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## **Design Fatigue Life of Welded Cruciform Joints**

Calcul de la durée de vie des assemblages soudés en croix

Lebensdauerberechnung von geschweissten Kreuzverbindungen

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## **SUMMARY**

The paper presents an attempt to identify a simple relationship between the phenomena describing the propagation of a plane crack subjected to constant amplitude loading (Paris law type). Experimental fatigue results for welded cruciform joints plotted on a classical log N-log S<sub>r</sub> scale are used. The objective is to determine the coefficients of the Paris law through the use of global S-N curves, and to establish a simple method, based on fracture mechanics models of fatigue, for the evaluation of the design fatigue life of cruciform welded joints.

## **RESUME**

Cet article se propose d'identifier une relation phénoménologique qui décrit la propagation d'une fissure plane sous cycles d'amplitude constante (du type loi de Paris) aux résultats d'essais globaux obtenus sur des assemblages soudés cruciformes, dont les efforts sont transmis par les cordons de soudures, et tracés sous la forme classique des courbes S-N. L'objectif est double: d'une part, déterminer les coefficients de la loi de Paris à partir de cette seule information globale, d'autre part, d'établir une méthode simple de vérification de la durée de vie de ces assemblages basée sur les concepts de la mécanique de la rupture.

## **ZUSAMMENFASSUNG**

In diesem Beitrag wird der Versuch unternommen, ein phänomenologisches Verhalten, das die Rissfortpflanzung unter konstanter Schwingbreite beschreibt (entsprechend der Paris-Gleichung) mit den Gesamtversuchsergebnissen geschweisster Kreuzverbindungen, welche durch Wöhlerkurven dargestellt sind, zu identifizieren. Es werden zwei Ziele verfolgt: Einerseits die Beiwerte der Paris-Gleichung aus diesen Gesamtergebnissen zu bestimmen und andererseits eine einfache, auf der Bruchmechanik gründende, Näherungsmethode für die Lebensdauer von ermüdungsbeanspruchten Kreuzverbindungen auszuarbeiten.



## 1. INTRODUCTION

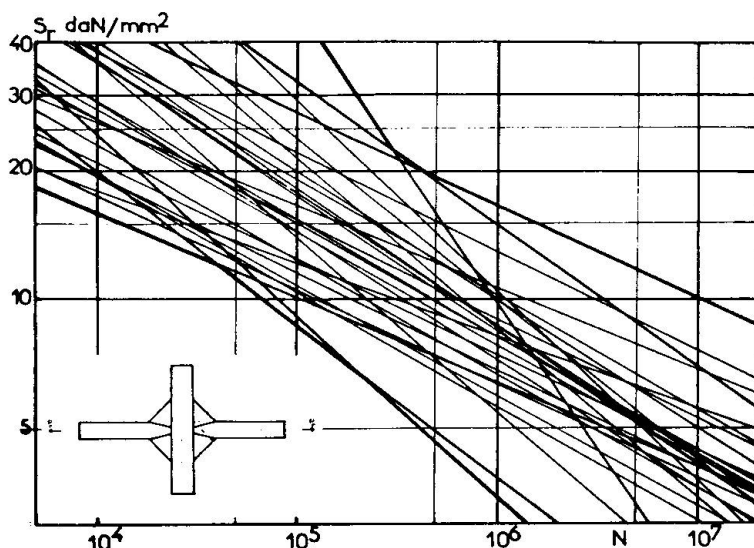
From a code specifications stand point, the design life of welded joints is checked on the basis of  $S_r$ -N Curves [1], [2], [3], which relate, for given classified details, the stress variation  $S_r$  (stress range) against the number of cycles N. This relation, known as the WOHLER line, is written as :

$$N = A S_r^b$$

A and b are two constants which are determined on an experimental basis. These coefficients depend in reality of many factors, whose influence are still not well established. This explains, the reason why fatigue test results show very wide dispersion. Figure 1, which gathers a set of S-N lines of various test series of specimens which belong to the same statistical population referred as load carrying welds of cruciform welded joints - type K4 in the conventional classification - shows the evidence for this statement.

The purpose of this paper is twofold : one is to show how the coefficients of the PARIS law, which is a fracture mechanic model of fatigue, may be deduced from global information as given by a  $S_r$ -N line, the other is to derive simplified formulae which gives the fatigue life with a reasonable good agreement for the type of welded joint considered. An example will illustrate the validity of this approximate formulation.

