Finite Element Analysis of Reinforced Concrete Beams with Special Regard to Bond Behaviour

Calcul des poutres en béton armé par la methode des éléments finis en tenant compte de l'adhérence entre le béton et l'armature

Die Berechnung von Stahlbetonbalken nach der Methode der finiten Elemente unter besonderer Berücksichtigung des Verbundes

GÜNTHER PLAUK
Dr.-Ing.
Federal Institute for Materials Testing (BAM)
Berlin (West), Germany

GEBHARD HEES
Prof. Dr.-Ing.
Technical University Berlin
Berlin (West), Germany

SUMMARY
A nonlinear finite element method has been developed and used to study the behaviour of reinforced concrete beams. The mechanical model takes into account the nonlinear properties of material, progressive cracking and local failure. For the first time realistic bond-slip relations were used in a finite element model. The validity of the method was studied by comparing analytical and experimental results which show an excellent agreement.

RÉSUMÉ
Une méthode de calcul par les éléments finis a été développée pour déterminer le comportement des poutres en béton armé. Le modèle mécanique a tenu compte de la fissuration progressive, des propriétés non-linéaires du matériau, de l'adhérence et de la rupture locale. Les examens effectués ont montré que dû à l'utilisation des relations réalistes relatives à l'adhérence on a obtenu une conformité excellente entre les résultats d'essais et les résultats de calcul.

ZUSAMMENFASSUNG
Ein Rechenverfahren nach der Methode der finiten Elemente wurde entwickelt und auf die Untersuchung des Verhaltens von Stahlbetonbalken angewendet. In das mechanische Modell einbezogen wurde die Rissbildung, das nichtlineare Werkstoffverhalten, der Verbund sowie das örtliche Beton- und Verbundversagen. Die durchgeführten Untersuchungen haben gezeigt, dass insbesondere durch Verwendung wirklichkeitsnaher Gesetze für den Verbund eine ausgezeichnete Übereinstimmung von Rechen- und Versuchsergebnissen erreicht wird.

1. INTRODUCTION

Recent development of the finite element method permits study of reinforced concrete structural member behaviour in the full range of loading. An excellent summarized presentation of existing finite element models used for analysing reinforced concrete is given in [1,2].

A critical review of published analytical solutions for reinforced concrete beams subjected to static loading [3,4,5,6] such as load-deflection curves, stress and strain distributions and crack patterns show only little agreement with experimental results. In finite element analysis of reinforced concrete it has become customary to assume that steel bars are ridgidly attached to concrete nodes. This treatment, however, is unrealistic and physically unjustified. The reason for the discrepancy in analytical and experimental results must therefore be seen in the fact that bond between concrete and reinforcement in mechanical models which are applied to study the structural response has mostly been neglected.

The finite element method presented in this paper permits to determine the internal stress and strain distribution as well as the crack patterns and deflections of reinforced concrete beams incrementally loaded from zero load to ultimate load. The proposed model used for analysis takes into consideration nonlinear material properties, progressive cracking, and bond between concrete and reinforcement. Included is also local crushing of concrete and bond failure. To fully account for the profound influence of bond on cracking and on the internal stress distribution different bond-stress slip relations for bond intervals near cracks, respectively in some distance from cracks, were implied into the finite element model.

2. FINITE ELEMENT APPROXIMATION

The finite element idealization of a reinforced concrete beam for two-dimensional analysis is shown in Fig. 1. To account for the varied material properties the beam is divided into concrete and steel elements. For both, concrete and steel reinforcement, conventional two-dimensional quatrilateral plane stress elements are used. Each element consists of four triangular sublements with

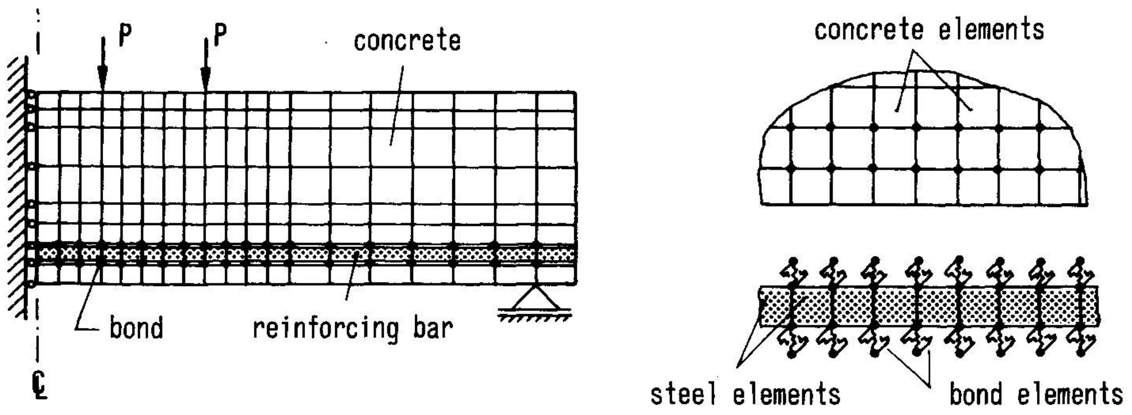


Fig. 1 Finite Element Idealization of a Reinforced Concrete Beam

constant stress and strain distribution. The unloaded central node of this type of element is eliminated which reduces the degrees of freedom.

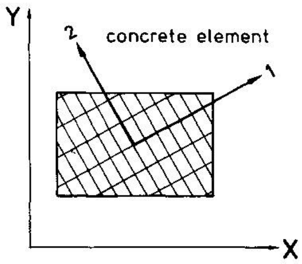
The finite element model contains separate nodes for concrete and reinforcing elements crossing each other. Bond between concrete and steel bars is represented by special bond-elements first created by Nilson [7]. Each bond element contains two springs, one acting parallel to the axis of the steel bar, the other acting perpendicular to it. The overlapping nodes of concrete and steel elements are directly connected by these linkage elements.

In regions of the beam where intensive crack propagation is expected a finer element mesh is used so that in the tension zone between the cracks a number of concrete elements remain uncracked. In connection with establishing a realistic model for bond behaviour this simple but necessary treatment permits to analize the true state of stress and strain between cracks so that the influence of bond on beam deflections is fully accounted in the model.

3. MATERIAL IDEALIZATION

3.1 Modelling of Concrete

The stress-strain relationship for concrete under biaxial state of stress used here for analysis are given in Fig. 2. The uncracked concrete is assumed to be a homogeneous material with properties differing in two directions perpendicular to each other. The axes of anisotropy are identical to the axes of the principle stresses. Furthermore, isotropic behaviour is assumed for the concrete element due to uniaxial loading σ_1 or σ_2, respectively shear loading τ as shown in Fig. 2. It should be noted that the four material properties E_1, E_2, ν_1, and ν_2 are not independent and could only be determined if test data were available. Neglecting the non-diagonal terms of the stress-strain relationship the values of the material properties E_1 and E_2 can be obtained from uniaxial stress-strain curve as a function of strains in the principle directions. The uniaxial stress-strain curve used for analysis is shown in Fig. 3.



$$\{\varepsilon\} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_2}{E_2} & 0 \\ -\frac{\nu_1}{E_1} & \frac{1}{E_2} & 0 \\ 0 & 0 & G_{1,2} \end{bmatrix} \{\sigma\}$$

