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## **Fatigue as a Design Limit State for Bridges**

Fatigue en tant qu'état-limite de dimensionnement pour les ponts

Ermüdung als Bemessungsgrenzwert für Brücken

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### **SUMMARY**

Current methods of design for fatigue life are part deterministic for load effects and part probabilistic for resistance. An estimate is made of risk of fatigue failure, which is found to be much higher than for other ultimate limit states. This difference needs to be reconciled through reference to inspection and maintenance, and the prevention of collapse by redundant load paths.

### **RESUME**

Les méthodes existantes de dimensionnement à la fatigue sont d'une part déterministes en ce qui concerne les charges et d'autre part probabilistes pour ce qui est de la résistance. On fait une estimation du risque de rupture par fatigue, lequel est trouvé beaucoup plus élevé que pour d'autres états-limites ultimes. Cette différence doit être prise en considération en ce qui concerne les inspections et la maintenance, ainsi que la prévention des ruptures sous le passage de charges répétées.

### **ZUSAMMENFASSUNG**

Gegenwärtige Verfahren für den Dauerfestigkeitsnachweis sind deterministisch für Lasteffekte und basieren auf wahrscheinlichkeitstheoretischen Überlegungen für die Festigkeit. Das Risiko eines Ermüdungsbruches wird abgeschätzt. Es stellt sich heraus, dass es wesentlich grösser ist als dasjenige anderer Traglastbemessungen. Dieser Tatsache muss durch Überprüfung und Wartung Rechnung getragen werden. Vorsorge gegen Versagen sollte ausserdem durch die Möglichkeit einer Kräfteumlagerung getroffen werden.



## 1. INTRODUCTION

This paper is an attempt to assess the implicit risk in designing structures for adequate fatigue life in accordance with modern specifications for structural steel design, [1,2,5,18] with the intention of reconciling fatigue design philosophy with limit state design philosophy in general [4,12]. An anomaly exists between design for fatigue and the design for other ultimate limit states, with the risk for the former far greater than that for the latter.

Fatigue design is based upon actual traffic for loads, and fatigue life curves for 95% [1,2] or 97.7% [5] survival at design life. The design concept is shown in Fig. 1 in which the estimated number of cycles,  $n$ , must not exceed the design limit,  $N$ , for a representative stress range,  $S_R$ . In practice there is a probability distribution function (PDF) associated with each parameter. Current design philosophy admits the PDF for  $N$ , but assigns a deterministic value to  $n$ .

These failure rates of greater than  $0.023$  are to be compared with nominal rates less than  $10^{-3}$  for other collapse limit states. Such high fatigue failure rates are not acceptable without qualification. It is necessary to review the load and resistance models used to obtain a better estimate of fatigue reliability. The role of redundant load paths and inspection programs in controlling risk must then be taken into account. These matters are considered in the following sections.

## 2. ESTIMATION OF LOAD EFFECTS

### 2.1 Design Vehicle or Train

Cumulative damage is accounted for by the Palmgren-Miner Rule [16], allowing for the reduction or abolition of the fatigue limit which occurs under variable amplitude loading with some amplitudes above the fatigue limit [13, 2]. Artificial stress amplitudes producing equivalent fatigue damage can be used because of the linear  $\log S - \log N$  relationship employed for fatigue resistance [see Eq. 1 in section 4]. If a stress amplitude other than  $S_{RRMS}$  or  $S_{RMMS}$  (MMS = Miner's mean or root mean cube) is used, then the actual number of stress cycles has to be modified to produce the equivalent fatigue damage [10,15,22,23].

A wide range of artificial stress amplitudes are used. BS5400 [5] uses a characteristic vehicle for the estimate of  $S_R$ , this vehicle weight being less than the maximum design vehicle for bridge capacity. The design value of  $n$  bears a close relationship to the actual value occurring. Since  $n$  depends on influence length or span, the value specified in codes is supported by a background analysis of the influence of axle spacing and load spectrum on cumulative damage, represented as a correction to  $n$ .

