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Local Strain Range and Fatigue Crack Initiation Life

Variation de l'allongement local et durée de vie jusqu'à l'initiation d'une fissure de fatigue

Lokale zyklische Dehnungsdifferenz und Lebensdauer bis zur Bildung eines Ermüdungsrisses

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SUMMARY
Approximate expressions are developed for the notch tip cyclic strain range in a metallic element under stress ratios, R = 0 and R = −1. Fatigue crack initiation life is then investigated with particular emphasis on the effect of stress ratio R and relationship to the metal's tensile properties. A generalised formula is derived and compared with test results in the literature.

RESUME
Des formules approchées sont développées pour le calcul de la variation de l'allongement local à l'extrémité de l'entaille située dans un élément métallique, ceci pour des rapports de contrainte R = 0 et R = −1. La durée de vie jusqu'à l'initiation d'une fissure de fatigue est ensuite déterminée en considérant plus particulièrement l'influence du rapport de contrainte R et celle des propriétés du métal à la traction. Une expression généralisée est développée puis comparée aux résultats d'essais tirés de la litérature.

ZUSAMMENFASSUNG
Eine Näherungsbeziehung zur Bestimmung der lokalen zyklischen Dehnungsdifferenz im Scheitelpunkt einer Kerbe in einem Metallprüfkörper wird vorgeschlagen, dies für Spannungsverhältnisse von R = 0 und R = −1. Die Lebensdauer bis zur Bildung eines Ermüdungsrisses wird untersucht und zwar unter besonderer Berücksichtigung des Einflusses des Spannungsverhältnisses R und der Zugeigenschaften des Metalles. Eine allgemeine Beziehung wird hergeleitet und mit Versuchsresultaten aus der Literatur verglichen.

1. INTRODUCTION

In the past decade much work has been devoted to predicting fatigue crack initiation life (FCIL) in notched elements. Of this, very little has considered the effect of stress ratio [1] [2] or the relationship to metallic tensile properties [3] [4].

Approximate formulae are developed from Neuber's rule [5] and Hollomon's equation [6] to give the notch tip cyclic local strain range which governs the prediction of FCIL. Then with the accepted local stress-strain concept for predicting FCIL [7] [8] an approximate expression of FCIL is derived. Examples are given for the prediction of FCIL in steels from their tensile properties.

2. CYCLIC LOCAL STRAIN RANGE IN NOTCHED METALLIC ELEMENTS

2.1. Basic equations

Although the nominal stress is below yield and the element as a whole elastic, plastic deformation may occur at the notch root from stress concentration. As a basis for calculating the notch tip local strain Neuber's rule [5] [7] [8] gives :

$$K_t^2 = K_\sigma K_\epsilon = (\sigma/S)\ (\epsilon/e)\ , \tag{1}$$

with : K_t (theoretical stress concentration factor), K_σ (stress concentration factor), K_ϵ (strain concentration factor), σ and ϵ (notch tip local stress and strain), S and e (nominal stress and strain in the net section). Using Hooke's law and E as Young's modulus we have :

$$e = S/E\ . \tag{2}$$

Then from eqs. (1) and (2) we obtain :

$$\sigma\ \epsilon = (K_t\ S)^2/E\ . \tag{3}$$

From Hollomon's equation [6] for stress-strain behaviour in the elasto-plastic state :

$$\sigma = K\ \epsilon_p^n\ , \tag{4}$$

with : n (strain-hardening exponent), K (strength coefficient) and ϵ_p (plastic component of the local strain).

From eq. (4) K may be approximated by :

$$K = \sigma_f/\epsilon_f^n\ , \tag{5}$$

with : σ_f (fracture strength), ϵ_f (fracture ductility).