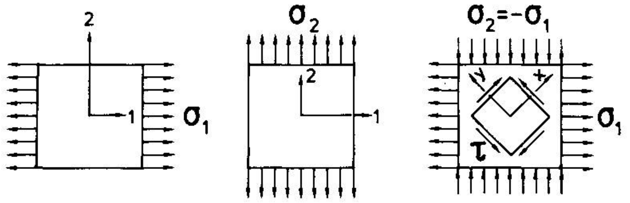


Fig. 2 Stress-Strain Relationship for Concrete under Biaxial State of Stress

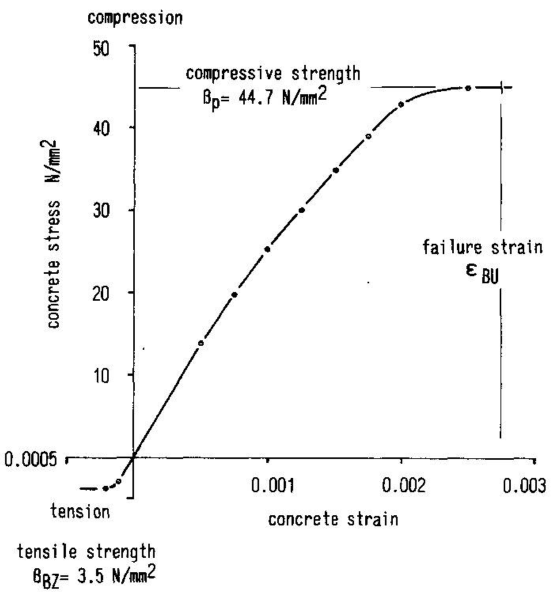


Fig. 3 Uniaxial Stress-Strain Curve for Concrete

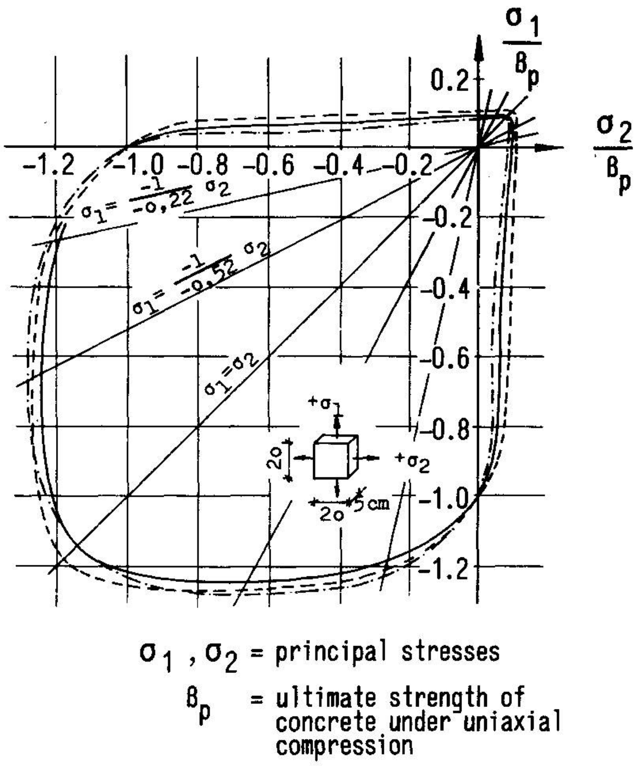


Fig. 4 Strength Envelopes for Concrete under Biaxial State of Stress [8]

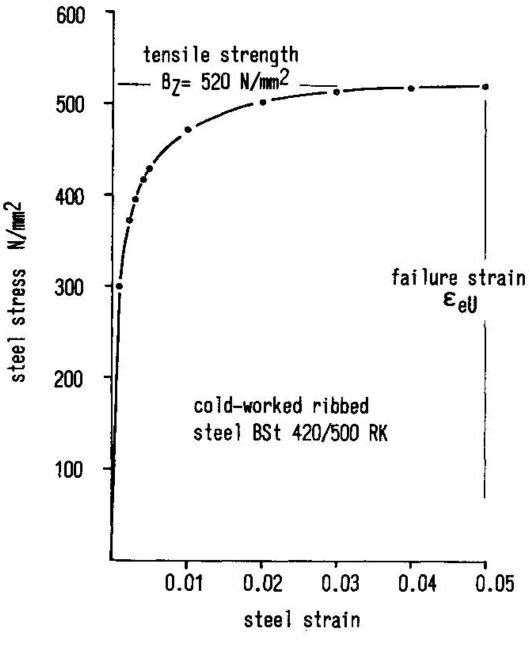


Fig. 5 Uniaxial Stress-Strain Curve for Reinforcing Steel

To take into account the failure of concrete elements under a combined state of stress a biaxial failure envelope (Fig. 4) derived from test data by Kupfer [8] is implied in the material model. Cracks are established in concrete elements when in one of the principle directions concrete strains exceed the maximum strain value obtained from the uniaxial stress-strain curve of Fig. 3.

3.2 Modelling of Steel

Similar treatment as for concrete is proposed for steel. The reinforcing bars of a beam are essentially in a state of uniaxial stress. Therefore, an uncoupling of the stress-strain matrix given in Fig. 2 is justifiable. Neglecting the influence of Poisson's ratio on element stresses which can be done without any loss of accuracy, the values of material properties E_1 and E_2 can be taken directly from the nonlinear stress-strain diagram of Fig. 5. The shown curve is valid for a particular reinforcing steel and was obtained from uniaxial tests. This steel was used for a test beam [10] which is selected here for analysis.

To determine failure of steel elements a maximum strain criterion was used. Failure occurs if the principle strains exceed the maximum value of strains corresponding to peak stress in tension given in Fig. 5.

3.3 Modelling of Bond

An intensive experimental study of bond between steel reinforcement and concrete was done for the last years by Eifler [9]. For the present analysis realistic bond-slip relations were obtained from these test results.

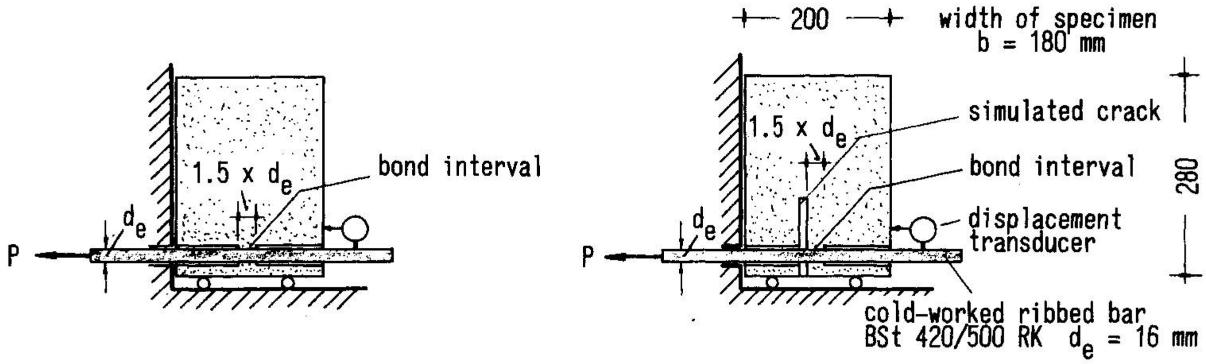


Fig. 6 Test Specimens for Bond Investigation Ref. 9

The specimens used for bond investigations are shown in Fig. 6. It is important to recognize that the midsection of the test specimens are very similar to beam sections of a short length. The steel bar is only over a short length of approximately 1.6 x bar diameter in connection with the surrounding concrete, so that in this case a uniform distribution of bond stresses could be assumed. It should be noted that this means a full correspondence between bond intervals in the test specimen and the bond idealization proposed in the finite element model. To study the influence of cracking on bond behaviour, a modified test specimen was developed and used with a simulated crack in front of the bond interval (Fig. 6b).

