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Reliability of Snow Roof Load Assessment

Fiabilité des hypothèses de charges de neige

Zuverlässigkeit bei der Schätzung von Schneelasten

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SUMMARY
The paper deals with uncertainties regarding roof snow assessment according to present codified procedures. It comments upon the statistical calculation of annual maximum snow depths with different mean return periods for a given site, upon the determination of the specific gravity of snow on statistical snow accumulation factors and the safety of the structures against snow load on a second moment format.

RESUME
Les charges de neige admises dans les codes actuels ne correspondent pas toujours à la réalité. L'article traite des valeurs statistiques de profondeur de neige maximum qui peuvent se rencontrer en un endroit donné, pendant une certaine période. D'autres paramètres sont pris en considération: poids spécifique de la neige, facteur d'accumulation de la neige, sécurité des structures vis-à-vis des charges de neige.

ZUSAMMENFASSUNG
Der Beitrag behandelt die Unsicherheiten einer Schätzung von Dach-Schneelasten aufgrund der heutigen Richtlinien. Die Autoren kommentieren die statistische Berechnung der maximalen jährlichen Schneehöhe mit verschiedenen mittleren Wiederkehrperioden für ein bestimmtes Gebiet, die Bestimmung des spezifischen Gewichtes von Schnee aufgrund statistischer Angaben, die Effekte von Schneeverwehungen ab Dächern, die Ansammlung von Schnee und die Sicherheit von Tragwerken bezüglich Schneelasten.

1. STATISTICAL DISTRIBUTION OF SNOW DEPTH ON THE GROUND

The statistical analysis makes use of the annual maxima of snow depth on the ground. They are considered to be independent and represent a random variabile, not a time stochastic process.

The annual extremes of snow depth for structural design are defined by their mean return period $\bar{T}$, in years, within a usual range of about two years (i.e.mean of annual maxima) and one hundred years. A probability of exceeding in one year corresponds to thease values:

$$P_{1\ year}(>) = \frac{1}{\bar{T}}$$

and in N years, in the hypothesis of annual maxima independence:

$$P_{N\ years}(>) = 1-(1-\frac{1}{\bar{T}})^N \qquad (1)$$

that is:

$\bar{T}$, years	2	1o	2o	3o	5o	1oo
$P_{1year}(>)$	o,5	o.1	o.o5	o.o33	o.o2o	o.olo
$P_{in\ 3o\ years}(>)$	1	o.958	o.785	o.638	o.455	o.26o
$P_{in\ 5o\ years}(>)$	1	o.995	o.923	o.816	o.636	o.395
$P_{in\ 1oo\ years}(>)$	1	o.999	o.994	o.966	o.865	o.634

Extreme values distributions for maxima of type I, or Gumbel, and of type II, or Fréchet, as well as lognormal distribution, Pearson type III, etc., are used in statistical analysis of annual of snow depth. The higher the values for $\bar{T}$, the greater the differences between the snow depth fractiles calculated in various distributions. As a rule, the more the distribution upper tail tends asymptotically more smoothly towards zero, the greater the values of fractiles defined with the same period $\bar{T}$.

For the mean return periods of more than 3o years, the fractiles calculated in the Fréchet distribution are greater than the ones in the Gumbel distribution, while the latter are greater than the ones calculated in lognormal distribution or Pearson type III, etc.

Usually there are only subiective reasons for preferring one to the other of the above mentioned distributions. The Gumbel distribution seems to be more adequate for the advantages of certain mathematical connexions and continuities in the analysis of safety against snow load. In this case, in terms of mean m_1 and coefficient of variation V_1 of annual maxima of snow depth on the ground, the fractiles x_p of snow depth defined by probability p of having smaller values than x_p in N years are calculated by the formula:

$$x_p = m_1\left\{1+\left[\left(\frac{-\ln\ln\frac{1}{p}}{1{,}282} - 0{,}45\right)+\frac{\ln N}{1{,}282}\right]V_1\right\}$$

respectively:

$$x_p = m_1(1+K_N V_1) \qquad (2)$$

where the values of K_N in terms of p and N are the following:

N, years \ 1-p	o.1o	o.o5	o.o2	o.o1
1	1.3o4	1.966	2.593	3.138
2	1.846	2.4oo	3.134	3.67o
5	2.56o	3.122	3.849	4.394
1o	3.1o1	3.662	4.389	4.964
2o	3.641	4.2o2	4.7o5	5.473
3o	3.958	4.519	5.246	5.79o
5o	4.356	4.917	5.644	6.29o
1oo	4.896	5.457	6.184	6.728

In order to use formula (2) and for the analysis of safety of the roof structure against snow load on a second moment format, it is necessary that meteorological information should be shown by means of two basic maps:

(i) the map of the mean of annual maxima of the snow depth, m_1;
(ii) the map of coefficient of variation of annual maxima of the snow depth, V_1.

With them, any values of load fractiles can be calculated directly in different sites by formula (2). The coefficient of variation V_1 and the mean m_1 are thus the basic indicators of climate severity of an area. It is to be noted that the values of V_1 can be very high, for example in Romania they are frequently higher than value o.45 estimatively codified by J.C.S.S. [2].

The effect of snow depth coefficient of variation on snow depth fractiles is shown in fig.1.

As a joined action of both wind and snow the result of snow depth measurements greatly depends on local topographical conditions and built environment in the vecinity of the area where the meteorological study is made.

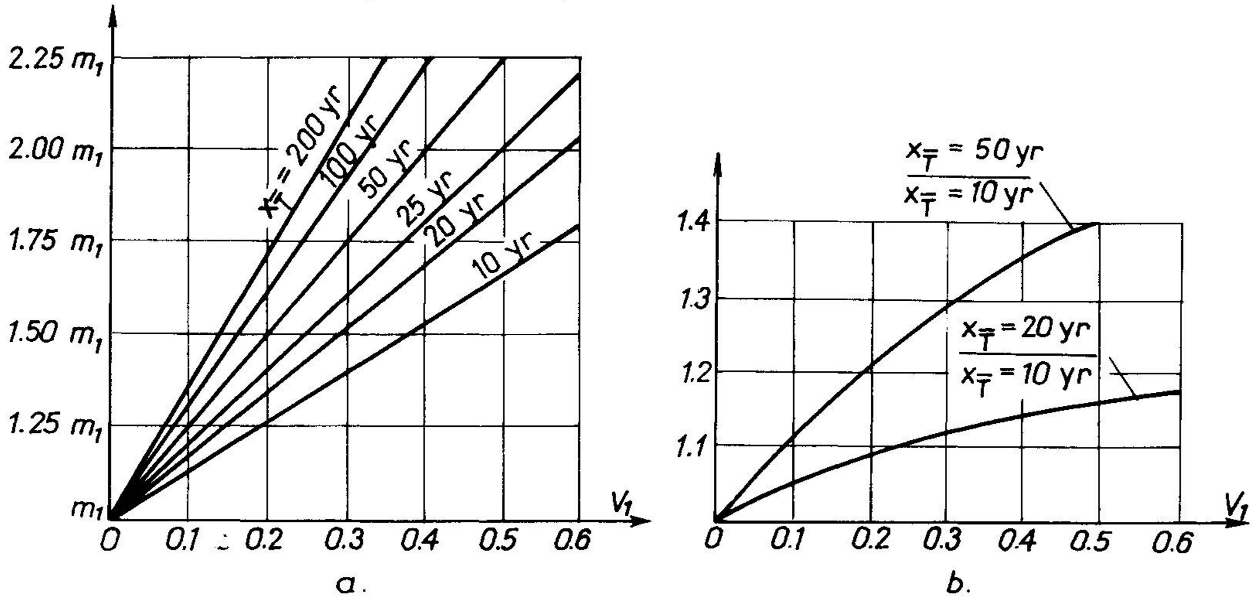


Fig.1 The fractiles of snow load as function of the coefficient of variation of annual maxima of snow depth.

