

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 37 (1982)

**Artikel:** Fatigue strength of joints with bolts in staggered patterns  
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**DOI:** <https://doi.org/10.5169/seals-28972>

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## **Fatigue Strength of Joints with Bolts in Staggered Patterns**

Résistance à la fatigue des assemblages boulonnés de type échelonné

Ermüdungsfestigkeit von Verbindungen mit versetzt angeordneten Schrauben

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## **SUMMARY**

In connections made with high-strength bolts in staggered patterns the fracture path, under fatigue loading, is frequently different from that under static loading. As a result of fatigue tests on a large number of specimens, with various stagger patterns, it was found that, irrespective of the pattern, the fatigue stress intensity calculated on the basis of the gross sectional area is almost constant. An allowable stress range for high-strength bolted connections based on the gross sectional area is recommended.

## **RESUME**

Dans les assemblages réalisés avec des boulons à haute résistance de type échelonné, le mode de rupture sous des charges de fatigue est souvent différent de celui sous des charges statiques. Les résultats d'essais de fatigue sur un grand nombre d'éprouvettes, avec différents types d'échelonnement, ont montré que, indépendamment du type, l'intensité de la contrainte de fatigue, calculée sur la base de la surface de la section brute, est presque constante. Une contrainte admissible dans de tels assemblages basée sur la surface de la section brute, est recommandée.

## **ZUSAMMENFASSUNG**

Bei hochfesten Schraubenverbindungen mit versetzt angeordneten Schrauben ist der Verlauf der Bruchlinie unter Ermüdungsbelastung oft verschieden gegenüber dem Bruchverlauf unter statischer Belastung. Als Ergebnis einer grossen Anzahl Versuche an Prüfkörpern mit verschiedener Schraubenanordnung wurde festgestellt, dass die Ermüdungsspannungsintensität, berechnet am Bruttoquerschnitt, unabhängig vom Verbindungstyp ist und praktisch konstant bleibt. Es wird demnach für hochfeste Schraubenverbindungen ein zulässiger Spannungsbereich, basierend auf der Bruttoquerschnittsfläche, empfohlen.



## 1. INTRODUCTION

According to the current design specifications, the stress intensity working in the sectional area of a tension member connected by rivets with a stagger pattern is checked on the basis of the net sectional area which is calculated by so-called V. H. Cockrane's Formula as follows; "The net width for any chain of holes extending progressively across the part shall be obtained by deducting from the gross width the sum of the diameters of all the holes in the chain and adding, for each gage space in the chain, the quantity;  $p^2/(4g)$  where  $p$ = pitch of any two successive holes in the chain, and  $g$ = gage of the same holes". This is currently applied also to the design of high-strength bolt joints in tension members. It has been used for a long time and may be reasonable in the case where loads are statically applied. It is, however, known that the type of fracture of a tension member connected with rivets or high-strength bolts with a stagger pattern under repeated loading is different from that under static loading. Under the fatigue loading crack proceeds generally from the bolt or bolts in the first row straightly in the direction perpendicular to the axial tensile stress.

The authors investigated the fatigue strength of specimens which were spliced with high-strength bolts arranged in a stagger pattern in order to find a formula that can be applied to such cases. In the test specimens the pitch between bolt rows and the gage between bolt lines were varied in order to clarify their influence on the fatigue strength.

## 2. SPECIMENS AND APPARATUS

The dimensions, designation and number of the specimens are shown in Fig 1. All of them are connected with a double splice. The specimens of Series A are provided with three bolt lines and the pitch between the first bolt row and the second one varies from 10 to 30 mm, while the gage between bolt lines is kept invariable, 35 mm. The specimens of Series B are provided with five bolt lines and the other conditions are identical with those of Series A. The specimens of Series S are similar to those of Series A, except that the gage is changed; 30 mm in S-1 and 40 mm in S-3, while the pitch between the first row and the second row is kept invariable, 20 mm. Together with A-3 the test results of this series were expected to reveal the influence of the gage length.