(fig.1)

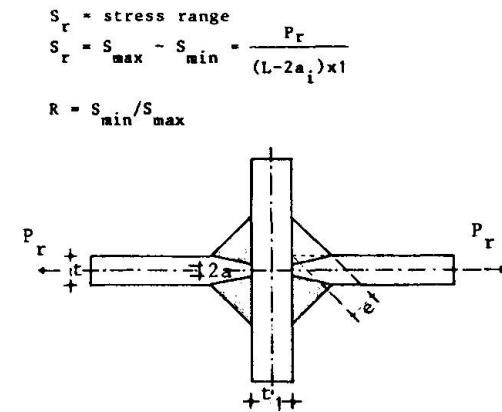


## 2. EVALUATION OF THE COEFFICIENTS OF PARIS LAW IN TERMS OF THE S-N RESULTS

### 2.1 Presentation of the joint geometry

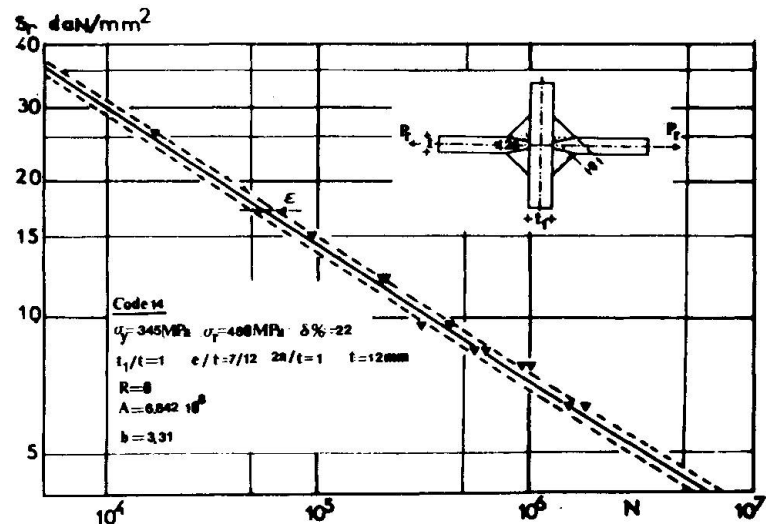
Several reasons have motivated the choice of a cruciform welded joint in which the applied load is transmitted through fillet welds (with or without lack of penetration) (fig. 2) to validate the underlined approach :

- this type of welded detail shows two crack initiation mechanisms, one at the root of weld which results from the design of the weldment, i.e., partial penetration welds, the other at the toe of the weld due to local stress concentration or defects.
- the fatigue life for such detail is mainly determined by the crack growth period (for which the PARIS law is applicable).
- and finally, for this type of welded joint, a sufficient amount of data exists in the literature, to enable us to study statistically the influence of various design variables (geometry, material, welding methods...).



load-carrying welded cruciform joint

(fig. 2)



(fig.3)

## 2.2 Calculation of the life duration on the basis of fracture mechanics analysis

In fracture mechanics models of fatigue, the initiation period is normally neglected. The crack growth from existing defects in welded joints is assumed to follow the PARIS law (the threshold value is disregarded) :

$$da/dN = D(\Delta K)^n \quad (1)$$

$da/dN$  : is the crack growth propagation from crack size  $a$ .

$\Delta K$  : is the amplitude of the stress intensity factor

$D$  and  $n$  : are two constants which depends on the material

When applying a crack propagation law, one must know the crack size from which this law is applicable. For load carrying welds (as in cruciform joints) where a lack of penetration is present, the size of the lack of penetration ( $2a$  in fig.2) is a good estimate of the initial crack size. But for a crack which originates at the toe of the weld (in case of full penetration welds) a crack size must be assessed [4]. As well, the parameters of the PARIS law  $D$  and  $n$  must to be known. The determination of these constants presents some difficulties, since the crack propagates generally in the HAZ, and this needs special fracture mechanic specimens which reproduce the HAZ of the welded zone to determine the  $D$  and  $n$  constants. To overcome this difficulty, we have used an engineering approach which consists in identifying the PARIS law with S-N lines in order to evaluate an average value of  $D$  and  $n$  (this will be explained in § 2.3).