For example, the following parallel values of snow depth on the ground have been noted at two meteorological stations in Bucharest, located respectively in the north-out of town and in the south-within the town, on a small elevation:

	Bucharest Baneasa(N)	Bucharest Filaret(S)
In January 1980	40 cm	52 cm
Mean of annual maxima	35	48
Absolute maximum	104	150
Annual maximum having:		
$\bar{T}$ = 10 years	65	92
$\bar{T}$ = 50 years	98	132

The following variations of the same snow of the 1980 winter (corresponding to mean climatic conditions) are also mentioned in two different areas in Bucharest [8]:

Zone	Snow depth	Snow load
(1)	17 - 29 cm	34 - 56 daN/m^2
(2)a	29 - 46	32 - 76
b	30 - 40	47 - 58

The effects of snowfalls blown off by the wind being strongly influenced by the local conditions, the snow depth measurements in different points of the same site are different and contain errors.

If h is annual maximum snow depth, h_m the same depth measured, and ε measurement error, then obviously:

$$h = h_m + \varepsilon \qquad (3)$$

ε being a random variable of zero mean.

The means of h and h_m are equal $m_h = m_{hm}$, while the actual snow depth coefficient of variation out to be greater than that of the measured snow depth:

$$V_h = \sqrt{V_{h_m}^2 + \frac{\sigma_\varepsilon^2}{m_h^2}} \qquad (4)$$

where σ_ε^2 is the variance of measurement error ε.

Therefore, when calculating snow depth fractiles with different mean return periods according to equation (2), coefficient V_h should be used for V_1.

2. SPECIFIC GRAVITY OF SNOW

Snow load is calculated at present by the product between depth, probabilistically defined with a specified mean return period, and the snow specific gravity expressed deterministically:

(i) either by a singular numerical value;

(ii) or by deterministic function of different parameters (most frequently of depth h); for example, according to J.C.S.S. suggestion [2] :

$$(h) = 300 - 200\, e^{-1.5h}$$

It is to be noted that for depth h>1 m, the specific gravity of snow $\gamma(h) > 250$ daN/m^3, which has also been proved by other studies and measurements [7], [1].

In accordance with this procedure, the random variability of snow load is derived exclusively from snow depth. This does not correspond to physical reality.

Snow load L, definedby the product between depth and specific gravity actually depends on both random variables, $\gamma(h)$ and h:

$$L = \gamma(h)\ h$$

The mean and coefficient of variation of load are thus calculated in terms of means and coefficients of variation of specific gravity and of depth with the relations:

$$m_L = m_{\gamma}(h)\ m_h \qquad (5)$$
$$V_L = \sqrt{V^2_{\gamma}(h) + V^2_h}$$

As the analytical expressions for $\gamma(h)$ are extremely different and reflect subjective approximations, the assessment of coefficient of variation of specific gravity should not be made analytically out of function $\gamma(h)$, but rather directly out of the measurements corresponding to various ranges of depth h.

The coefficients of variation of the mean specific gravity of snow for various ranges of depth, $V_{\gamma(h)}$ may have orders of size comparable with those of depth h, but they are likely to be smaller:

$$V_{\gamma(h)} < V_h$$

The monthly maximum specific gravities of snow in Bucharest for a period of about lo years, analysed irrespective of snow depth, are characterized by a coefficient of variation

V_{γ}=o,52 [11] . Obviously, the order of size of V_{γ} is greater than that for $V_{\gamma(h)}$:

$$V_{\gamma} > V_{\gamma(h)}$$

The previous remarks show that uncertainities concerning the assessment of random snow load are much greater than those that appear out of considering as random variable only the snow depth and only in one point at one meteorological station.

3. SNOW BLOWN OFF BY THE WIND

The factors of changing the snow depth on the ground into the snow depth on the roof are estimated in the present codes between o.8 and o.6, depending on the wind exposure of the building. Such values generally apply to roofs with a relatively small surface placed in areas that are relatively free of obstacles, out of towns as a rule.

In towns, the snowfalls on roofs are determined by the effects of wind covering various "random" configurations of built volumes or of relief. Under such conditions it is almost impossible to select one single clear factor of passing from the depth of the snow fallen quite uniformly on the ground out of town to the snow depth on flat roofs in town.

Parallel measurements of snow depth on the ground and on the roof made in the winter of 198o in two different sites in Bucharest have shown that [8] :

(i) with roofs of limited areas and without obstacles in their vecinity or on the roof, the coefficient of blown off snow had values of about o.7 - o.8 ;

(ii) with roofs of large areas and with obstacles at their back no effect of blowing off appeared;

(iii) the snow depth measured on the ground in several various points in urban sites was in all cases different(larger or smaller) from the snow depth measured on the ground out of town at a meteorological station.