All the base plates and splice plates are 6 mm thick and their physical and chemical properties conform to the requirements of JIS-SS 41. According to the static test on the coupon specimens the yield point and the ultimate strength are about  $34 \text{ kg/mm}^2$  and  $46 \text{ kg/mm}^2$ , respectively. Plates of 108 mm in width and 16 mm in thickness were attached to the base plates by welding so as to fit the grip of the testing machine. The surface of the base plates and the splice plates were shot-blasted in a roughness of  $50\mu$  on average. High-strength bolts of F10 T and 12 mm in nominal diameter were used. They were tightened by a torque wrench with an axial tension up to 6.3 tons.

In the fatigue test, the specimens were tested by a 100 ton-fatigue machine in a tension range from  $2 \text{ kg/mm}^2$  to a maximum at a rate of 600 cycles per minute. Prior to the fatigue test, the static test was conducted on two specimens from each kind.

## 3. TEST RESULTS

### 3.1 Static Test

Because the main purpose of the investigation conducted this time is the fatigue

behavior, the results of the static test will be briefly presented only for comparison to the fatigue test results. There are roughly two kinds of fracture type according to the difference of geometric pattern of bolt arrangement; straight type and zigzag type. Fig 2 shows the classification of all the specimens into the two fracture types. Figs 3, 4 and 5 give the ultimate stress intensity on the basis of the net sectional area calculated by Cockrane's Formula, the one on the basis of the fracture sectional area and the one based on the gross sectional area, respectively. (Photos 1 and 2)

### 3.2 Fatigue Test

Crack started from the bolt hole or holes on the first row in the direction perpendicular to the applied force, that is, at right angles with the plate surface and the side edge. In some of the specimens the crack passed throughout across the whole section. (Photos 3 and 4)

According to some fatigue tests conducted by other investigators, however, the crack did not started from the hole edge, but from the portion of the base plate just under the front edge of a bolt-washer.

Figs 6, 7 and 8 show the relations of all the specimens between the number of loading cycles of fracture formation and the stress intensity. In Fig 6 the stress is expressed on the basis of the net area calculated by Cockrane's Formula, in Fig 7 on the basis of the fracture area (the net area across the first row of holes) and in Fig 8 based on the gross sectional area. The correlation coefficients in reference to the straight line drawn by means of the least squares method are 0.565 in Fig 6, 0.886 in Fig 7 and 0.867 in Fig 8.

Figs 9, 10 and 11 give the relations between the stagger angle  $\theta$  (See Fig 1) and the fatigue strength in two million cycles of loading repetition. The areas referred to for the stress calculation in Figs 9, 10 and 11 are the same as those in Figs 6, 7 and 8, respectively. In Figs 9, 10 and 11 the results of test series previously conducted by the authors on the specimens provided with only one bolt line, designated as TA, and those with four bolts in each row, designated as TB, are added for comparison.

## 4. DISCUSSION

### 4.1 Static Test

In the static test the ultimate stress intensity shown in Fig 3 decreases as the stagger angle  $\theta$  increases. It means that the Cockrane's Formula is not quite adequate, but at least up to about 45 degrees the values are considerably higher than the ultimate strength obtained from the coupon test specimens,  $46 \text{ kg/mm}^2$ , thus giving conservative values.

Fig 4 shows a similar tendency to Fig 3, but there is a great discrepancy between the result of Series A and that of Series B. Moreover, this method can not be applied to an actual design, because the fracture path can not be determined before the test is done.

Fig 5 show that the values are not constant with variation of  $\theta$  and, moreover, considerably lower than  $46 \text{ kg/mm}^2$ . Therefore, the stress calculation disregarding the deduction of the sectional area due to bolt holes is not adequate.

In conclusion, it will be better to develop a new calculation method for the static strength of the connection with high-strength bolts with a stagger pattern. But in order to formulate a new one, much more test should be conducted, varying



the conditions such as bolt patterns, plate width, plate thickness and stress.