In the AASHTO and AREA specifications [1,2], the maximum design vehicle for bridge capacity is used. This necessarily produces a larger  $S_R$  than  $S_{RRMS}$  or  $S_{RMMS}$ , so that  $n$  must be reduced below the actual number of cycles to produce the equivalent fatigue damage. Even more extreme is the UIS proposed  $\lambda_T$  for railway bridges [19]. In this the whole train is reduced in its effect to a single stress cycle  $S_R$  ( $= \lambda_T S_{R,UIS}$ ), producing the same damage as the actual spectrum of stress cycles.

The use of a characteristic vehicle helps to separate load effects from resistance. It locates the design  $S_R$  on the S-N curve at a point corresponding to actual stresses. Changes in the design S-N curve which future experience and research might bring can be accommodated without adjustment of the load effect,  $S_R$ . Secondly, fatigue damage assessment remains independent of the design

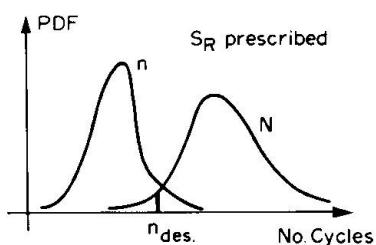
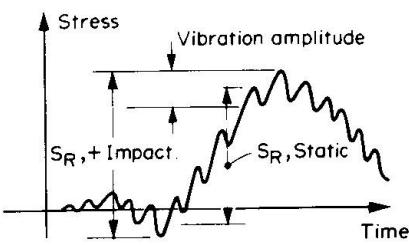
Fig. 1 Load effect,  $n$ , and Resistance,  $N$ .

Fig. 2 Vibration and Impact.

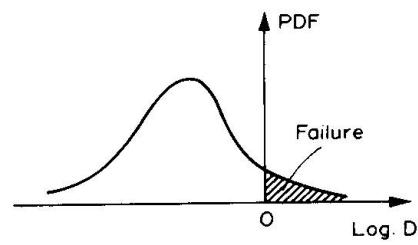


Fig. 3 Damage Function

vehicle used for ultimate load capacity, which can be changed to meet changing legislation on vehicle limits. Fatigue damage is not directly related to such changes.

Essentially, the load effect is the estimated *mean* value of stress amplitudes and frequency of occurrence associated with a given traffic density. This enables the calculation of actual fatigue damage. It is in no way equivalent to the upper 95 percentile characteristic load of floor live loads, for example.

## 2.2 Traffic Density

One mitigating factor leading to an overestimate of  $n$  is the use of categories to define traffic. Whether for cranes or bridges, the effect of using categories, with quantum steps in  $n$  between classes of use (motorway vs. arterial road, heavy vs. light crane duty, etc.) is to reduce the mean vs. design traffic density. The effect is similar to that of having discrete member sizes, standard sections, available for selection of design member resistance. The implications of load categorization will be included in the general statement later.

## 2.3 Correlation of computed with actual load effects

The simplifying models used for structural analysis are sometimes inadequate for calculating realistic values of  $S_R$  under specified loads. Many instances have been cited where measured stresses have been less than predicted by theory [7,13,17]. Where this occurs through the participation in structural action of elements normally ignored, such as cladding, topping, timber decks, ballast, sleepers and rails, then the result is wholly beneficial. Byers suggests a mean ratio of 0.93 of actual to computed stress range, with a coefficient of variation of 0.15 [7] for railway girders of longer span. Moses suggests lower ratios for shorter spans [17].

Sometimes the reduction in actual stress can be attributed to load sharing which has been ignored or underestimated by the designer. Cross girders, bracing or diaphragms might then experience stresses greater than calculated. Fatigue failures associated with these details in the past can frequently be attributed in part to a neglect of load sharing in the analysis. There is a case for allowing for the difference between calculated and actual values of  $S_R$ . Current codes based upon the  $S_R$  concept do not take advantage of this factor.