The total local strain ϵ is the sum of the elastic and plastic components :

$$\epsilon = \epsilon_e + \epsilon_p\ , \tag{6}$$

obtaining the elastic component ϵ_e from Hooke's law and eq. (4) thus :

$$\epsilon_e = \sigma/E = K\ \epsilon_p^n/E\ . \tag{7}$$

Subsituting eqs. (4), (6) and (7) into (3) also gives the total local strain as :

$$\epsilon = [1 - \frac{\epsilon_e}{\epsilon}]^{\frac{-n}{1+n}} [\frac{1}{E\,K} (K_t\ S)^2]^{\frac{1}{1+n}} , \tag{8}$$

or, altenatively, the plastic component ϵ_p as :

$$\epsilon_p = [1 + \frac{K}{E}\ \epsilon_p^{n-1}]^{\frac{-1}{1+n}} [\frac{1}{E\,K} (K_t\ S)^2]^{\frac{1}{1+n}} . \tag{9}$$

Given S and K_t eq. (9) may be solved by iteration. The elastic component may be found from eq. (1) and hence the total local stain.

If a macrocrack initiates at notch root when ϵ_p reaches the value of ϵ_f and, for structural steel, we assume $\epsilon_f \geq 0.40$ and $\sigma_f \simeq 0.01$ E, then, after rearranging eq. (9), the first term becomes :

$$(1 + \frac{K}{E}\ \epsilon_p^{n-1})^{0.5} = (1 + \frac{\sigma_f}{E\ \epsilon_f})^{0.5} \simeq 1.0 ,$$

and we have the criterion for macrocrack initiation as :

$$K_t\ S = \sqrt{E\ \sigma_f\ \epsilon_f} . \tag{10}$$

Using these basic equations, and the argument of Barsom et al [9] that strain in the notch root plastic zone is governed by the surrounding elastic zone displacements, the variation of local strain with nominal stress may be analysed.

2.2. Pulsating stress cycles (R = 0)

The variation of the local strain with nominal stress is shown in the first quadrant of figure 1. On loading, the local strain increases to ϵ_B, along the line OAB as the nominal stress increases to S_B. Phase OA is elastic, and AB plastic with point A the elastic limit. In the plastic phase, the local strain is calculated using eqs. (9) and (7).

On unloading, the nominal stress returns to zero. The displacement of the elastic zone surrounding the notch root also reduces to zero. Thus, the local strain at the notch tip is forced to return to zero along line BCO, phase BC being elastic recovery, and CO the "forced plastic recovery".

From the above model and eq. (8), the cyclic local strain range may be expressed as :

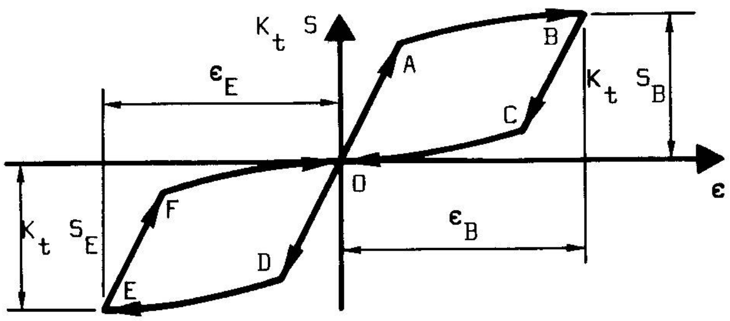


Figure 1 :
Variation of the local strain at the notch tip with the nominal stress.

$$\Delta\varepsilon = [1 - \frac{\Delta\varepsilon_e}{\Delta\varepsilon}]^{\frac{-n}{1+n}} [\frac{1}{E\ K}(K_t\ \Delta S)^2]^{\frac{1}{1+n}}, \tag{11a}$$

where $\Delta S = S_B = S_{max}$ represents the applied nominal stress range.

2.3. Full stress reversal (R = - 1)

From Figure 1 and based on the same argument, the cyclic local strain range becomes :

$$\Delta\varepsilon = 2[1 - \frac{\Delta\varepsilon_e}{\Delta\varepsilon}]^{\frac{-n}{1+n}} [\frac{1}{E\ K}(K_t\ \frac{\Delta S}{2})^2]^{\frac{1}{1+n}}, \tag{11b}$$

where $\Delta S = 2\ S_B = 2\ S_{max}$ is again the applied nominal stress range.