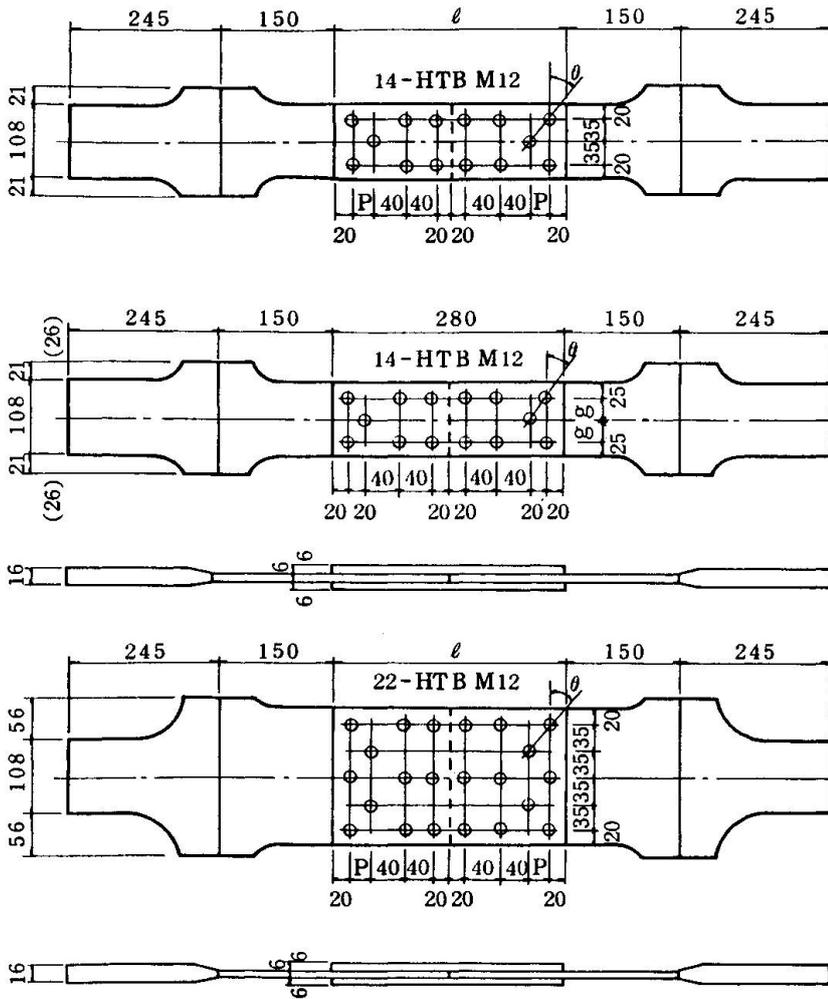
#### 4.2 Fatigue Test

In the fatigue test, while the values in Fig 6 have a considerable scattering ( $c = 0.565$ ), those in Figs 7 and 8 both have a small scattering ( $c = 0.886$  and  $0.867$ , respectively). In Fig 9 the fatigue strength is not constant, but decreases as the stagger angle  $\theta$  increases. It may be concluded from both Figs 6 and 9 that Cockrane's Formula is not adequate to predict the fatigue strength of bolt connection with a stagger pattern.

Although the values in Fig 7 (based on the fracture area) scatter to almost as small an extent as those in Fig 8 (based on the gross area), the values in Fig 11 (based on the gross area) are more constant than those in Fig 10 (based on the fracture area), only  $2 \text{ kg/mm}^2$  ( $17.5$  to  $19.5 \text{ kg/mm}^2$ ) in variation, irrespective of the stagger angle  $\theta$ , including the values obtained from the test specimens with a square pattern. It is, however, about two thirds of the fatigue strength of a plain plate. It reveals that the stress is raised in the neighbourhood of the bolts.

#### 5. CONCLUSION

- In the bolt connections with a stagger pattern that can be applied to a practical design, the fatigue fracture type is different from the static one. Even in the case where a specimen fractures in a zigzag type under statical loading, it fractures across the bolt holes (or the portion under the washer edge) in the direction perpendicular to the axial stress.
- Irrespective of the stagger pattern that can be applied to practical design of connection, the difference of fatigue strength of two million cycles is found to be as small as  $2 \text{ kg/mm}^2$  ( $17.5$  to  $19.5 \text{ kg/mm}^2$ ), if it is expressed as a stress on the gross sectional area. In conclusion it is recommended that the stress intensity should be checked based on the gross sectional area in reference to a suitable fatigue allowable unit stress, say,  $15 \text{ kg/mm}^2$ .
- According to the results of static test conducted by the authors, Cockrane's Formula which have been currently used is found to be conservative but not sufficiently adequate. However, in order to establish any new better formula that can be more universally applicable, much more tests will be required.