Furthermore, the influence of a plastic steel strain on bond behaviour was investigated. Therefore, before carrying out pull-out-tests the casted steel bar in bond with the concrete of the test specimen was stretched to a certain amount of plastic steel strain by applying tension to the projecting ends of the bar.

The bond-slip curves obtained from first test results are shown for bond in uncracked regions of a beam or in some distance of cracks in Fig. 7 and for bond near crack faces in Fig. 8.

Several points are of interest to observe in the bond-slip-curves presented. In general, a significant influence of plastic steel strain on bond stiffness and peak bond stress is obvious. Comparing the two sets of bond-slip-curves it can be clearly seen that bond stiffness and bond stresses near cracks are significantly lower than in some distance from the crack face or in uncracked regions for equal values of bond slip.

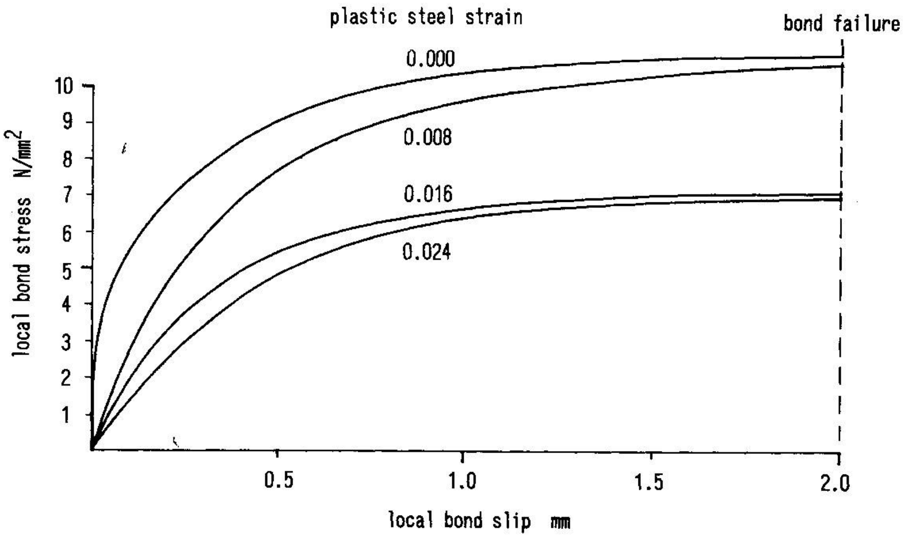


Fig. 7 Bond Stress-Slip Relations for Bond in Uncracked Regions or in some Distance from Cracks

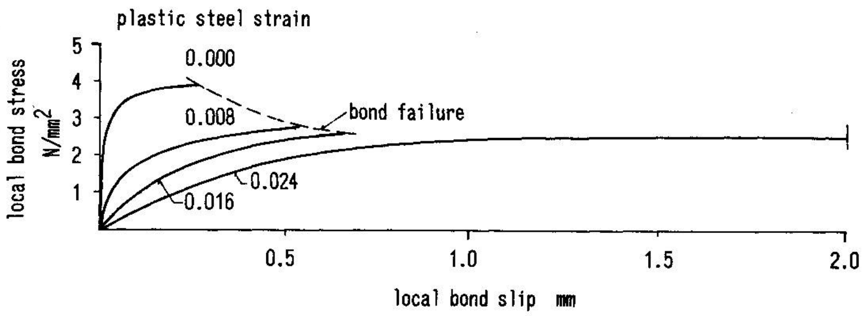


Fig. 8 Bond Stress-Slip Relations for Bond near Cracks

The different bond properties near cracks and between cracks can be easily incorporated into the finite element model as shown in Fig. 9.

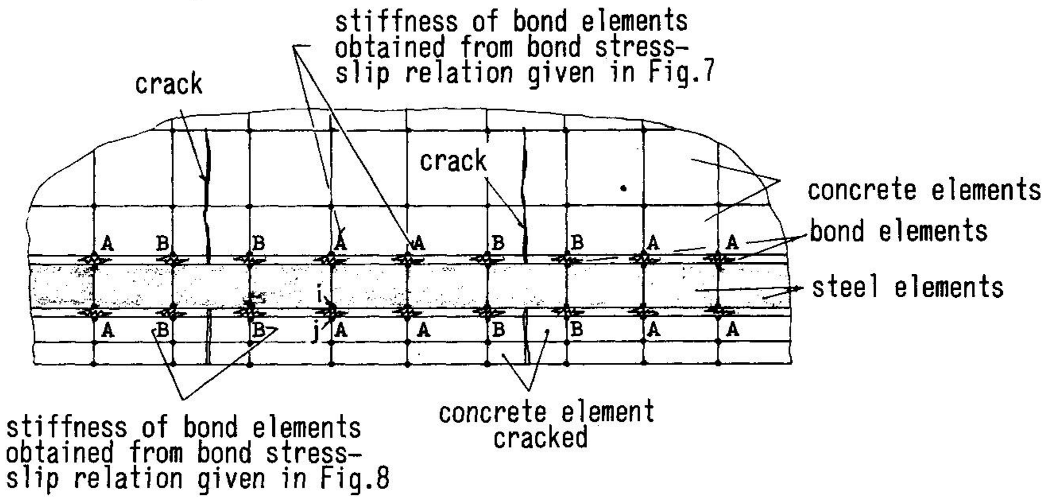


Fig. 9 Representation of Bond in a Cracked Reinforced Concrete Beam

4. NONLINEAR ANALYSIS AND COMPUTATIONAL ASPECTS

A nonlinear analysis of reinforced concrete beams accounting for all important effects on internal stress distribution, crack propagation and external deformations can only be realized by a step-by-step solution procedure. Therefore, the finite element method proposed involves an incremental loading and two different iteration procedures to satisfy equilibrium and constitutive relations within each load increment.

The solution process starts with incrementation of external loads. In each load increment the increments of nodal deflections and internal stresses are first obtained by using a global stiffness matrix containing tangential values of beam stiffness reached in the previous load step. After computing the resulting stress and

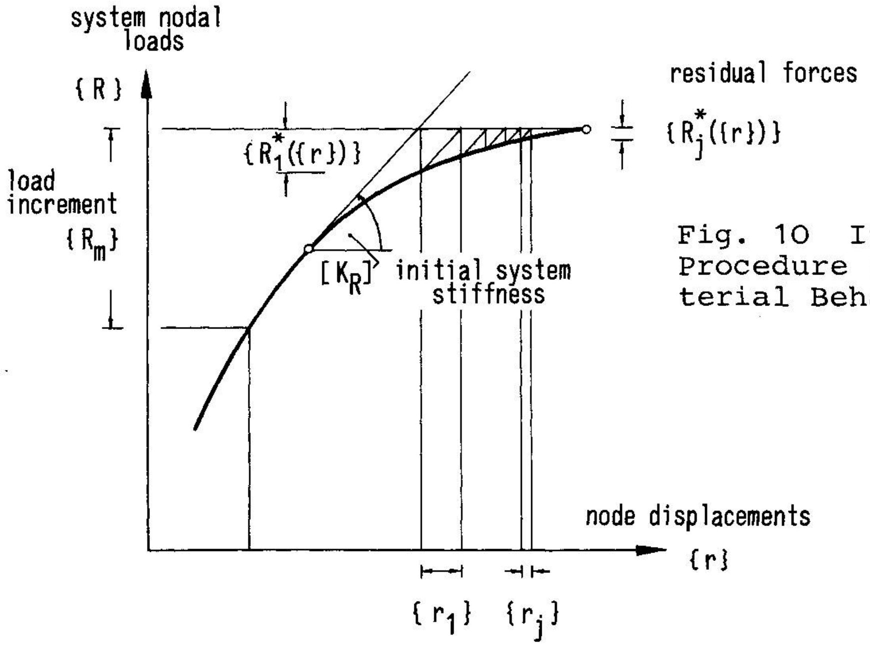


Fig. 10 Iterative Solution Procedure for Nonlinear Material Behaviour

strain increments the total stresses of elements are not corresponding to the stress state obtained from constitutive relations for total strains. The "unbalanced" stresses are now integrated to receive "unbalanced" nodal forces which are applied to the element nodes in the next iteration step. To fully account for nonlinear material behaviour to structural stiffness an iteration process is now started as shown in Fig. 10 until constitutive relations are satisfied. It should be noted that the global stiffness matrix remains unchanged in this case.