Therefore, the factors of passing from the snow depth on the ground to the snow depth on the roof seem to matter generally only for out of town sites, as for the sites in town suc such factors cannot be correctly appreciated.

As a matter of fact, for the experimental determination(on natural scale or on models) of the factors of snow accumulation on roofs of various shapes, these factors cannot be determined but with respect to snow depth on the ground near the roof. As a result, the blowing off effect cannot be separated from the accumulation effect, either in measurements or in designing.

4. FACTORS OF SNOW ACCUMULATION ON ROOFS

Owing to (i) the geometrical variety of structural shapes of roofs and to (ii) the conditions of exposure to the joined action of both wind and snow of various sites, the determination of snow accumulation factors can be only informative.

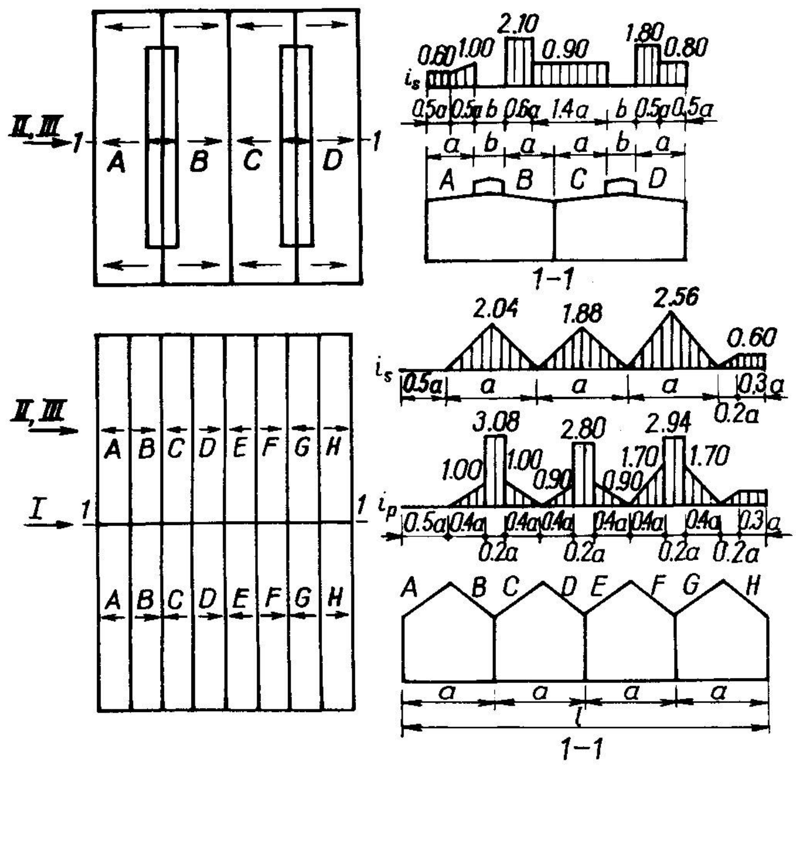
Accumulation factors can be defined:

(i) on the surface;

(ii) linear;

(iii) punctual.

The values of some of these factors, for the basic geometrical forms of roofs, are codified and verified by confronting them with the results of measurements on natural scale. The extrapolation of these results on new shapes of roofs is difficult and unreliable. As a result, it is only the modelling of snow fall in wind tunnel, that can provide designing guiding lines. Such results for different shapes of roofs are shown in [6] and are illustrated in fig.2.