According to the definitions given in figure 2, the life duration (or number of cycles to rupture) for crack initiating at the root may be derived from equation (1) :

$$N = \frac{P_r^n}{D} \int_{a_i}^{a_f} \frac{da}{(\Delta K)^n} P_r^{-n} \quad (2)$$

In this law the stress intensity factor ( $\Delta K$ ) may be written as :

$$\Delta K = S_r \cdot K_t \cdot \sqrt{\pi a} \cdot f(a) \cdot f(p) \quad (3)$$

$K_t$  is the stress concentration factor (S.C.F.)  $\Delta\sigma/S_r$ ,  $\Delta\sigma$  is the variation of the principal maximum stress at the notch. The evolution of the S.C.F. in terms of



the crack length may be approximated by the following relation :

$$K_t = K_o + K\left(\frac{12}{t}\right)^\alpha a^\alpha \quad (4)$$

and the geometrical correction factor  $f(a)$  which depends on the geometry of the cruciform joint, and on the size of the propagating crack may be approximated by the following relation

$$f(a) = f_o + F\left(\frac{12}{t}\right)^\beta a^\beta \quad (5)$$

In both relations, (4) and (5)  $t$  is in mm, and  $K_o$ ,  $K$ ,  $f_o$  and  $F$  have been calculated by F.E.M. [5]. In relation (3),  $f(p)$  is the correction for the existing plastic zone, which may be evaluated from [5] :

$$f(p) = \left\{ 1 - \frac{1}{2} f(a_i) \frac{1}{(1-R)^2} \frac{k_t S_r^2}{\sigma_e} \right\}^{-1/2} \quad (6)$$

It has been observed that the value of  $f(p)$  which is, in most case, close to 1 does not have a significant effect on the life duration.

Taking into account (4) and (5), (2) becomes

$$N = \frac{1}{D \cdot f^n(p)} \int_{a_i}^{a_f} \frac{da}{\left\{ \left( \frac{\sqrt{\pi a}}{L-2a} \right) \left( k_o + K\left(\frac{12}{t}\right)^\alpha a^\alpha \right) \left( f_o + F\left(\frac{12}{t}\right)^\beta a^\beta \right) \right\}^n} P_r^{-n} \quad (7)$$

Knowing the coefficients  $D$  and  $n$  of the PARIS law, the relation (7) allows us to evaluate the number of cycles to failure.

### 2.3 Evaluation of $D$ and $n$ through an identification procedure

A classical representation of fatigue test results is represented on figure 3, each point defined by the couple of values  $(S_{ri}, N_i)$  indicates a test result. Because of the significant dispersion which exists in the experimental results, a statistical treatment of the data is required to estimate the fitting curve. Generally the data are plotted on a log-log scale, and a simple linear regression is used to evaluate the unknown parameters  $(\log A, b)$  through a minimization procedure of the sum of squares errors  $\epsilon_i$  :

$$\log N = \log A + b \log S_r + \epsilon_i \quad (8)$$

The results of such statistical treatment on a large series of test results, performed on the same type of welded cruciform joints was already shown in figure 1. With the help of the definition stated in figure 2, equation (8) is written as :

$$N = \{A/(L-2a_i)^b\} \cdot P_r^b \quad (9)$$

By equating the relations (7) and (9), the coefficients of PARIS law may be evaluated as :

$$n = -b$$

$$D = \frac{(L-2a_i)^b}{A \cdot f^n(p)} \int_{a_i}^a \frac{da}{\left\{ \left( \frac{\sqrt{\pi a}}{L-2a} \right) \left( k_0 + K \left( \frac{12}{t} \right)^\alpha a^\alpha \right) \left( f_0 + F \left( \frac{12}{t} \right)^\beta a^\beta \right) \right\}^n} \quad (10)$$

In the previous relation, the values of  $b$ ,  $A$  are obtained through statistical analysis,  $L$  and  $a_i$  are experimental values, and  $k_0$ ,  $K$ ,  $f_0$ ,  $F$ ,  $\alpha$  and  $\beta$  are numerical values (see § 2.2).