Cracking of concrete causes a sudden change of element stiffness and needs, therefore, a modified treatment. In a system with a compatible state of stress and strain the proposed failure criterion for concrete in tension is satisfied in a more or less greater number of adjacent concrete elements or possibly in the total area of tension zone. To help the system adjust in the correct manner only in elements with a maximum value of tensile stresses in principal directions - exceeding tensile strength - cracks are established as shown in Fig. 11. The stiffness of cracked elements satisfying this criterion is now modified and reduced to zero in a direction perpendicular to cracks.

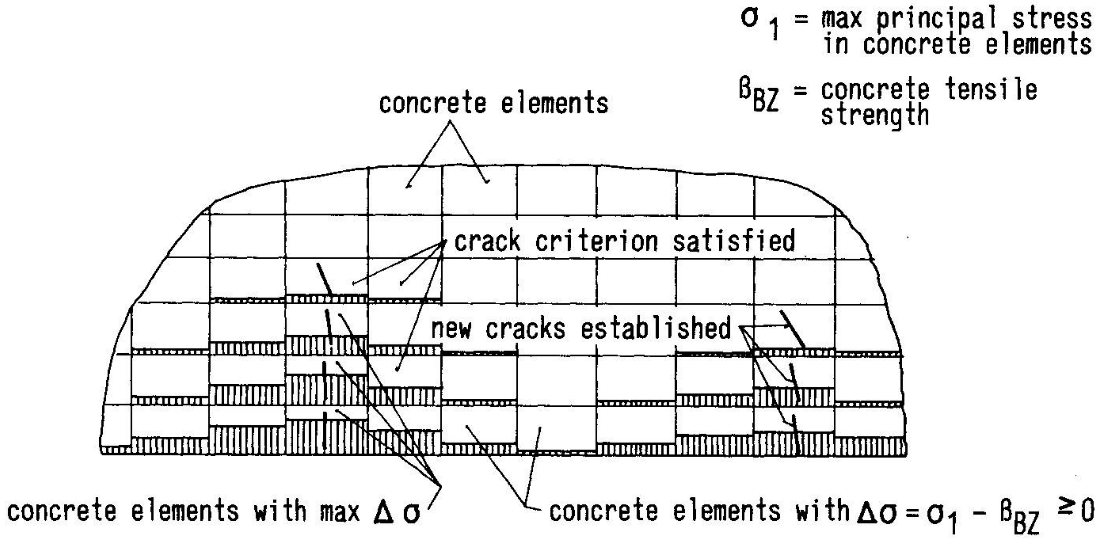


Fig. 11 Crack Propagation in a Seperate Step of Iteration

Tension stresses causing cracks are removed and transferred to adjacent uncracked concrete elements. This is done by sets of nodal forces as illustrated in Fig. 12. To simulate the process of successive cracking a similar iteration process as shown in Fig. 10 is proposed, but with a global stiffness matrix changing in each iteration step. This iteration is stopped if the constitutive relations of materials are violated. In this case iterations corresponding to Fig. 10 are performed until the equilibrium of the system is re-established.

All load increments are treated in the same manner until the ultimate load has been reached and failure in concrete or steel occurs.

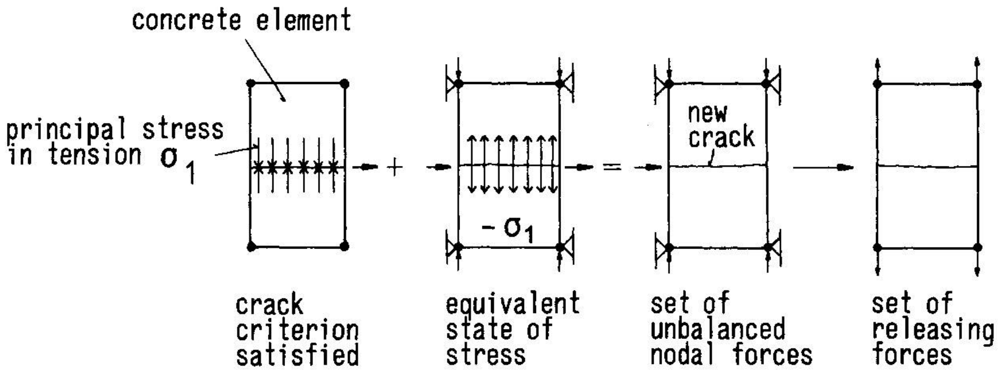


Fig. 12 Removing of Tension Stress in a Cracked Concrete Element

5. RESULTS

5.1 Analysis of a Reinforced Concrete Beam

The method is applied to study the behaviour of a reinforced concrete beam previously tested by Eifler [10]. Crack propagation, internal stress and strain distribution and load deformation response due to vertical single loads were traced through the elastic and inelastic ranges. The test beam and the finite element idealization used for analysis is shown in Fig. 13. One half of the symmetrically loaded beam was divided into a finite element mesh with a total number of 514 elements and 1056 degrees of freedom. In the region of proposed progressive cracking finer grid dimensions were chosen.

In the following the analytical solutions are presented and compared if possible with the experimental results.