Test results for notched cylindrical specimens in [10] are shown in figure 2 with calculated values from eq. (11). As may be seen from figure 2, good agreement exists between the calculated curves and the experimental results. As the notch tip would be in biaxial tension a combined stress concentration factor from [11] was used for K_t. The use of tensile instead of cyclic properties in the calculation of cyclic local strain range is acceptable as, according to Barsom et al [9], the cyclic strain hardening/softening has a negligible effect on local strain range. This fact has been experimentally verified by Kremple [10].

In investigating the effect of stress ratio it can be shown that eqs. (11a) and (11b) can be rewritten as :

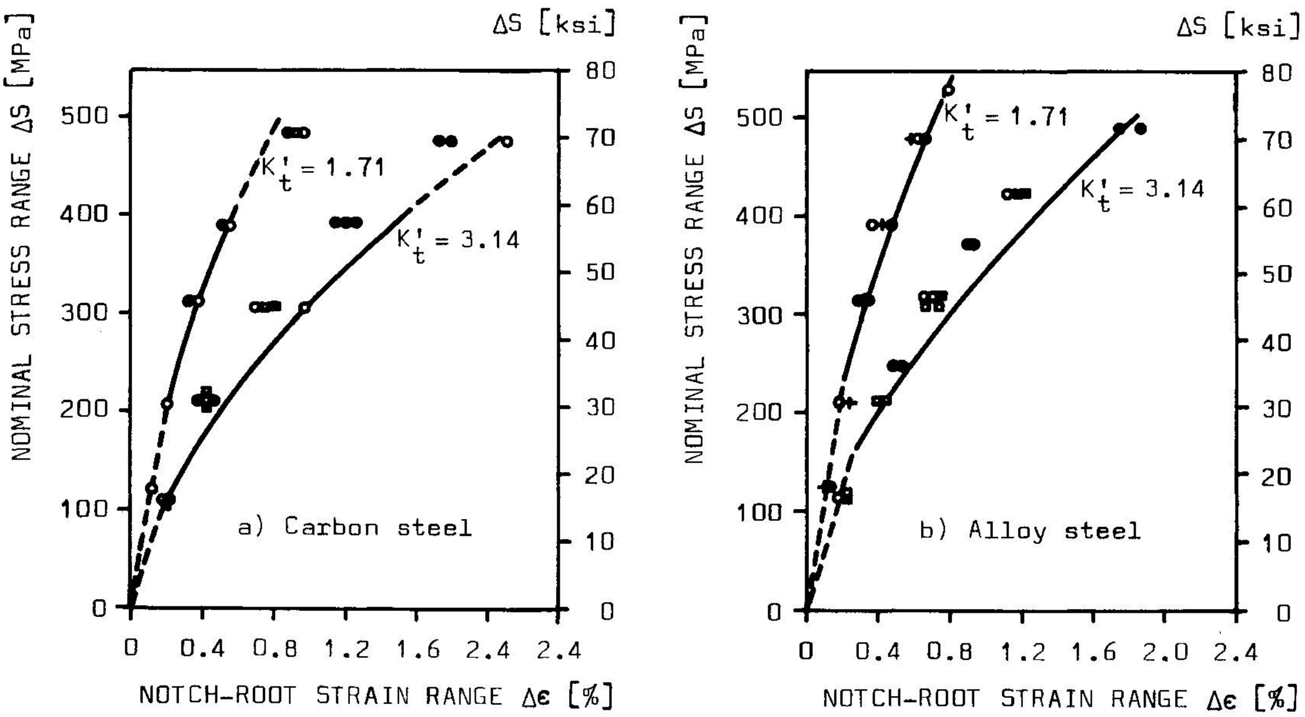


Figure 2 : Calculated local strain range (eq. 11b) and test results [10] of notched cylindrical specimens. a) Carbon steel ($E = 2.06 \cdot 10^5$ MPa, $K = 694.2$ MPa, $n = 0.199$) b) Alloy steel ($E = 2.06 \cdot 10^5$ MPa, $K = 838.3$ MPa, $n = 0.193$)