**SERIES A**

	No. of Test			
	p	$\theta$	S	F
A-1	30	40.6	4	5
A-2	25	35.5	3	5
A-3	20	29.7	2	5
A-4	15	23.2	2	5
A-5	10	15.9	2	5

$g = 35\text{mm}$ , constant

**SERIES S**

	No. of Test			
	g	$\theta$	S	F
S-1	30	33.7	2	5
S-2*	35	29.7	2	5
S-3	40	26.7	2	5

$p = 20\text{mm}$ , constant

\* S-2 is the same as A-3

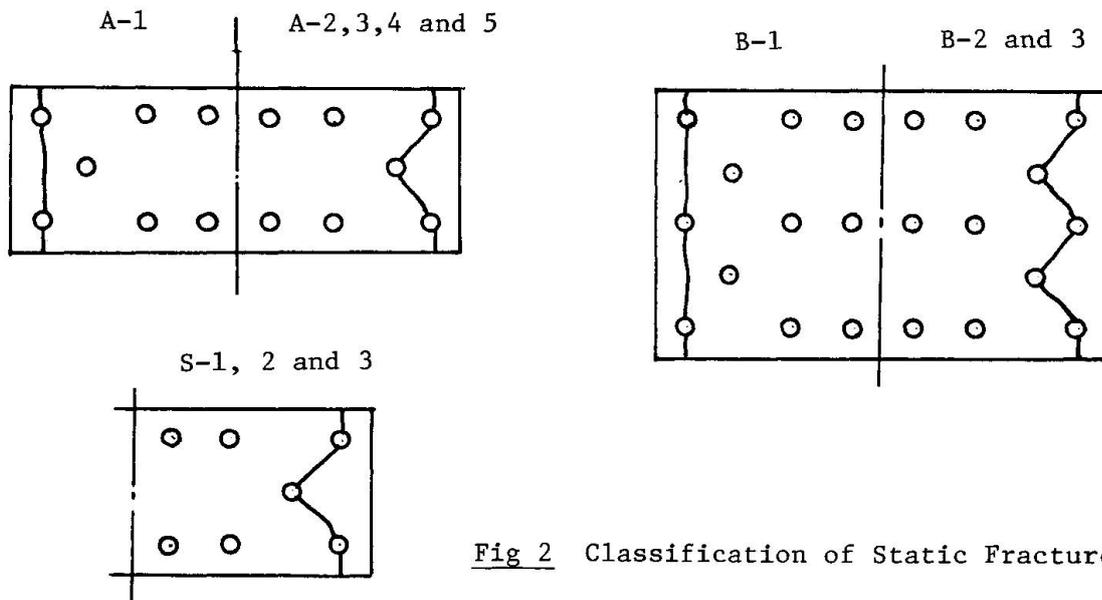
**SERIES B**

	No. of Test			
	p	$\theta$	S	F
B-1	30	40.6	2	5
B-2	20	29.7	2	5
B-3	10	15.9	2	5

$g = 35$ , constant

S ; Static Test  
F ; Fatigue Test

**Fig 1** Designation, Configuration and Number of Test Specimens



**Fig 2** Classification of Static Fracture Types

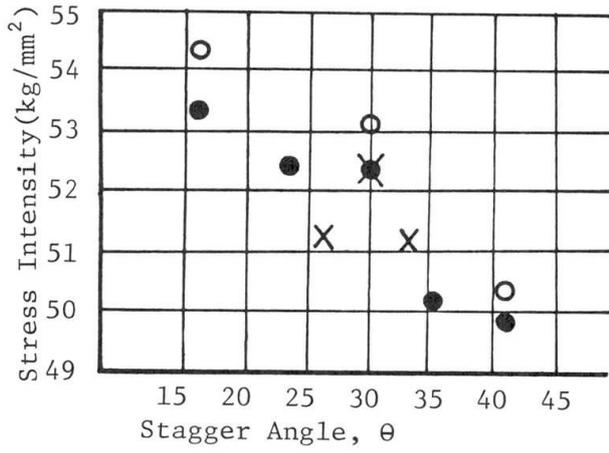


Fig 3 Relation between Stagger Angle and Static Stress based on Cockrane's Formula