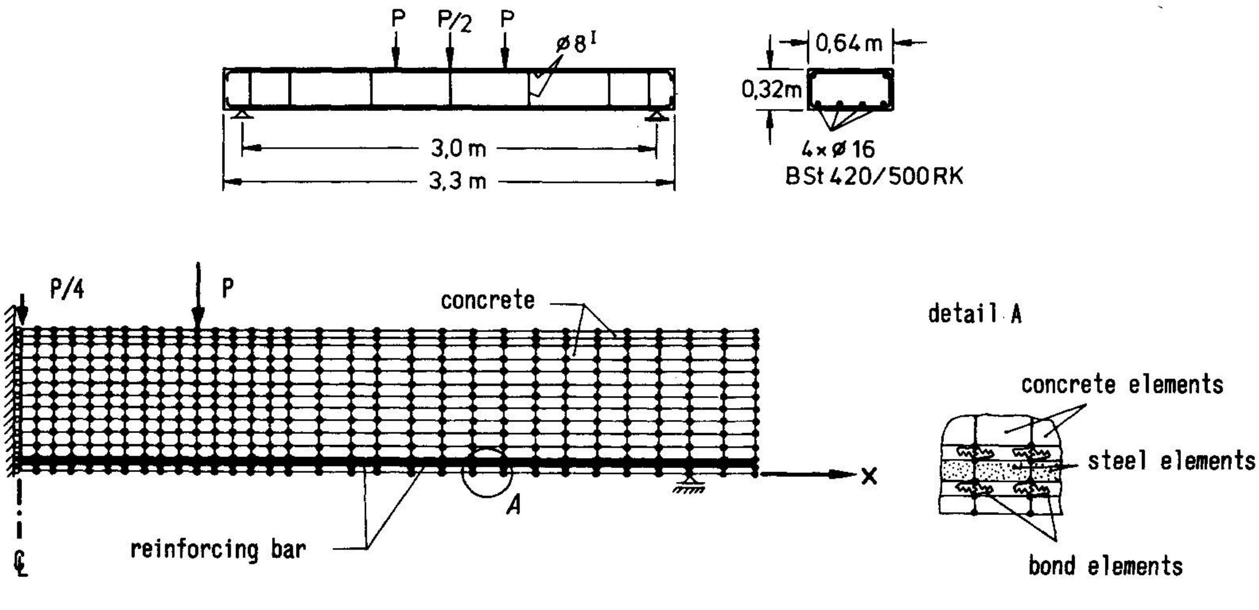


Fig. 13 Test specimen [10] and Analytical Beam Mesh Layout

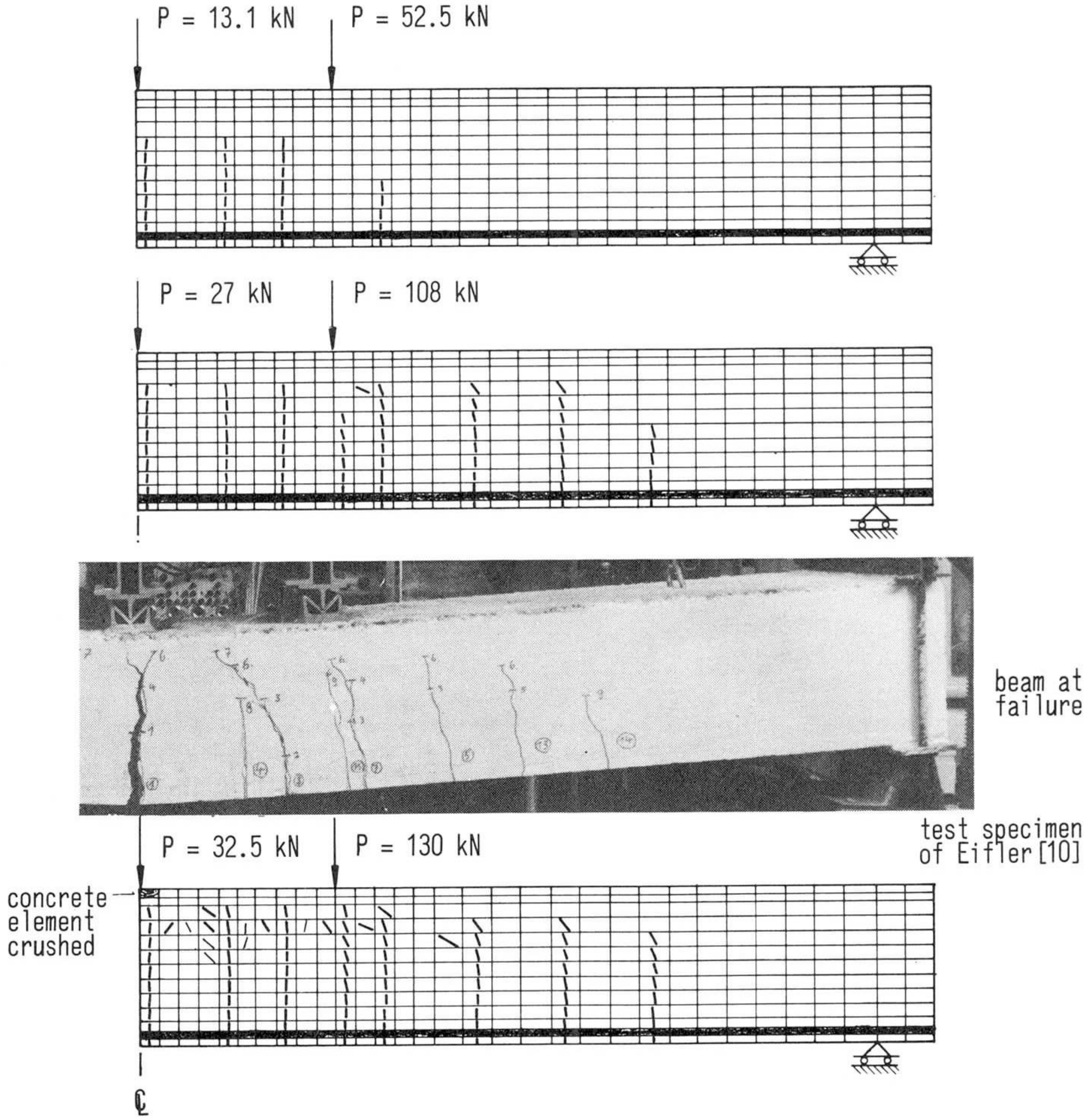


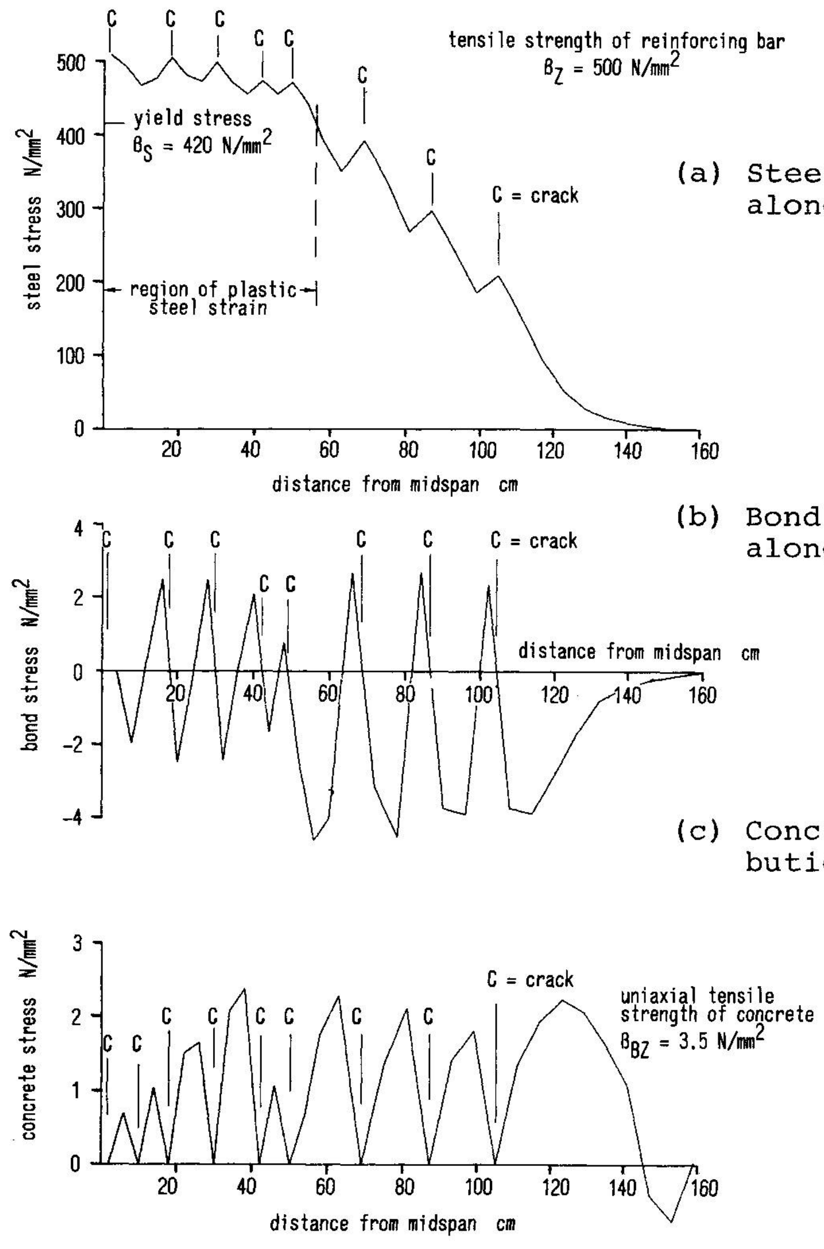
Fig. 14 Comparison of Analytical and Experimental Crack Patterns