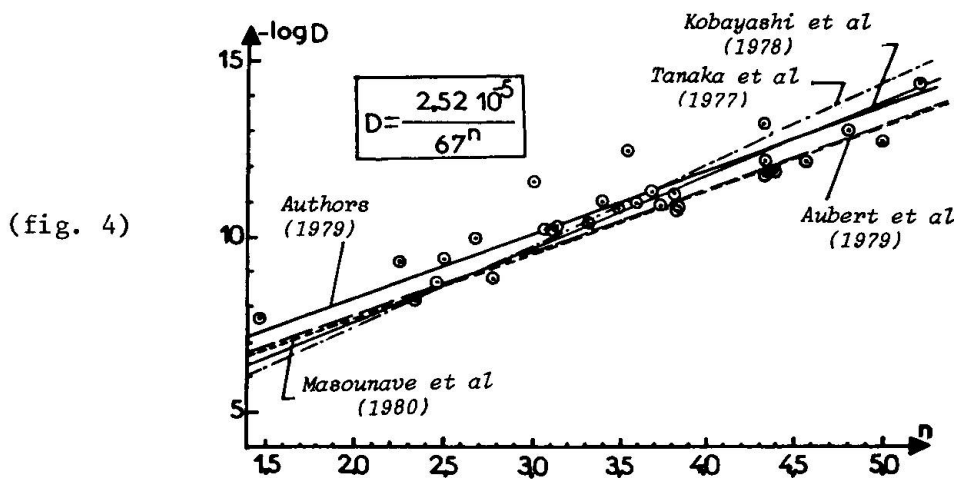
## 2.4 Results of the statistical analysis

Through the procedure that has been explained above, a set of  $D$ , and  $n$  coefficients were evaluated. Several variances analysis were performed in order to assess the significance of influencing parameters on  $D$  and  $n$  values. Results of these variance studies are summarized hereafter [5] :

- the material properties of steel (yield point, max stress, percent elongation) do not have any influence.
- the  $R$  ratio, has a slight influence which appears only when  $R > 0$ , (this conclusion is not surprising because the specimens are small).
- there exists a linear relation between  $\log D$  and  $n$ , which does not seem to depend on the grade of steel, but on the type of failure mechanism (root or toe initiation). For welded load carrying cruciform joints the relation is the following.

$$D = \frac{2,52}{(67)} n \cdot 10^{-5} \quad (\text{units : daN, mm}) \quad (11)$$

Figure 4 gives a comparison of test results and proposals from various published sources with the relation given by equation (11). The use of this equation reduces to a great extent the scatter in fatigue data.



## 3. A SIMPLIFIED FORMULAE BASED ON FRACTURE MECHANIC CONCEPTS TO EVALUATE THE LIFE DURATION OF LOAD CARRYING WELDED CRUCIFORM JOINTS.

### 3.1 Derivation of a simplified formulae

For design purposes, the evaluation of the life duration through the direct use



of the equation (7) may be too cumbersome. For an engineering approach, there is still a need for simplified formulae, which do not have the handicap in the classical approach which, for a given detail, defines "probabilistic" safe S-N lines from test results with a wide scatter. The WOHLER line concept fails in determining the influence of purely deterministic design variables, like geometry and failure mechanism for example.

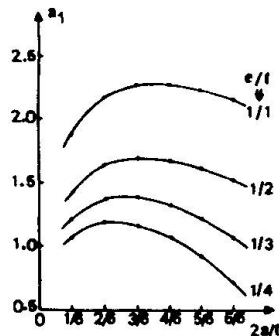
We have determined for load carrying fillet welded cruciform joints that mean life may be predicted with a reasonable agreement according to the following expression

$$N = \frac{1}{D} \cdot \bar{I} \cdot P_r^{-n} \quad (12)$$

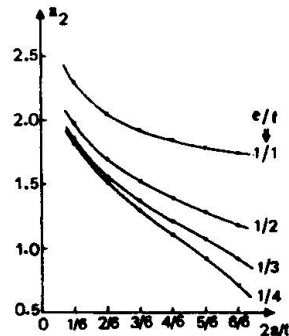
in which :

a)  $\bar{I}$  = is a factor which depends on the geometry, and  $n$ , and is given by the empirical relation :  $\bar{I} = (t/12)^{(n/2 + 1)} I_1$ , (13), with  $I = \exp(a_1 + a_2 n)$  (14)

Constants  $a_1$  and  $a_2$  are functions of the non dimensional geometrical characteristics of the welded joints (fig.2)  $t/t_1$ ,  $e/t$  and  $2a/t$  which are determined from the charts of figures 5 and 6.