$$\Delta\varepsilon = 2^{\frac{-n}{1+n}}\, 2\left[1 - \frac{\Delta\varepsilon_e}{\Delta\varepsilon}\right]^{\frac{-n}{1+n}} \left[\frac{1}{E\,K}\left(\sqrt{\frac{1}{2\,(1-R)}}\,K_t\,\Delta S\right)^2\right]^{\frac{1}{1+n}} \quad \text{for } R = 0. \qquad (12a)$$

$$\Delta\varepsilon = 2\left[1 - \frac{\Delta\varepsilon_e}{\Delta\varepsilon}\right]^{\frac{-n}{1+n}} \left[\frac{1}{E\,K}\left(\sqrt{\frac{1}{2\,(1-R)}}\,K_t\,\Delta S\right)^2\right]^{\frac{1}{1+n}} \quad \text{for } R = -1. \qquad (12b)$$

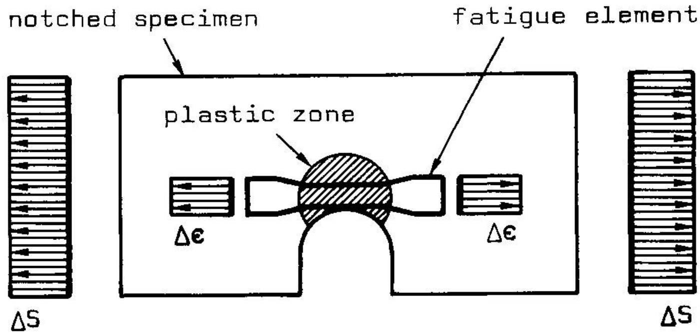
Although the cyclic local strain range is still not given by a unique formula it becomes more a function of the parameter $\sqrt{1/2\,(1-R)}\,K_t\,\Delta S$ as n becomes smaller, thus showing the significance of stress ratio.

3. EFFECT OF STRESS RATIO ON THE FATIGUE CRACK INITIATION LIFE

Crack initiation may be assumed to occur due to the fracture of a hypothetical, uniaxial fatigue element located at the notch root in figure 3. The fatigue life of a similar smooth test specimen can thus indicate the FCIL of a notched element if the "fatigue element" encounters the same stress-strain history.

Figure 3 :

Plastic zone and hypothetical, uniaxial fatigue element at notch root.



From [12] the fatigue life of a smooth specimen can be expressed as follows :

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon_f'\,(2\,N_f)^c\,, \qquad (13)$$

where $\Delta\varepsilon_p$ is the plastic component of the cyclic strain range, c the fatigue ductility exponent and ε_f' the fatigue ductility coefficient. According to [12], this latter may be approximately set equal to ε_f.

Also, in low-cycle fatigue behaviour, the plastic component of cyclic strain range may be considered approximately equal to the total local strain range, i e, $\Delta\varepsilon_p \simeq \Delta\varepsilon$, and $(1 - \Delta\varepsilon_e/\Delta\varepsilon)^{-n/(1+n)} \simeq 1$.

Thus substituting eqs. (12) and (5) into eq. (13) and replacing N_f the fatigue life of the "fatigue element", with N_i, the FCIL of the notched element, we have the following expressions of FCIL :

$$2\,N_i = 2^{-0.5n\cdot k_p}\, C_p\left[\sqrt{\frac{1}{2\,(1-R)}}\,K_t\,\Delta S\right]^{k_p} \quad \text{for } R = 0, \qquad (14a)$$

$$2\,N_i = C_p\left[\sqrt{\frac{1}{2\,(1-R)}}\,K_t\,\Delta S\right]^{k_p} \quad \text{for } R = -1, \qquad (14b)$$

where : $k_p = 2/c\,(1 + n)$, (15)

$C_p = \sqrt{E\,\sigma_f\,\varepsilon_f}$. (16)

As may be seen from eq. (14b), if the product of $K_t\,\Delta S/2$ reaches the value of $\sqrt{E\,\sigma_f\,\varepsilon_f}$, the first cycle of loading will already create crack initiation (see eq. 10), hence $N_i = 1$. Therefore, the constant value of 2, on the left hand side of eqs. (14), should be omitted. In doing so, the predicted values of C_p will correspond better to the experimentally determined data [3] [4].