## 2.4 Impact Factor

The use of the maximum impact factor required for static strength on all loads in the calculation of fatigue load effect naturally leads to overestimates of damage. Average impact factors, which are much less than maximum values, should be used. To achieve the maximum a serious imperfection in the riding surface is required. A common dynamic response of a bridge to a passing vehicle is a vibration arising from the interaction of vehicle suspension, track roughness and bridge deflection. This vibration is superimposed on the basic static



response, amplifying the static  $S_R$  by the amplitude of the vibration, and adding many cycles of stress of the amplitude of the vibration (Fig. 2). Usually, the amplitude of the vibration is small compared with the basic static  $S_R$ , so that the contribution of the vibration to fatigue damage is not great. The probability distribution of stress amplitudes is, however, changed from the Raleigh distribution assumed in the AREA specification, [10] to some form of exponential distribution, which measurements confirm [3].

Only in BS5400 with the standard fatigue vehicle for highway bridges is the question of impact factor circumvented in damage calculations. Statistical data on impact factor should be used in formulating the load to be used in assessing fatigue damage. Byers estimated the additional load on railway bridges from impact to be 14% with a coefficient of variation of 0.61 [6]. For cranes the picture is more obscure. Oscillations associated with acceleration and deceleration of moving parts can be very significant, and sensitive to damping, so that quantitative estimates of impact factors and cumulative damage for cranes can be very elusive.

### 3. ESTIMATION OF FATIGUE RESISTANCE

#### 3.1 Statistical Properties of Fatigue Life

To obtain the lower confidence survival curves used in fatigue design, the laboratory data must be evaluated. A problem here lies in reconciling test results from different sources. Each series may have a low coefficient of variation,  $V_N$ , on  $\log N$  but differing mean values, leading to a higher  $V_N$  when the results are taken together. Fisher et al. [8] find  $V_N = 0.101$  for their own tests for fatigue life at welded cover plate ends, but the scatter with other test results taken into account is much larger [8,14,20]. Diversity in fabricating and testing procedures leads to this systematic difference in results between laboratories, and it is proper that fatigue life characteristics should be based upon several sources in order to include this diversity in the statistical characteristics. There is a serious possibility that the data base does not include adverse environmental effects of actual structures in service. Tests do show differences between field and laboratory [11].

Only BS5400 publishes the statistical assumptions on which the design curves are based.  $V_N$  varies from 0.179 to 0.251, values which are much higher than for some individual test series. The design curves of the American and British Codes are fairly consistent on the same details. More detailed comparison is not possible on the published data.

#### 3.2 Identification of Detail

It is important to place a fatigue sensitive detail in the correct fatigue category. Placing the detail in the adjacent category to the correct one can lead to a large overestimate or underestimate of life. The maximum error can be about 60% reduction from estimated life to true life using AASHTO and 42% reduction using BS5400. The difference arises from the fact that the American specifications use four fatigue categories for the same details described in seven categories in the British Code. The error applies to the constant  $K$  in the expression for fatigue life (Eq. 1). In all specifications the increments in  $\log K$  between categories are unequal so that the consequence of incorrectly identifying a fatigue detail are statistically variable.

Some allowance should be made for the designer making an error in classifying detail. This might be described as a professional error [12, 21] in the estimate of  $K$ .

### 3.3 Size Effect

A specific factor in transferring laboratory data to field application lies in the usually larger size of the design structure compared with the test pieces used for the data base. Fisher has proposed a Category E' lower than E, where  $K_{E'} = 0.44 K_E$ , for the same cover plate details as E where plate thicknesses exceed 19 mm [11]. This is a large change in fatigue life, solely due to size effect. A similar question must be asked about the size effect for other welded details, but more research is required to answer it.