The extent of concrete cracking at various stages in the loading history of the beam is shown in Fig. 14. The crack patterns, e.g., number, direction, and distance of cracks obtained from analytical solution agree with the experimental results. The analytical failure consisted in a crushing of concrete elements in the midspan cross section of the beam. It occured at a bending moment of M = 124 kNm which corresponds to the experimental values. It may now be observed in Fig. 14 that in the tension zone between cracks a number of concrete elements remained uncracked which shows the validity of the bond model.

For each load increment the internal stresses and strains of the beam were calculated. To demonstrate the influence of cracking and bond the distribution of concrete stresses in the cover of reinforcement, the steel stresses along the bar and the bond stresses between concrete and steel for the analytical ultimate load are given in Fig. 15. It may be of some interest to show the steel stresses along the bar for this case. Peak steel stresses were found in the cracked cross sections. Between cracks the decrease of steel stresses is obviously caused by bond and indicates the force transfer between concrete and reinforcement.

The bond stress curve is characterized by an antisymmetric stress distribution between cracks with peak bond stress near to crack surfaces and zero bond stress approximately in the middle of the concrete blocks. At ultimate load bond failure occured only adjacent to the first crack near the midspan section of the beam.

To show the stress and strain distribution at each cross section of the beam which can also easily be obtained from analytical results a concrete block between two cracks is regarded in Fig. 16.



(a) Steel stress distribution along the reinforcing bar

(b) Bond stress distribution along the reinforcing bar

(c) Concrete stress distribution at bottom fiber

Fig. 15 Longitudinal Variation of Stresses at Ultimate Load

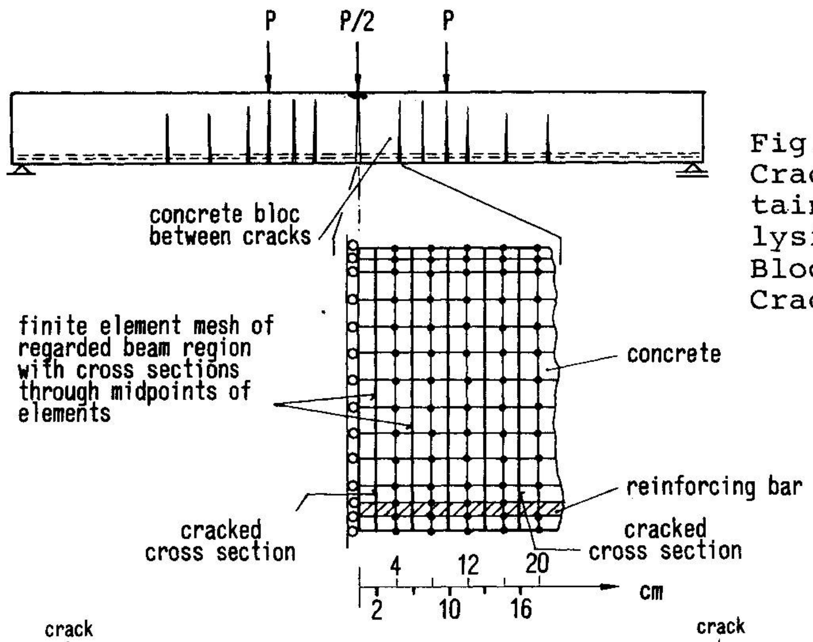
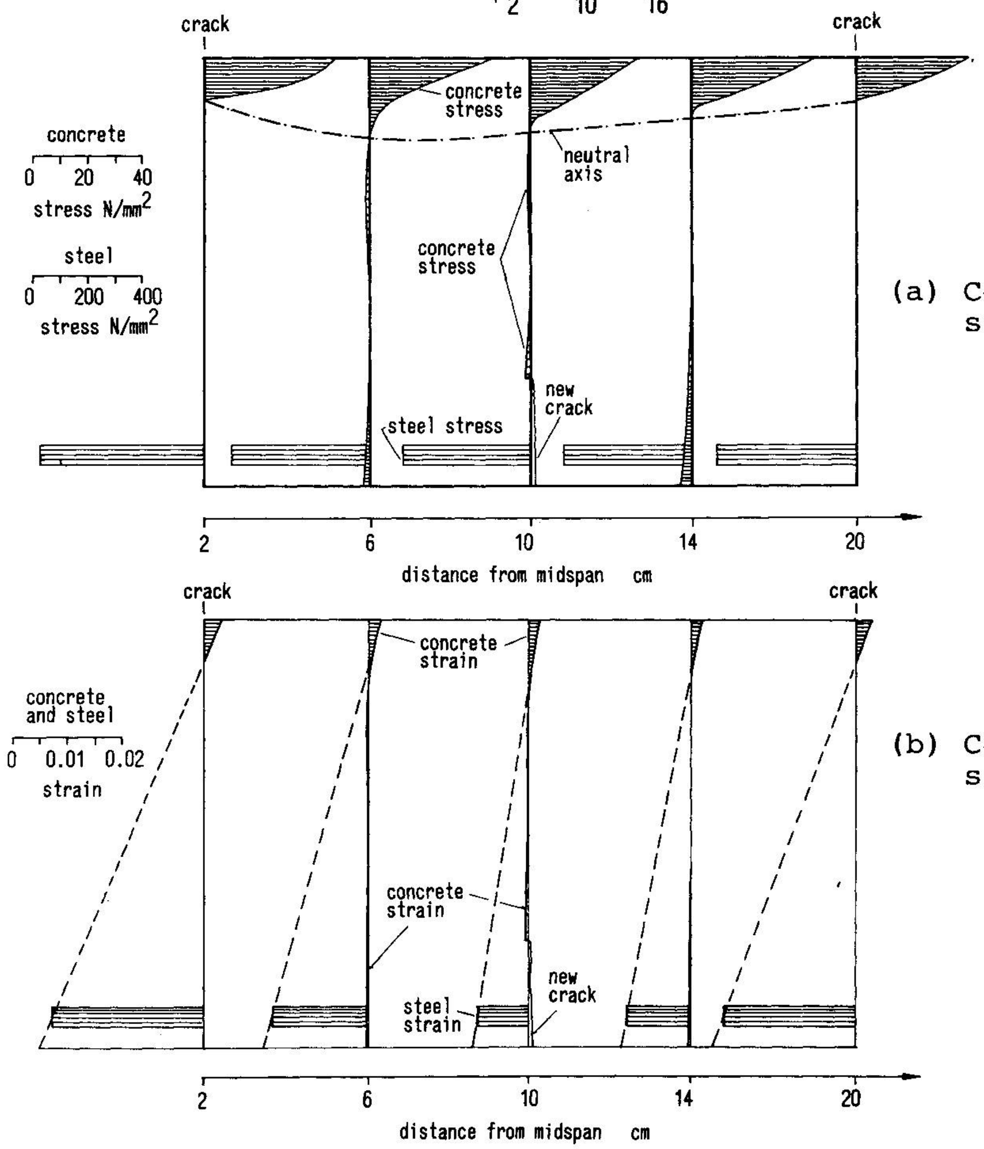


Fig. 16 Idealized Crack Pattern obtained from Analysis and Concrete Block between Cracks



(a) Concrete and steel stresses

(b) Concrete and steel strains

Fig. 17 Longitudinal Stress and Strain Distribution at Cross Sections between Cracks at Ultimate Load

The concrete and steel stresses due to ultimate load for cross sections through the midpoints of concrete and steel elements are given in Fig. 17. The longitudinal concrete stress distribution at any cross section is almost nonlinear. It should be noted that the concentration of concrete stresses above cracks causes a fluctuation of the neutral axis position. The variation of steel stresses along the reinforcing bar is nonlinear as mentioned before with peak steel stresses at both ends of the block. The resulting strain distribution is shown in Fig. 17 b. It may now be observed from Fig. 17 b that cross sections of cracked reinforced concrete members do not remain plane as normally assumed in simplified analysis.