a. Roof with skylight

b.Folded plate type roof

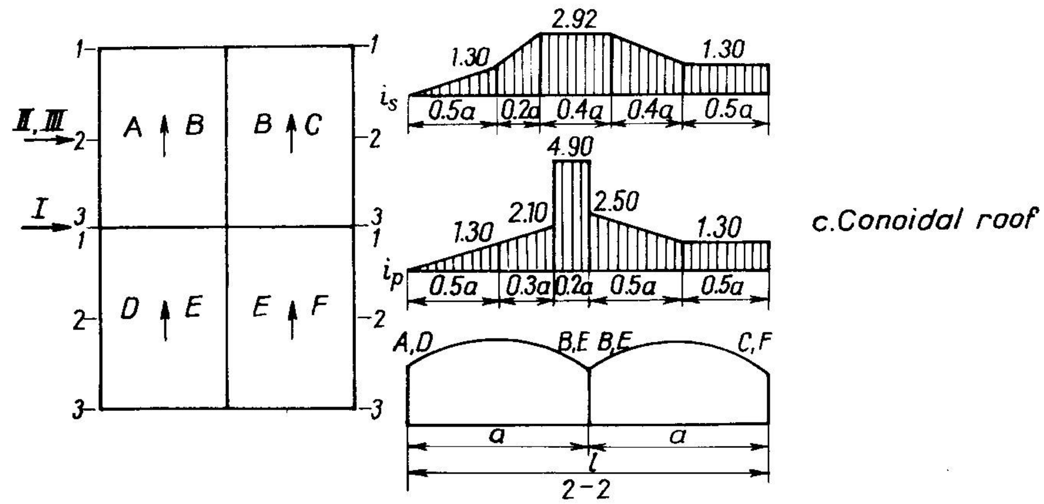


Fig.2 Within test snow accumulation coefficients

It must be noted that generally, the values of accumulation factors in the Soviet loading code are coupled in the calculation of snow loading with snow depths defined with small return periods(2-5 yr.), while the Canadian and American building codes use larger return periods (3o-1oo yr.). The maximum values of codified accumulation factors are usually ≤3, fig.3; however, parallel measurements of snow depth on the ground and on the roof in the conditions of roofs with moderate subsidence behind some obstacles (skylight) were amplified on the roof up to 4-5 times the mean depth measured on the ground [8] .

As a rule, the maxima of structural effects of snow load are achieved on an asymmetrical load scheme of the roof,thus expressing the wind effect. The responsability of the choice of accumulated snow load scheme on the roof for the designing of the structure and of its parts belongs entirely to the designer and cannot,therefore,be transfered to codes and code-producing committees.

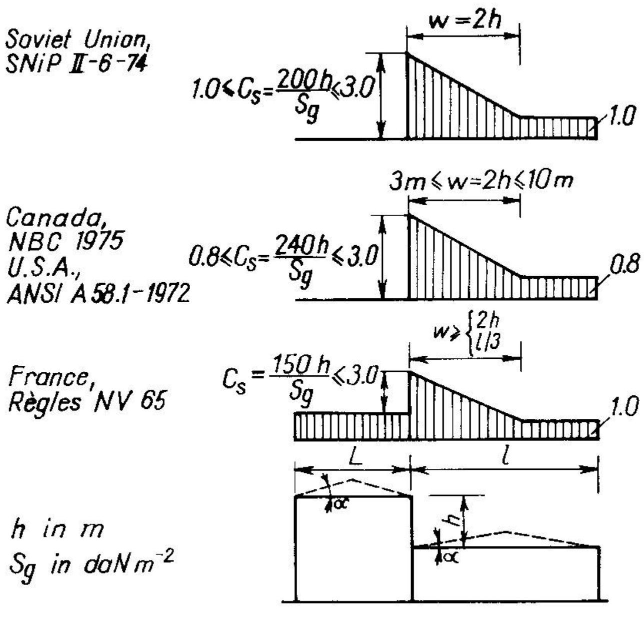


Fig.3 Snow accumulation coefficients for the lower level of multilevel roofs.

5. THE SAFETY OF STRUCTURES WITH SNOW LOAD

To analyse the safety of the roof structure with snow load and not to determine this load is the ultimate purpose in the activity of a civil engineer.

Including the meteorological information in terms of probability, the analysis of the safety of metal or reinforced concrete buildings can be made on a second moment format.

Considerind the annual maxima of snow load L, Gumbel distributed for maxima and characterized by mean m_1 and coefficient of variation V_1, the maxima in N years of load, L_N are also Gumbel distributed and characterized by the mean:

$$m_{L_N} = m_L\left[1+0{,}78(\ln N)V_1\right] > m_1 \qquad (6)$$

and the coefficient of variation:

$$V_{L_N} = \frac{V_1}{1+0{,}78(\ln N)V_1} < V_1 \qquad (7)$$

Let S_N be the sectional (or unitary) effect of permanent and snow loads on a member of the roof structure ,expressed in terms of them by a linear relationship:

$$S_N = a\,D + b\,L_N$$

where the deterministic factors a and b depend on the geometry of the structure, calculation method, etc.