In the ensuing analysis it is assumed that size effect is considered in the design so that there is no shift in the mean value of K due to it. However, some uncertainty of modeling must be attached to this assumption.

### 3.4 Design Life

There is no agreement to be found on design life, which is 120 years in BS5400, 80 years in the AREA specification, and unstated in the AASHTO specification, which instead specifies the number of stress cycles to be endured. Whether design life is a genuine target for survival or merely a strategem for achieving some adequate life less than stated has never been elucidated. The concept is fundamental to fatigue design, which is based on a probabilistically non-stationary process, unlike most other limit states, but it is beyond the scope of this paper to attach probabilistic significance to it.

## 4. ESTIMATE OF RELIABILITY OF PRACTICAL FATIGUE DESIGN

### 4.1 Damage Equation

The usual fatigue life relationship

$$N = K_2 S_r^{-m} \quad (1)$$

where  $K_2$  is a constant yielding an appropriate confidence limit for survival (97.7% for two standard deviations), can be transformed to a damage expression

$$D = K^{-1} n S_r^m \quad (2)$$

where  $n$  is the actual number of cycles applied. When the damage,  $D$ , exceeds unity, failure occurs. All terms of Eq. 2 are random functions with appropriate probability distributions.

Current design philosophy obtains what is superficially an expected value of  $n$  and  $S_r$ , and these are used deterministically to define a load effect,  $S$ . The scatter in fatigue resistance,  $R$ , is accommodated by modifying  $K_0$  to  $K_2$  - a form of partial resistance factor [21]. To bring fatigue design into line with other limit state criteria requires an evaluation of the statistical significance of all the factors mentioned (and no doubt others overlooked) to establish the reliability on a first order basis [4].

Using lognormal distributions, which fit at least some of the data quite well,  $\log D$  becomes a direct measure of reliability (Fig. 3), with all values of  $\log D$  greater than zero representing failure.

### 4.2 Estimation of Reliability of Current Design Procedure

The reciprocal of  $D$  is a measure of safety or reliability. It is merely necessary to tabulate the influence of the various parameters in terms of mean values and coefficients of variation as follows:

Item	Parameter	Mean	$V(\log)$
1. Design Vehicle/Train Specification	$S_{Rd}/S_R$	1.0	0.030
2. Traffic density	$N_d/N$	0.75	0.076
3. Actual vs. computed $S_R$	$S_{Rd}/S_R$	0.93	0.150
4. Impact factor	$S_{Rd}/S_R$	0.88	0.031
5. Fatigue life of detail	$N_d/N$	0.398	0.200
6. Fatigue category identification	$K_o/K_d$	1.0	0.100
7. Size effect	$K_o/K_d$	1.0	0.100

These figures, arbitrary at times, are derived as follows:

- Item 1: Allows for misrepresentation of the load effect by the design vehicle or train. It could include errors due to deficiencies in Miner's rule and the method of counting stress cycles.
- Item 2: Assumes design traffic density is 95% confidence limit with mean value in the middle of the traffic range to the next lower traffic range - consistent with BS5400 Highway Traffic designations.
- Item 3: Uses Byers' figures for railway bridges [7].
- Item 4: Uses Byers' figures of 1.14 impact factor [7] compared with typically 1.30 used in design [BS5400], and Byers value of  $V_S$ .
- Item 5: These values are chosen to link the determination of D with the Code design point, N, of Mean minus two standard deviations of log N. The value of  $V_N = 0.2$  is an average figure for a range of fatigue categories, and  $\log 0.398 = -2 \times 0.2$  for consistency.
- Item 6: The value of  $V = 0.1$  is a compromise between (a) an assumed 80% probability of selecting the correct fatigue category with an average factor of 0.631 to the next category in BS5400 and (b) an assumed 85% probability of correct selection with an average factor of 0.501 in the AASHTO specification.
- Item 7: Lack of information prevents an estimate of mean ratio other than 1.00 being made, and  $V_K$  corresponds to a category identification error.