The load-deflection curves (which are not shown here) obtained for each load increment permit to determine approximately a moment-rotation relation for the plastic hinge forming in the middle of the beam. The moment-rotation curve in Fig. 18 shows a sufficient agreement with the test results.

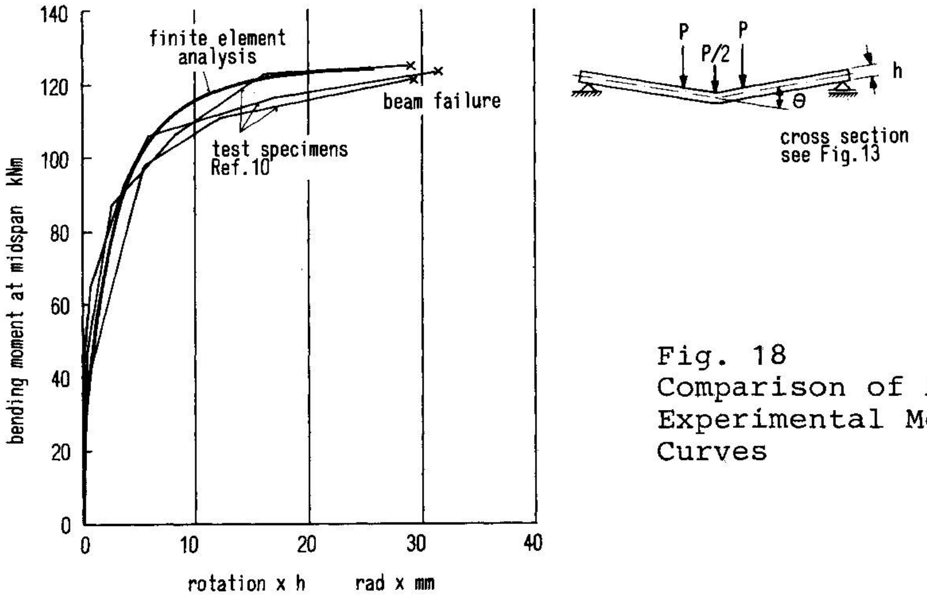


Fig. 18
Comparison of Analytical and Experimental Moment-Rotation Curves

5.2 Moment-Curvature Relation

The analytical results also permit to establish a realistic moment-curvature relation for the beam. Therefore, the concrete block of Fig.16 between two cracks is regarded again. For this block the strain distribution at one cracked cross section and at the midsection between both cracks is given for selected load increments in Fig. 19. It can be easily seen in Fig. 17 b and Fig. 19 that the slope of a line connecting the maximum concrete strain with the steel strain represents the rotation of a "plane" cross section. As indicated in Fig. 17 b the rotation of cross sections is decreasing with increasing distance from the crack surface which clearly reflects the influence of bond.

If a constant moment region is assumed between two cracks a mean value of curvature can be calculated approximately by integrating the rotations of the individual cross sections.

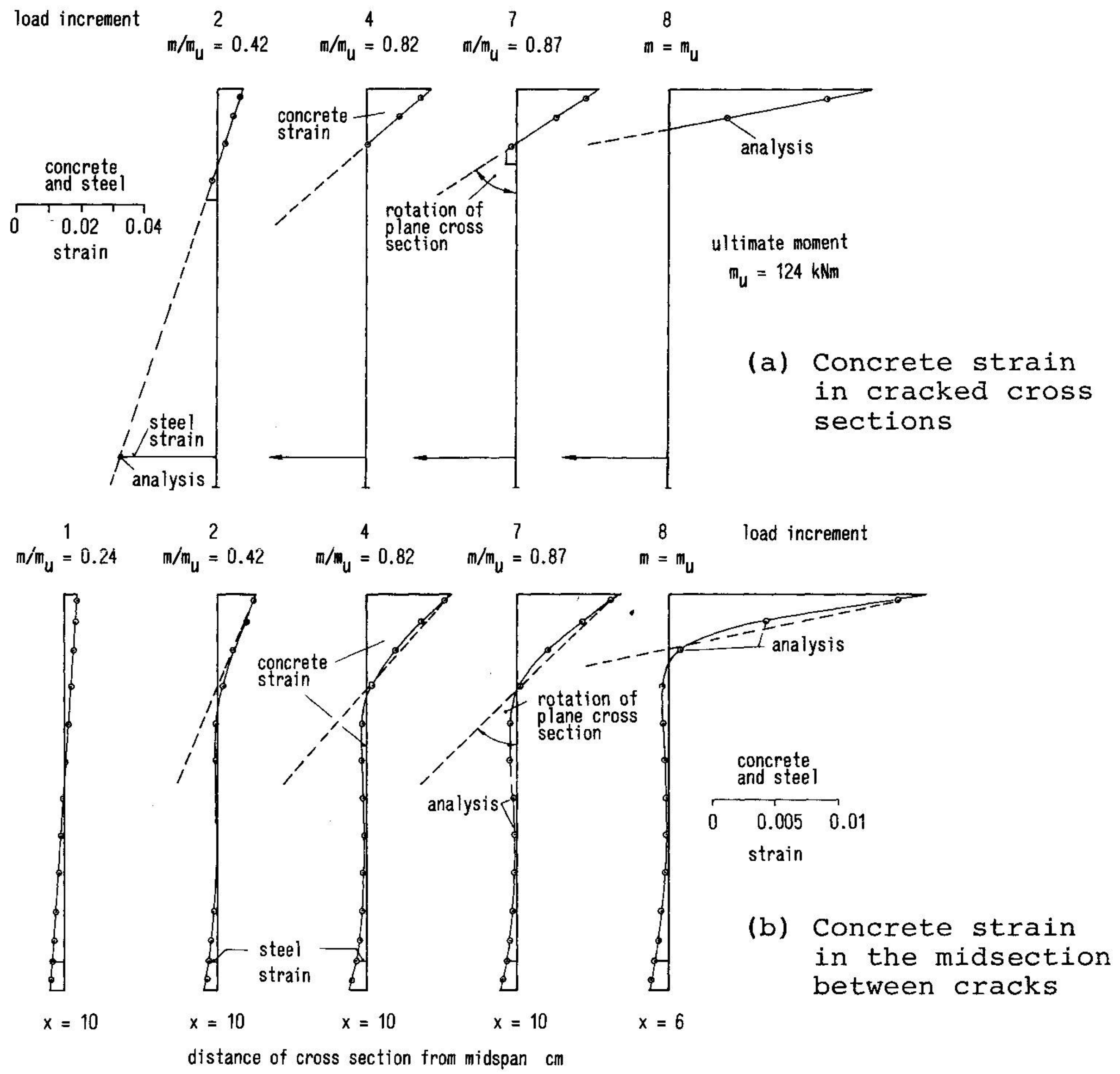


Fig. 19 Longitudinal Strain Distribution at Cross Sections for Several Load Increments

