the von Mises criterion and hence to the octahedral stress criterion.
Various modified versions of this criterion have appeared in recent german literature. Most of them are mentioned in (3). Generally, they are devised in order to reduce the calculational work (of solving eq. 4 numerically) and to include the σ_o/τ_o - ratio as a material parameter. In most cases, the validity of these versions is restricted to loadcases of two dimensional stress states.

3. RESULTS

The predictions of the above 4 methods have been compared to 224 fatigue tests with multiaxial loading reported in the literature. All these tests were performed with smooth, steel specimens. The deviation between predictions and measurements is denoted "ERROR" and is measured in per cent. Positive "ERROR" means predictions on the conservative side.
The results are shown in figs. 1-4. It is seen that the internal friction criterion and the shear stress intensity criterion are the best ones.

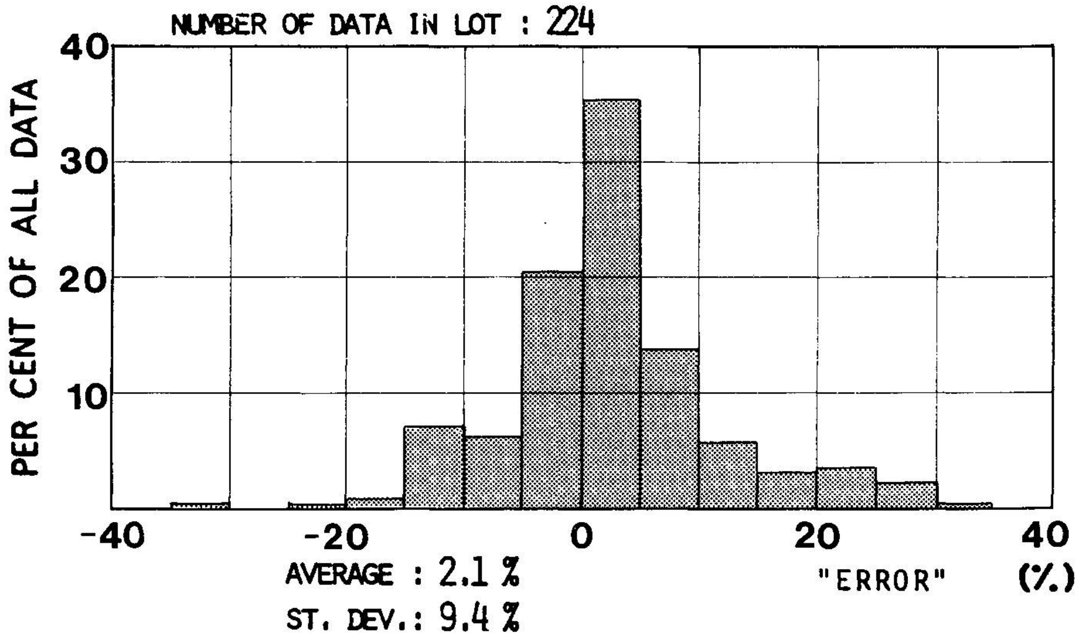

Fig. 1 Distribution of "ERROR" for octahedral stress criterion

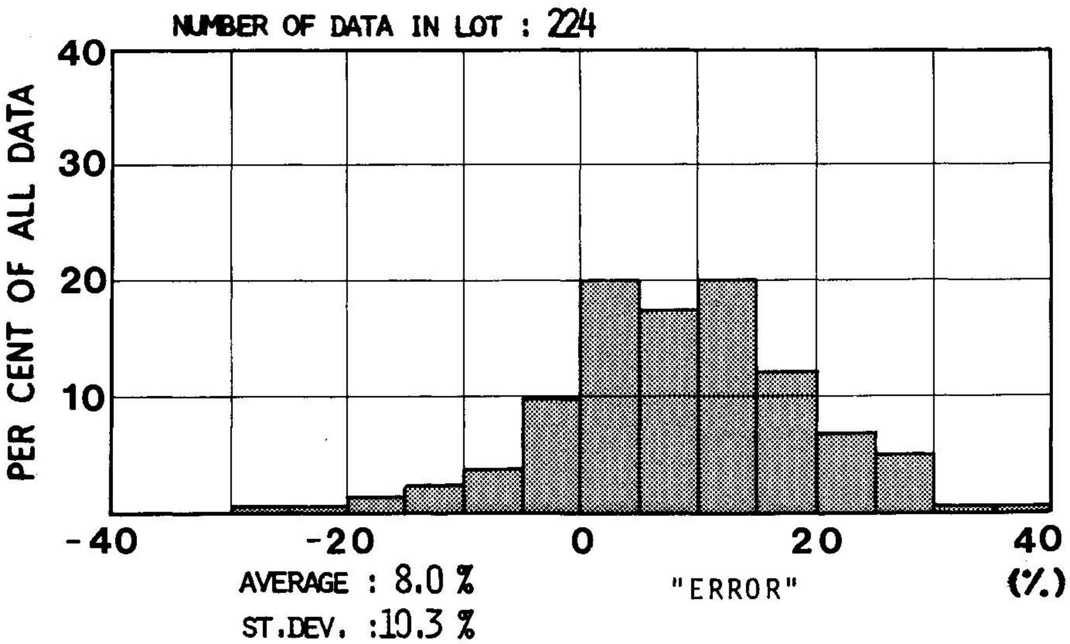

Fig. 2 Distribution of "ERROR" for the shear stress criterion.

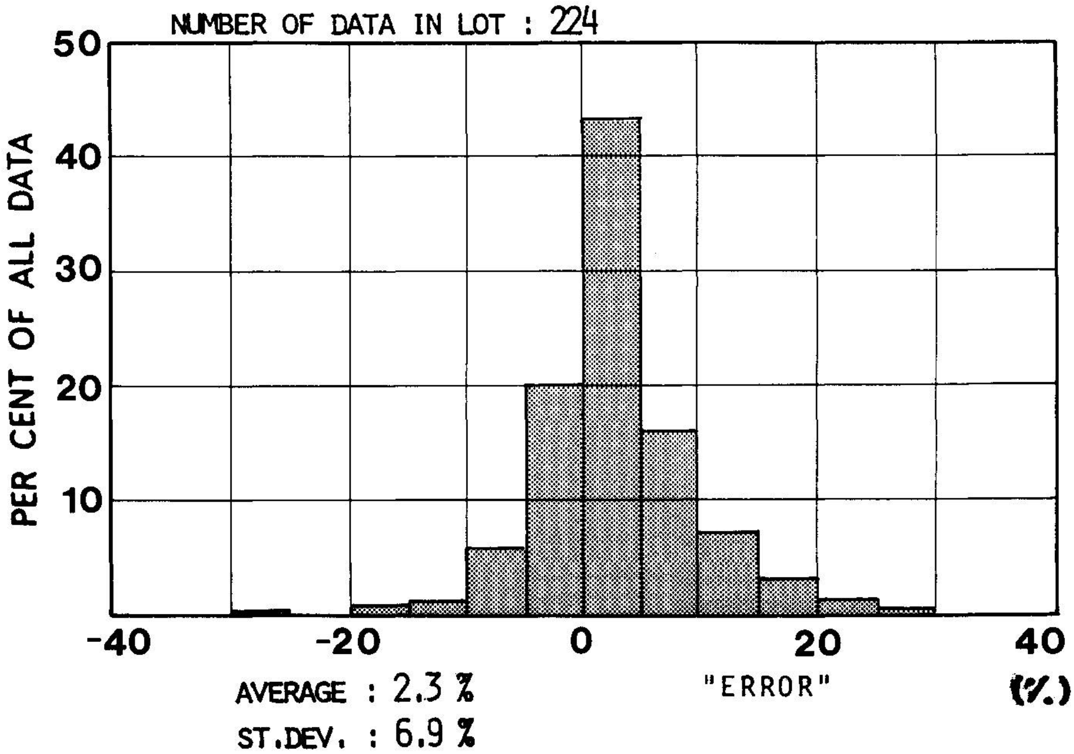

Fig. 3 Distribution of "ERROR" for internal friction criterion.