Similarly the constant 2 exp $(-0.5n \bullet k_p)$ in eq. (14a) tends to unity as n becomes small, as in structural metals, so that an approximate, but unique, expression for FCIL is :

$$N_i = C_p\left[\sqrt{\frac{1}{2\,(1 - R)}}\,K_t\,\Delta S\right]^{k_p} . \qquad (17)$$

After logarithmic transformation equation (17) will be represented as a straight line of log $[\sqrt{1/2\,(1 - R)}\,K_t\,\Delta S]$ against log N_i. Figure 4 shows the least square fit to test data in [13] for stress ratios varying between 0.05 and 0.45. Regression analysis yields the following constants : $C_p = 2.87 \bullet 10^{17}$ and $k_p = -4.89$; the correlation coefficient is $r = -0.951$ and the standard deviation $s = 0.084$. As may be seen eq. (17) represents well the test data.

Fatigue crack initiation may also be described using fracture mechanics parameters [14]. Dowling [15] pointed out that, in some cases, the following relationship exists :

$$K_t\,\Delta S = \frac{2}{\sqrt{\pi}}\,\frac{\Delta K_I}{\sqrt{\rho}} , \qquad (18)$$

where K_I is the fracture mechanics stress intensity factor and ρ the notch root radius.

Substituting eq. (18) into eq. (17), we find :

$$N_i = C'_p\left[\sqrt{\frac{1}{2\,(1 - R)}}\,\frac{\Delta K_I}{\sqrt{\rho}}\right]^{k_p} , \qquad (19)$$

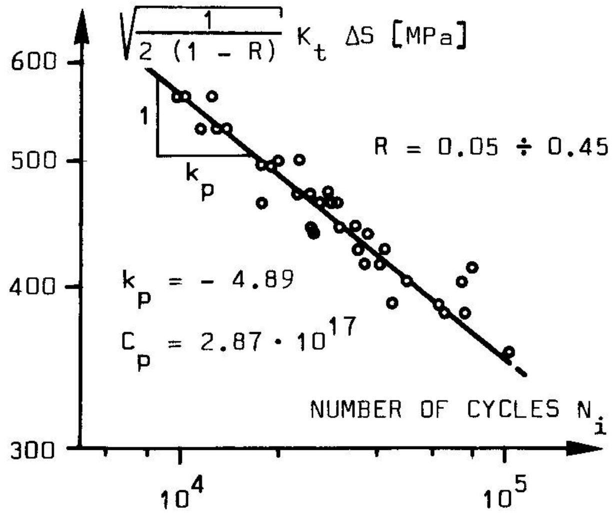


Figure 4 :

Regression analysis of test results of FCIL at various values of stress ratio (Al-alloy 2024-T3 [13]).

$$\text{where} \quad : \quad C'_p = [0.5 \sqrt{\pi E \sigma_f \epsilon_f}]^{-k_p} . \tag{20}$$

Therefore, FCIL can also be expressed as a power function of $\Delta K_I/\sqrt{\rho}$ which has been experimentally verified by different authors [3] [4] [16] [17] [20].

4. PREDICTION OF FCIL IN STEEL

The main difficulty in the prediction of FCIL lies in the evaluation of the exponent k_p (eq. 15). Once this is known, the coefficient C_p or C'_p (eqs. 16 or 20) may be calculated and subsequently introduced into eqs. (17) and (19) to yield the fatigue crack initiation life N_i.

4.1. Basic equations

To compute the exponent k_p, the values of c and n are needed. Manson [1] gives an experimental formula for c, using basic tensile properties :

$$c = 0.52 - \frac{1}{4} \log \epsilon_f + \frac{1}{3} \log [1 - 82 (\frac{\sigma_u}{E}) (\frac{\sigma_f}{\sigma_u})^{0.179}] . \tag{21}$$

From Hollomon's equation, n may be approximated as :

$$n = \log (\sigma_f/\sigma_{ys}) / \log (500 \epsilon_f) . \tag{22}$$

The true fracture values of σ_f and ϵ_f are given in [18] as :

$$\sigma_f = \sigma_u (1 + RA) , \tag{23}$$

$$\epsilon_f = - \ln (1 - RA) . \tag{24}$$

Here, σ_u is the ultimate stress, σ_y the yield stress, and RA the reduction in area at fracture in the tension test.