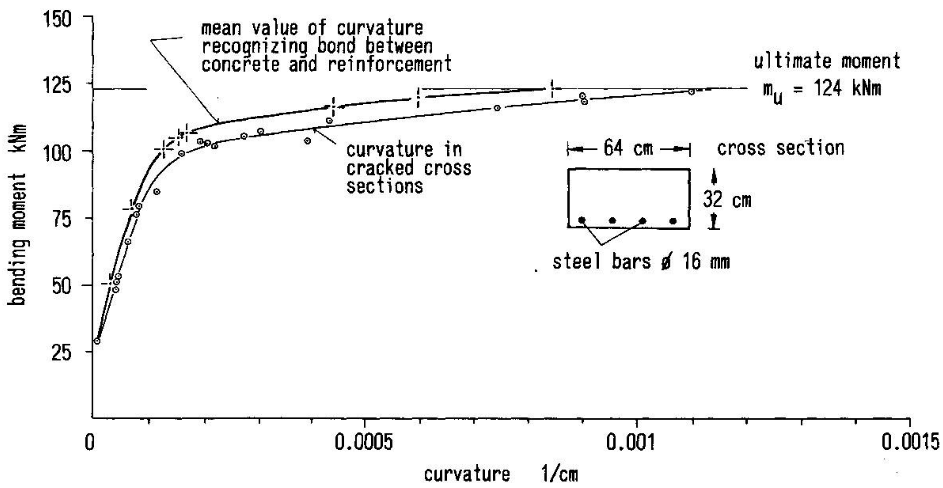


Fig. 20 Analytical Moment-Curvature Relation for the Reinforced Concrete Beam [10]

The mean value of curvature versus bending moment is shown in Fig. 20 and is compared with the moment-curvature curve resulting alone from the rotation of the cracked sections. It may be of some interest to realize that for equal bending moments the mean curvature is less than the curvature of cracked sections. The difference between both curves is growing with increasing plastic steel strains at cross sections.

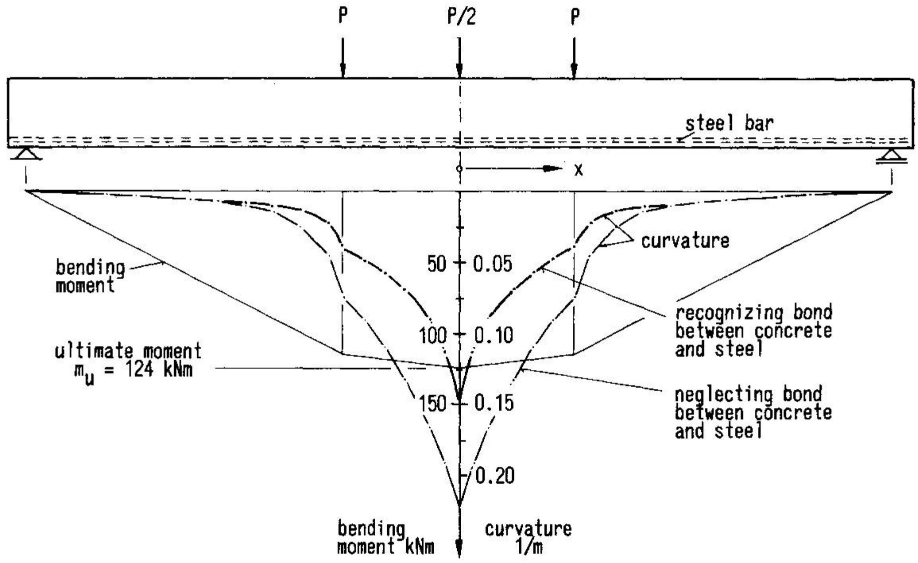


Fig. 21 Variation of Curvature along Beam Axis for Ultimate Load

The variation of curvature along the beam axis for the ultimate loading stage using both moment-curvature relations given in Fig. 20 is presented in Fig. 21. It can be seen by comparing both curves that the mean curvature which is recognizing the influence of bond on steel deformations gives a much more realistic basis for the calculation of beam deflections than the commonly for cracked sections used relation (deeper curve) which is neglecting this effect.

6. CONCLUSIONS

The finite element concept has been used for the development of an analytical method for reinforced concrete beams due to static loading which permits a simulation of beam behaviour through the entire range of loading. The accuracy of results show the validity of the proposed mechanical model which includes all important effects. It can be concluded from the results given in this paper that the true state of stress and strain as well as the actual crack pattern and deflections of a reinforced concrete beam can only be obtained from analysis if a realistic concept for bond is used recognizing a different bond-slip behaviour near cracks and in some distance from cracks respectively in uncracked regions of the beam. The moment-curvature relation obtained from analytical results shows furthermore the significant influence of bond on internal deformations which cannot be neglected without essential loss of accuracy if such a relation is used as a basis for simplified calculation of beam deflections.

REFERENCES

1. EIBL, J., IVÁNYI, G.: Studie zum Trag- und Verformungsverhalten von Stahlbeton, Deutscher Ausschuß für Stahlbeton Heft 260 (1976)

2. PLAUK, G.: Ermittlung der Verformungen biegebeanspruchter Stahlbetonbalken mit der Methode der Finiten Elemente unter besonderer Berücksichtigung des Verbundes zwischen Beton und Stahl, Dissertation Berlin TU,Berlin (1977)

3. SAUGY, B.: Contribution à l'étude théorique de comportment non linéaire des structures massives en béton armé sous charges rapides, Bulletin Technique de la Suisse Romande No. 22, Nov. 1969

4. FRANKLIN, H.A.: Nonlinear Analysis of Reinforced Concrete Frames and Panels, SESM Report No. 70-5, University of California, Berkeley, March 1970

5. LASSKER, A.J.: Nonlinear Behavior of Reinforced Concrete Beams by the Finite Element Method, Ph.D. Thesis, Department of Civil Engineering, Oklahoma, State University, 1972

6. NAM, C.H.: A Finite Element Study on the Nonlinear Behavior of Reinforced Concrete Flexural Members due to Short-Time Loading, Ph.D. Thesis, Civil and Environmental Engineering, University of Wisconsin, 1973

7. NILSON, A.H.: Finite Element Analysis of Reinforced Concrete Ph.D. Thesis, University of California, Berkeley, 1967

8. KUPFER, H.: Verhalten des Betons unter zweiachsiger Beanspruchung - Lehrstuhl für Massivbau TH München, Bericht Nr. 78 (1969)

9. EIFLER, H.: Drehfähigkeit plastischer Gelenke in biegebeanspruchten Stahlbetonkonstruktionen, Bericht Teil C: Verbunduntersuchungen an Rippentorstahl unter bauteilähnlichen Verhältnissen als Grundlage zur theoretischen Ermittlung der Drehfähigkeit plastischer Gelenke in Stahlbetonkonstruktionen, Bundesanstalt für Materialprüfung (BAM), Berlin 1974

10. EIFLER, H.: Bericht über Voruntersuchungen für die Ermittlung des Werkstoff- und Verbundverhaltens im Bereich plastischer Gelenke von Stahlbetonplatten bei statischer Belastung Bundesanstalt für Materialprüfung (BAM), 2.2/119 31 (1969)