The mean and coefficient of variation of loads effect S_N are obvionsly:

$$m_{S_N} = a\,m_D + b\,m_{L_N} \qquad (8)$$

$$V_{S_N} = \frac{\sqrt{a^2 m_D^2 V_D^2 + b^2 m_{L_N}^2 V_{L_N}^2}}{m_D + m_{L_N}} \qquad (9)$$

where m_D and V_D are the mean and coefficient of variation of permanent load D.

Let R be the sectional (or unitary) random strength opposed to S_N having mean m_R and coefficient of variation V_R depending on basic strength of material M by an usually linear or quasi linear relationship:

$$R = c\,M$$

respectively:

$$m_R = c\,m_M \qquad V_R \cong V_M \qquad (10)$$

For the further calculation schematizing the distributions of random variables R and S as being of an asymmetrical lognormal type (positive asymmetry), the dimensioning relationship is the following [3]:

$$c_{necessary} = \frac{m_R\ \text{necessary}}{m_{S_N}} = e^{\beta\sqrt{V_{S_N}^2+V_R^2}} \qquad (11)$$

and that of checking:

$$\beta_{effective} = \frac{\ln\dfrac{m_R}{m_{S_N}}}{\sqrt{V_{S_N}^2+V_R^2}} \qquad (12)$$

The correspondences between the reliability index β and the probability of failure of the roof during N years $P_{f,N}$ are the known ones:

$P_{f,N}$	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
β	2,32	3,09	3.72	4.27	4.75	5.20

etc.

By means of relations (11) and (12), the results of the dimensioning and checking based on probability can be compared to the results of the analysis according to a deterministic or semiprobabilistic procedures. Generally it can be estimated that:

(i) The sectional areas provided by probabilistic calculation appear to be greater than those obtained by applying the codified procedure; with low reliability of the order of P_f=10^{-3} the differences are only of about 20%, but with probabilities $P_f \cong 10^{-5}$ such differences come up to over 50%;

(ii) It can be appreciated that for light roofs where snow load represents more than half of the total load of the roof, to apply the codified procedure may lead to smaller reliabilities than those accepted by standards for other types of structures and loads;

(iii) In order to prevent such situations, many specifications use additional overload factors for lightweight roofs. These factors are defined in terms of the ratio between the snow load and the total load of roofs (snow included) and generally have values ranging from 1.0 to 1.3; these values are not selected on probabilistic bases and consequently the degree of safety cannot be strictly appreciated;

(iv) With roofs where snow load has the main weight in dimensioning the structure, the use of calibration of the semiprobabilistic design on a second moment format is a necessary way of improving the current procedures of designing.

6. CONCLUSIONS

1. Snow depth in a given site depend on the climatic severity of the site, as well as on the local topographical conditions and the built environment in the neighbourhood of the area where the measurements are made.

2. The engineer's uncertainities concerning the assessment of random snow load depend not only of the random character of stratum depth but also on the random character, for a given depth, of the specific gravity of snow.

3. In determining experimentally, both on models and on natural scale, of roof snow accumulation factors, the effect of blown off snow cannot be separated from the accumulation effect.

4. For determining the factors of snow accumulation on roofs of special shapes, the extrapolation of load schemes from known shapes is unreliable.

5. The use of certain additional deterministic safety factors for designing lightweight roofs does not give a realistic image of their safety, which can de quantized only based on probability.

NOTATIONS

- C central safety factor
- D dead load
- h snow depth on ground
- K_N factor for calculating snow depth fractiles
- L snow load
- m mean of a random variable
- N structure life, years
- $P_{f,N}$ probability of failure during N years
- R sectional or unitary strength

$\underline{S}$ sectional or unitary load effect
T mean return period of snow depth
V coefficient of variation of a random variable
x_p snow depth fractile
β safety index
γ specific gravity of snow

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