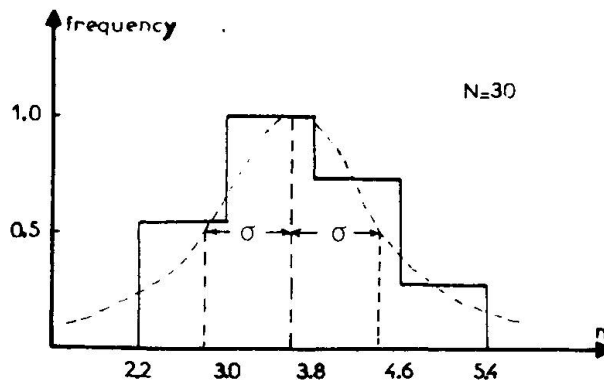
(fig. 5)



(fig. 6)

b)  $D$  is dependent on  $n$  according to relation (11)

c) the exponent  $n$  may be chosen as the mean value of the slopes of the experimental S-N lines for the sample. It has been found through statistical analysis of data provided from the literature that, for small specimens the mean value of  $n$  is 3.6 and the standard deviation is 0.8. The frequency diagram of the  $n$  values is given in figure 7 along with the associated Gaussian distribution.



(fig. 7)

d) Numerical example. As an example let us evaluate the S-N lines of the load carrying welds of two cruciform joints, whose geometries are defined in table I.

Table I

Ref	t mm	l mm	t/t <sub>1</sub>	e/t	2a/t	Crack initiations
1	25	19	1	0,54	1	weld root
2	25	22	1	0,62	1	weld root

## - Joint 1 :

from charts 1 and 2 of figures 5 and 6

for  $2a/t = 1$   $a_1 = 1,58$

$e/t = 0,54$   $a_2 = 1,23$

then equation (14) gives  $I = \exp (1.58 + 1.23 \times 3.6) = 401$

and the number of cycles equ. 12 :

$$N = \frac{(67)^{3.6}}{2.52 \times 10^{-5}} \left( \frac{25}{12} \right)^{\frac{3.6}{2} + 1} \times 401 \times P_r^{-3.6} = 4.65 \times 10^{14} P_r^{-n}$$

$P_r$  can be expressed in term of the stress range  $\sigma_r$

$$P_r = t \times l \times \sigma_r = 25 \sigma_r$$

then :

$$N = 4.432 \times 10^9 \sigma_r^{-3.6}$$

## - Joint 2 :

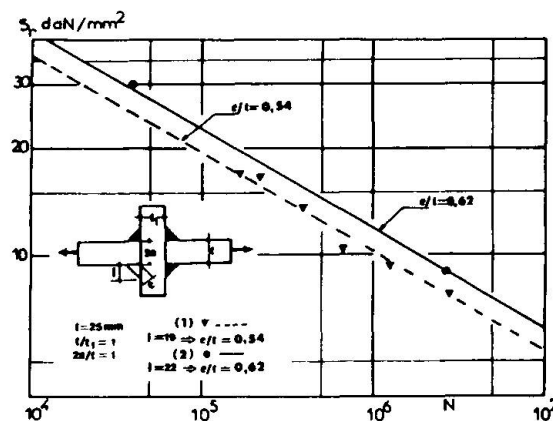
we have from figures 5 and 6

$a_1 = 1.68$  ,  $a_2 = 1.32$

and finally :

$$N = 6.60 \times 10^9 \sigma_r^{-3.6}$$

Figure 8 gives a comparison between test results and the previous calculated above S-N lines from the simplified equation 14.



(fig. 8)

## 4. CONCLUSIONS

The principal outcome of the simplified approach for the evaluation of the fatigue life of welded joints, based on fracture mechanic concepts as presented here, is the possibility of taking into account variables which are ignored by the classical WOHLER line type of approach and which may influence the fatigue life. These





variables are the geometry of the joint (thickness sizes...) and the failure mechanisms.

An identical study (not reported here) [5] has been made to evaluate the fatigue life of cruciform welded joint with non load-carrying welds. In those type of joints (referred in the literature as type K2) the crack occurs at the weld toe, partly due to stress concentration, partly due to surface defects. In such a case the applicability of fracture mechanics is bounded by the knowledge of the initial crack size. This initial crack size may be assessed through a probabilistic criterium or by assuring a good calibration of the fatigue life test results and fatigue life as evaluated by the method developed in paragraph 2. We have found [4] that the initial crack size at the weld toe may be taken as :

$a_i = 0,02 - 0,03$  mm for a submerged arc process

$a_i = 0,015 - 0,02$  mm for TIG welding

$a_i = 0,01$  when the automatic arc welds have been treated by shot-peening.

It is thought that the fracture mechanic concept may serve to derive simplified rules for evaluating fatigue life of welded joints which could improve present fatigue codes mainly based on the definition of classified S-N lines.

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Note :  $1 \text{ daN/mm}^2 = 10 \text{ N/mm}^2 = 10 \text{ MPa}$