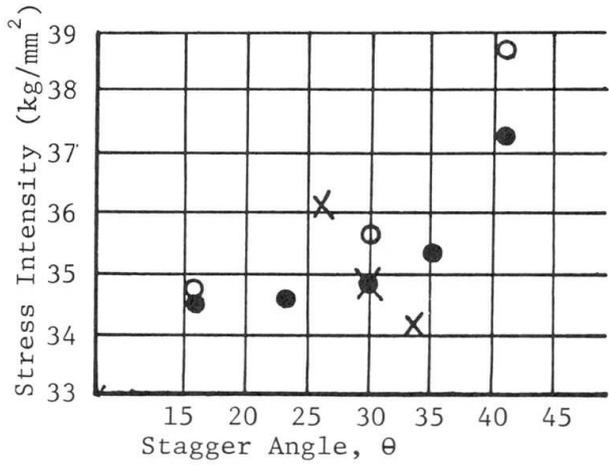


Fig 5 Relation between Stagger Angle and Static Stress based on Gross Sectional Area

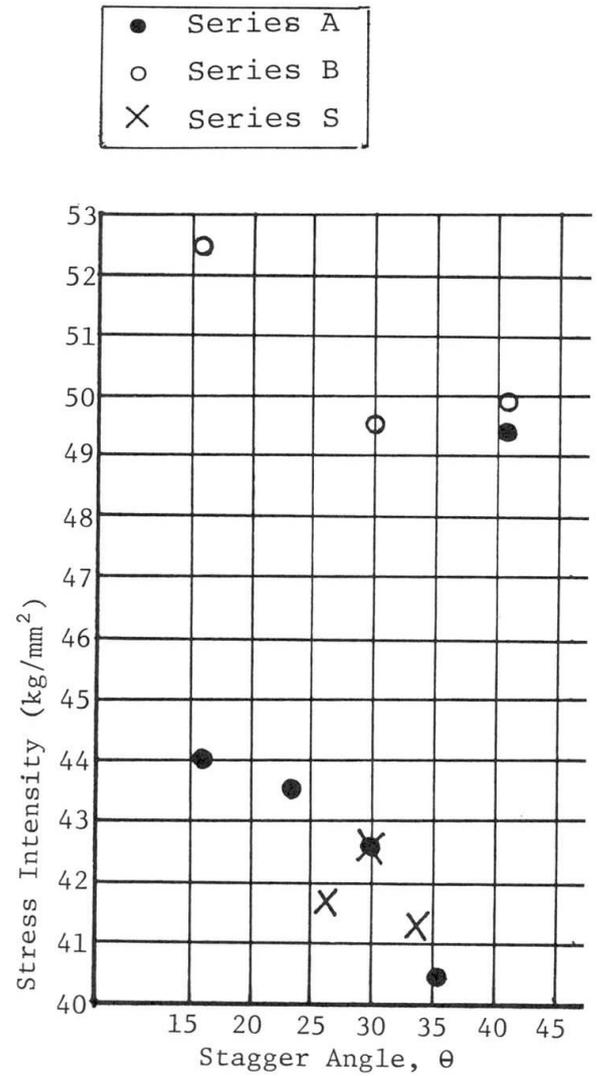


Fig 4 Relation between Stagger Angle and Static Stress based on Fracture Sectional Area