From the tabulated values, assuming  $m = 3$ ,

$$\bar{D} = 0.75 \times 0.93^3 \times 0.88^3 \times 0.398 = 0.164$$

$$V_D = [3(0.03^2 + 0.15^2 + 0.031^2) + 0.076^2 + 0.2^2 + 0.1^2]^{1/2} \\ = 0.373$$

$$\beta = \frac{-\log \bar{D}}{V_D} = \frac{0.786}{0.373} = 2.11$$

For comparison the present semi-probabilistic practice would only consider item 5 of the table, with  $\bar{D} = 0.398$ ,  $V_D = 0.200$ ,  $\beta = 2.0$ .

The above result could have been couched in the standard  $R - S$  formulation, with the first four items modifying  $S$  and the last three modifying  $R$ , with essentially the same result.

#### 4.3 Discussion

A  $\beta$ -index value of 2.11 is low compared with values accepted for other forms of collapse, where  $\beta$  ranges above 3.1 and is typically 3.5.  $\beta = 2.11$  represents a nominal probability of failure of 0.0221, compared with 0.000 233 for  $\beta = 3.5$ . Such a low  $\beta$ -value is unacceptable by conventional standards. Account must be taken of three factors so far ignored, if it is to be justified. These are:

1. The significance of inspection and maintenance in preventing collapse, and the cost-benefit implications.
2. The time-dependent nature of the risk of failure, such that an economically useful life is to be expected before collapse or repair.
3. The significance of redundant load-path design in preventing collapse in spite of individual fatigue failures.

It can be shown for a design life exceeding forty years with  $\beta \approx 2$  that even if the cost of failure includes replacement of the structure the marginal cost of modifying the structural details to improve the fatigue category one level must be very low for it to be economically justified. This assumes that there is no risk to human life in the fatigue failure. For this to be possible either inspection sufficient to detect all major fatigue cracks in time must be guaranteed, or the structure must have redundant load paths.

All specifications disregard the role of inspection and maintenance in the formulation of fatigue design rules. In the Ontario Highway Bridge Design Specification [18] redundant load paths are mandatory, thereby rendering the current rules with  $\beta \approx 2$  acceptable, but not necessarily optimum. Only in the AASHTO specification is the risk of fatigue failure in non-redundant load path structures recognised by an approximate reduction in  $S_R$  by 40%. Based on the tabulated parameters above, this leads to an estimate of  $\beta = 3.89$ , a figure quite acceptable by limit state design standards.

The above generalised reliability estimate can be invalid in particular circumstances. For example, using BS5400 to design a girder where maximum traffic is clearly anticipated, e.g. in the slow lane of a motorway, using the design vehicle for fatigue and a rigorous analysis for stresses could lead to the mean correction factors in the table for items 2, 3, and 4 being converted to unity. This would lead to  $\bar{D} = 0.398$ ,  $V_D = 0.373$ ,  $\beta = 1.073$ ;  $p_f = 0.142$ . Such a high risk is likely to be unacceptable, although it could be justified on economic grounds if failure of the bridge as a whole could be prevented in the event of an individual fatigue failure.

## 5. CONCLUSIONS

5.1 Present practice in fatigue design which treats stress amplitudes and the number of load cycles deterministically falls short of an adequate limit state format.

5.2 There is lack of consistency in treating factors affecting load effects, such as correlation of measured with theoretical stress amplitudes, impact factor and traffic density. This prevents fatigue design of consistent reliability for different structures. Ideally, the fatigue resistance characteristics of structural steel is the same for all steel structures, and independent of the load effects.

5.3 There is need to evaluate a professional factor allowing for designers' errors in applying the design rules.

5.4 This study indicates an effective safety index,  $\beta$ , not much more than 2.0 in current practice. This represents an unacceptably low reliability unless inspection and maintenance are considered, and/or redundant load path design is employed to prevent catastrophic collapse.



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