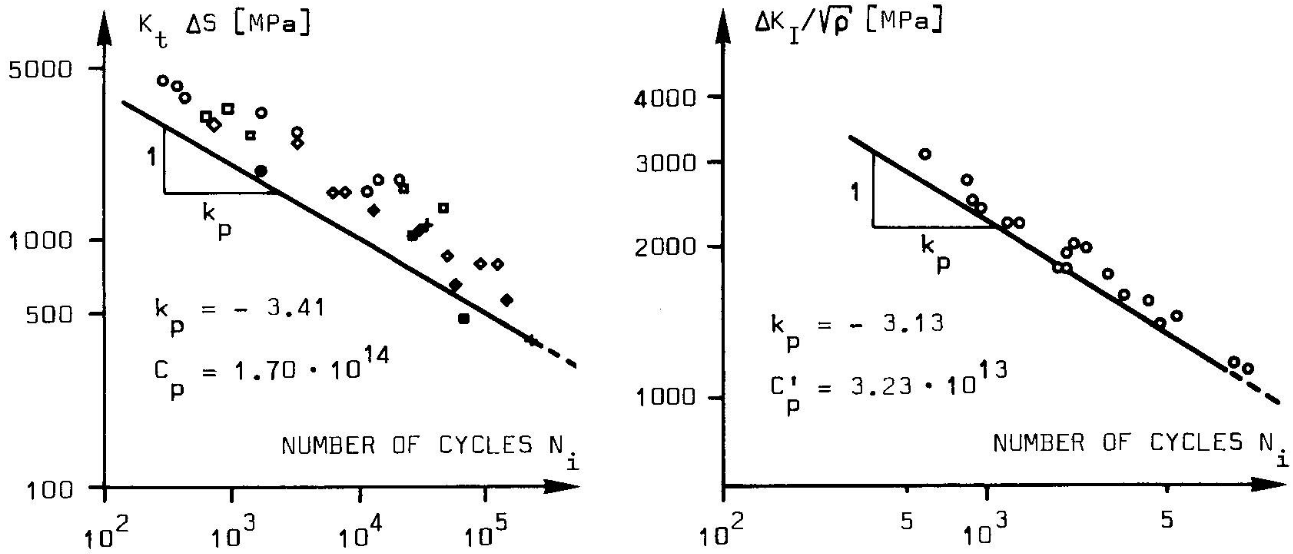
4.2. Notched specimens

Calculated values of the exponent k_p and the coefficient C_p of C'_p of three steels are listed in table 1. As shown in figure 5, the resulting curves of FCIL agree

Table 1 : Examples of tensile properties [20] [21] [22] and predicted values of k_p, C_p and C'_p for three steels.

MATERIAL		σ_u [MPa]	σ_{ys} [MPa]	RA [%]	k_p eq. (15)	C_p (modified) eq. (16)	C'_p (modified) eq. (20)
Cast steel	[21]	619	423	48.0	- 3.41	$1.70 \cdot 10^{14}$	
High-strength steel	[20]	1161	1053	52.6	- 3.13		$3.23 \cdot 10^{13}$
Fe 510	[22]	530	350	60.0*	- 3.19	$2.79 \cdot 10^{13}$	

* estimated value



a) Cast steel, R = 0.1 [16]. b) High-strength steel, R = 0 [20].

Figure 5 : Comparison of the predicted curves for the fatigue crack initiation life N_i with test results.

well with the experimental results [15] [20] although they give, to some extent, shorter lives. It should be noted that the values of C_p and C'_p in table 1 although calculated from eqs. (16) and (20) were further modified by a multiplication factor $[\sqrt{1/2\ (1-R)}]^{k_p}$. This is necessary when the stress ratio is the same value for all test.

These comparisons indicate a link between FCIL and tensile properties although additional work is necessary to give a more accurate correlation between them and k_p since the FCIL is sensitive to a number of other factors, in particular, the surface condition of the notch root [16] [18].