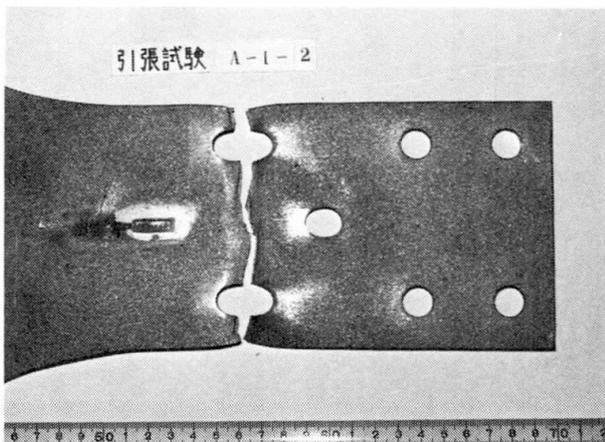


Photo 1 Static Fracture Example 1

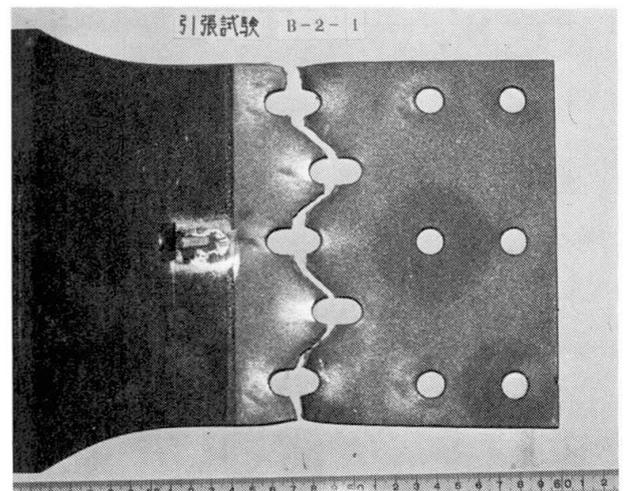


Photo 2 Static Fracture Example 2

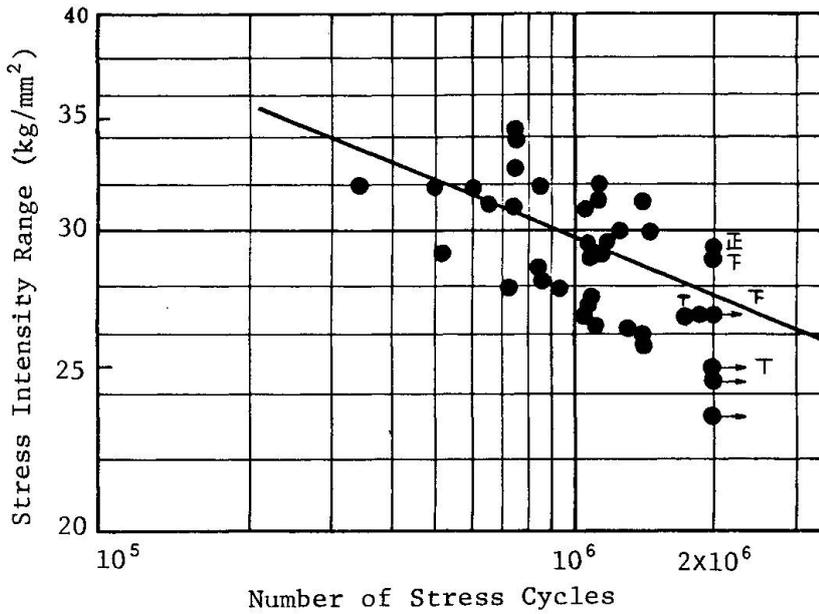


Fig 6  
Relation between Number of Loading Cycles and Fatigue Stress based on Cockrane's Formula