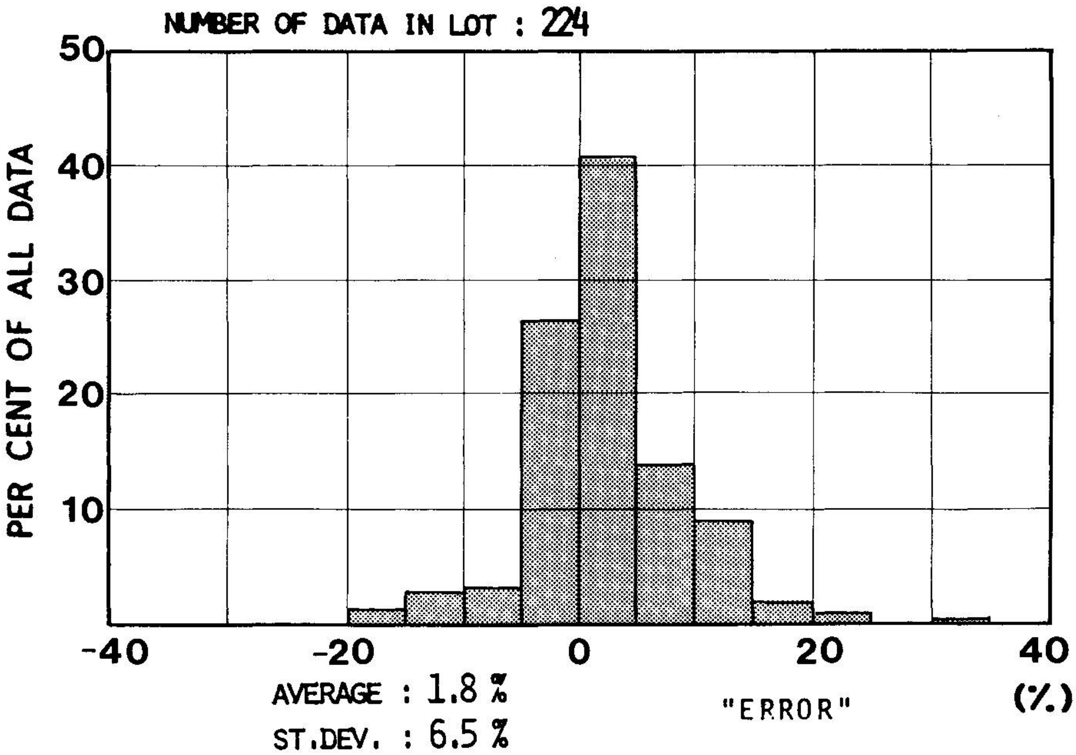

Fig. 4 Distribution of "ERROR" for shear stress intensity predictions

4. THE σ_o/τ_o RATIO

Based on the experimental data, the improvement in predictions obtained from including the σ_o/τ_o - ratio as a material parameter was investigated. This was done by performing the comparison between experimental and predictional results for two versions of the internal friction criterion, one with variable α and one with the fixed value $\alpha = 0.16$. The two approaches gave surprisingly small differences in the overall results, and it was concluded that the effort of measuring the fatigue strength in torsion is justified only in special cases of multiaxial fatigue evaluation.

5. MEAN STRESS EFFECT

In 70 of the experimental results, a static stress was included in the loading. This gave a proper background for evaluating criteria for equivalvent static stress under multiaxial stress conditions.
Two criteria are dominating in the literature here, namely the Sines criterion defining the first stress invariant ($\sigma_x + \sigma_y + \sigma_z$) as the equivalent static stress, and the von Mises criterion. The results clearly show that the latter gives best correlation between predictions and measurements.

6. REFERENCES

1 Nøkleby: Fatigue under multiaxial stress conditions
MD 81001, Div. of Machine Elements,The Norwegian Inst. of Technology, Trondheim, Norway.

2 Novozhilov: Theory of Elasticity
Pergamon Press, London, 1961

3 Simbürger:Festigkeitsverhalten Zäher Werkstoffe bei einer mehrachsigen, phasenverschobenen Schwingbeanspruchung mit körperfesten und veränderlichen Hauptspannungsrichtungen.
LBF-Bericht FB 121, 1975
Laboratorium für Betriebsfestigkeit, Darmstadt, Germany