4.3. Welded elements

Yamada and Hirt [23] give the fatigue life of gusset specimens similar to the specification of Stress Category D in AASHTO [24]. Using the values in Table 1 for Fe 510 base plate material and an average stress ratio of 0.158, eq. (17) gives :

$$N_i = 2.79 \cdot 10^{13}\,(K_t\,\Delta S)^{-3.19}\ . \tag{25}$$

Stress measurements on similar welded elements [25] show stress concentration factors at the crack initiation location largely in excess of 2 so that FCIL of gusset specimen becomes, for example $K_t = 3.0$:

$$N_i = 8.39 \cdot 10^{11}\ \Delta S^{-3.19}\ , \tag{26}$$

where ΔS is the nominal stress range defined as Δσ in [23].

The total fatigue life of the gusset specimens, made of Fe 510 and other structural steels, along with calculated FCIL from eq. (26) are shown in figure 6. It may be seen that the ratio of FCIL to the total fatigue life, N_i/N_f, is about

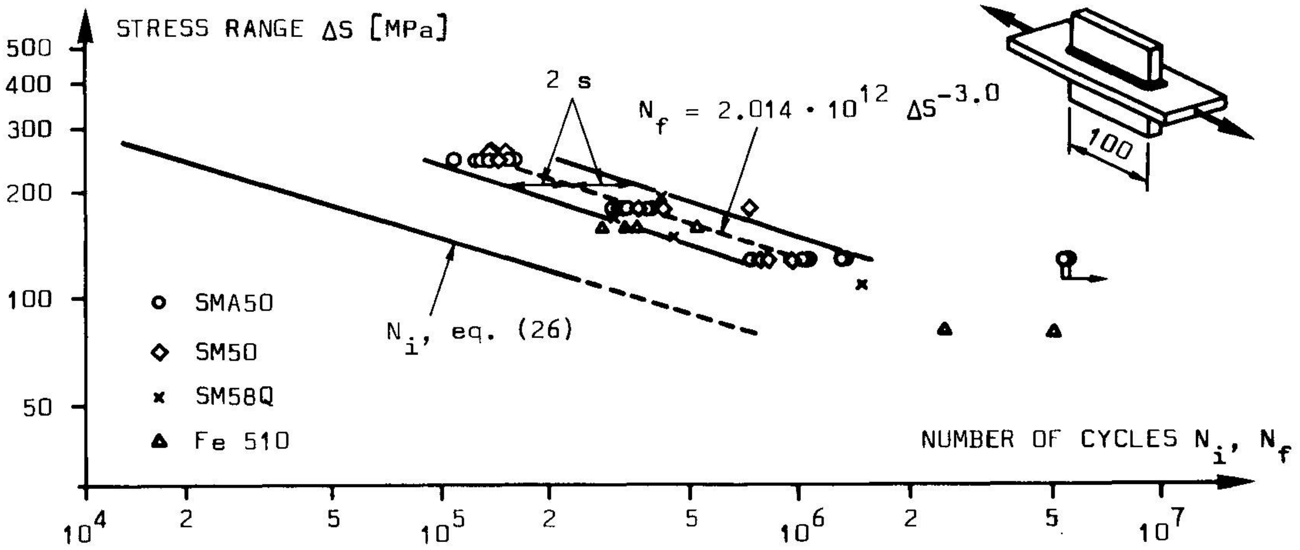


Figure 6 : Predicted FCIL and total fatigue life of gusset specimen [23].

0.16 at ΔS = 150 MPa. This agrees well with that given in [23], where the number of cycles to initiate and propagate a crack to 0.2 mm depth is considered as FCIL.

5. CONCLUSIONS

Based on the analysis developed and the comparison with test results, the following summarizing remarks may be made :

1.- Approximate formulae are derived to calculate the cyclic local strain range at the notch tip of a notched element for stress ratios of R = 0, and - 1.

2.- A generalized approximate expression for FCIL is given to indicate the effect of stress ratio on FCIL.

3.- It is possible to predict the FCIL of structural steel based on their tensile properties.

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