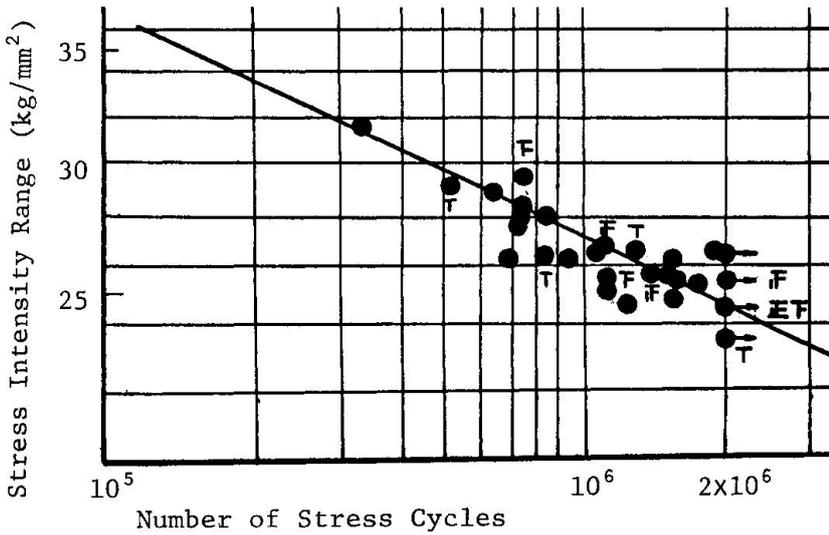


Fig 7  
Relation between Number of Loading Cycles and Fatigue Stress based on Fracture Sectional Area

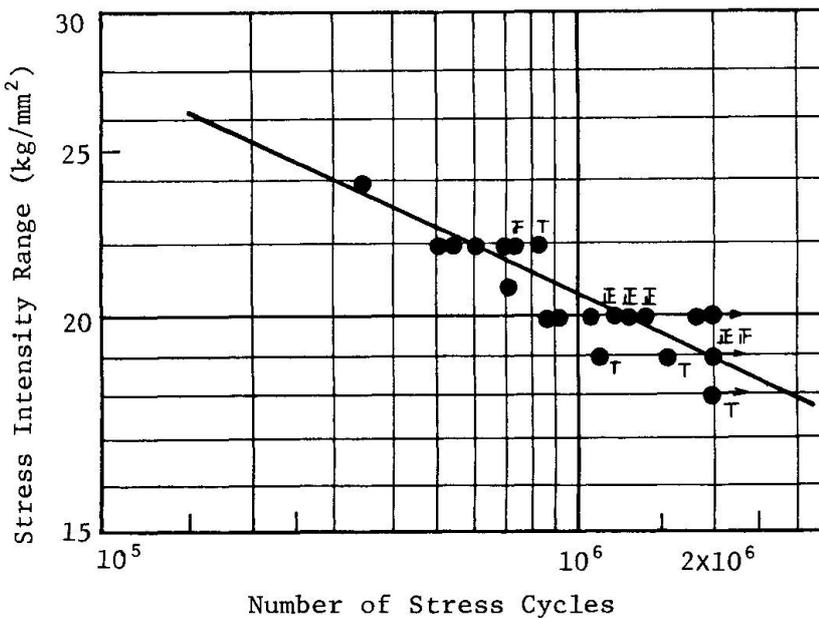


Fig 8  
Relation between Number of Loading Cycles and Fatigue Stress based on Gross Sectional Area

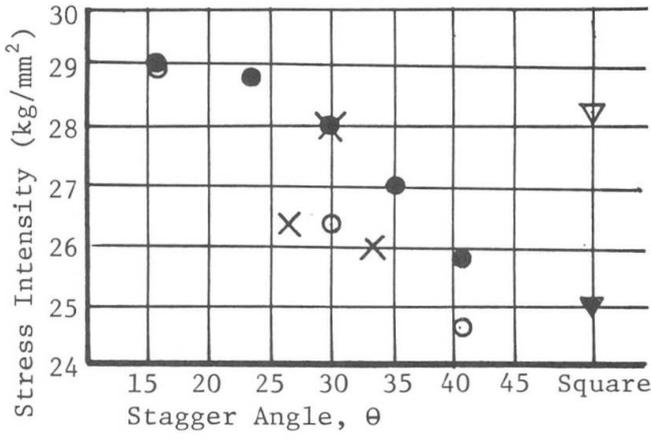


Fig 9 Relation between Stagger Angle and 2-Million Fatigue Stress based on Cockrane's Formula

●	Series A	} Stagger Pattern
○	Series B	
×	Series S	} Square Pattern
▼	Series TA	
▽	Series TB	

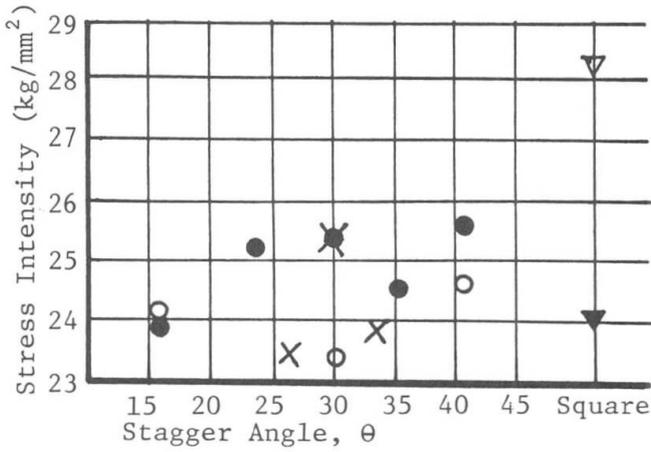


Fig 10 Relation between Stagger Angle and 2-Million Fatigue Stress based on Fracture Sectional Area

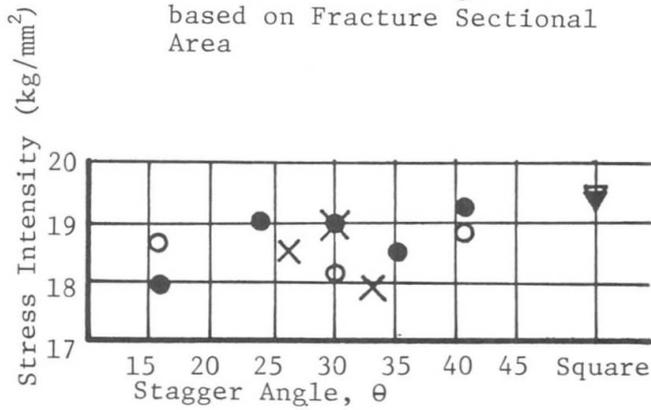


Fig 11 Relation between Stagger Angle and 2-Million Fatigue Stress based on Gross Sectional Area

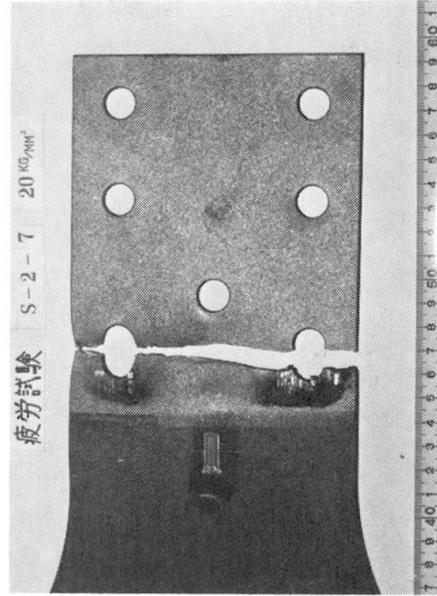


Photo 3 Fatigue Fracture Example 1

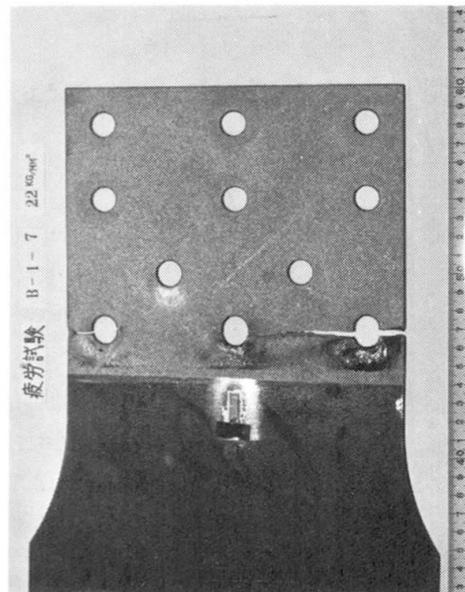


Photo 4 Fatigue Fracture Example 2