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Evaluation of Tension Stiffening Effects in Reinforced Concrete Linear Members

Détermination des effets de "tension stiffening" dans des éléments linéaires en béton armé

Reststeifigkeit von stabförmigen Stahlbetonbauteilen

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SUMMARY
The stiffness of reinforced concrete beams is increased by tensile concrete stresses between cracks. Methods to evaluate this stiffening effect are described and results obtained using these methods are compared with test results. A satisying agreement is found using several methods for first loading. For the case of reloading a calculation procedure is proposed.

RÉSUMÉ
Entre les fissures dans une poutre en béton armé existent des contraintes de traction. Ces contraintes contribuent à la raideur. Cette communication a pour objet la considération et le calcul de l'augmentation du raideur. Les résultats ont montré une bonne concordance avec les essais.

ZUSAMMENFASSUNG
In Stahlbetonbalken bestehen zwischen den Rissen Betonzugspannungen, die einen Beitrag zur Steifigkeit liefern. Methoden zur Berücksichtigung dieser Steifigkeitserhöhung werden beschrieben und die mit ihnen ermittelten Resultate mit Versuchsergebnissen verglichen. Dabei kann mit verschiedenen Methoden gute Übereinstimmung für Erstbelastung erzielt werden. Für den Fall der Wiederbelastung wird ein Berechnungsverfahren vorgeschlagen.

1. INTRODUCTION

Reinforced concrete sections are usually designed for bending moments and axial forces neglecting any resistance of concrete to tensile stresses. This approximation can well be made for cracked sections. Between cracks, tensile stresses are transferred to the concrete by means of bond stresses between reinforcement and adjacent concrete. Thus, the steel stresses are reduced between cracks and the stiffness is increased with respect to the cracked state without any contribution of concrete in tension. This effect which has been called the "tension stiffening effect of concrete" is of considerable interest in members with a small amount of reinforcement.

Numerous methods have been proposed in the literature for the treatment of tension stiffening effects, many of them being fundamentally different from each other. Since there have been at least ten new proposals ([1] to [9] , [17]) in the last three years, it is now necessary to study the results of the existing procedures and to compare them with test results in order to determine whether further refinements and new proposals are still necessary.

2. METHODS FOR THE EVALUATION OF TENSION STIFFENING EFFECTS FOR FIRST LOADING

2.1 Complete description of steel stress distribution

By integrating the differential equation of bond with the help of an appropriate bond-slip relationship and boundary conditions, a complete description of steel and concrete stresses and strains along the axis of reinforcement can be developed. NOAKOWSKI [3] has used this procedure for the determination of effects due to temperature gradients, where it can be assumed that between cracks remain undisturbed regions.

PLAUK [8] has developed an iterative procedure to determine the steel stress distribution by assuming a parabolic distribution of bond stresses. According to his proposal the maximum value of bond stress depends on a bond-slip relationship derived from pullout tests.

2.2 Determination of average steel stress

The average steel stress σ_m is calculated from

$$\sigma_m = \sigma_s - \Delta\sigma_s \qquad (1)$$

where σ_s is the steel stress in a cracked section without any contribution of concrete tensile stresses and $\Delta\sigma_s$ is the average reduction of steel stress due to tension stiffening effects. From equilibrium considerations, $\Delta\sigma_s$ can be determined as

$$\Delta\sigma_s = c\, f_{tc}\, \frac{A_{ct}}{A_s} \qquad (2)$$

with:
- c = empirical factor
- f_{tc} = concrete tensile strength
- A_{ct} = area of concrete in tension
- A_s = area of reinforcement cross-section

YU, WINTER [1o] have expressed the tensile strength of concrete in eq.(2) in terms of the compressive strength and propose the equation

$$\Delta\sigma_s = o{,}o2\ f_c^{2/3} \ \frac{b\ h\ (h - x)}{A_s\ z} \tag{3}$$

where: f_c = compressive strength of concrete
b = width of cross-section in the tension zone
h = total height of section
x = height of compression zone
z = lever arm of internal forces

In eq. (3), $\Delta\sigma_s$ and f_c must be expressed in MN/m^2.

By studying the experimental evidence, RAO [11] has found that the reduction $\Delta\sigma_s$ due to tension stiffening is reduced when the applied forces increase. A good fit of experimental data was found with the expression

$$\Delta\sigma_s = o{,}18\ \frac{\sigma_{sr}}{\sigma_s}\ f_{tc}\ \frac{bd}{A_s} \tag{4}$$

where: σ_{sr} = steel stress in a cracked section immediately after cracking
σ_s = steel stress in a cracked section at the load level considered
d = effective height.

Eq. (4) was further modified for the CEB-FIP Model Code for Concrete Structures [12] :

$$\Delta\sigma_s = c\ \frac{\sigma^2_{sr}}{\sigma_s} \tag{5}$$

where c is a factor depending on bond characteristics and type of loading.

2.3 Fictitious stress-strain relationship for steel

Already in 195o, MURASHEV proposed to multiply the elastic modulus of steel by an empirical factor to take into account the stiffening effect of concrete in tension. The appropriate factor was determined from experiments in function of concrete strength, percentage of reinforcement, steel stress and loading characteristics.

Recently, GILBERT, WARNER [2] have made a refined proposal for a fictitious stress-strain relationship to be used in the analysis of slabs (fig.1).

2.4 Fictitious stress-stain relationship for concrete in tension

In analogy to the consideration of average steel stresses along the beam axis, also average concrete stresses in the tension zone of reinforced concrete beams can be considered. In the last few years, there have been several proposals to relate these to the average tensile strains in the tension zone. In figure 2, such fictitious relationships used by SCANLON, MURRAY [13] , LIN, SCORDELIS [15], CHITNUYANONDH et al. [5] and COPE et al. [7] are compared. It can be concluded that there is very little agreement on the shape of such a stress-strain function. Other proposals have been made by MACCHI, SANGALLI [6] (constant concrete tensile stress in the tension zone), QUAST [9] (analogous stress-strain relationship as in compression), and GILBERT, WARNER [2].

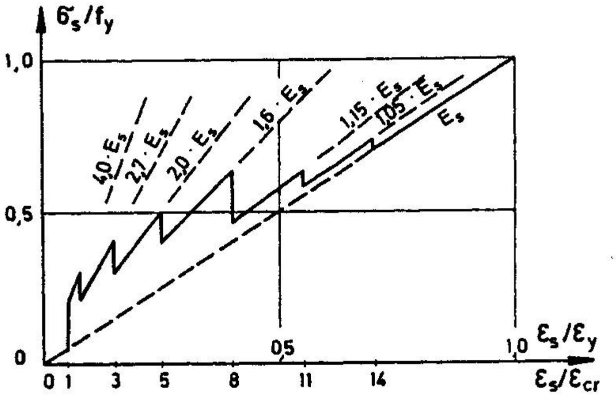

Fig.1: Fictitious stress-strain relationship for steel proposed by GILBERT, WARNER [2]

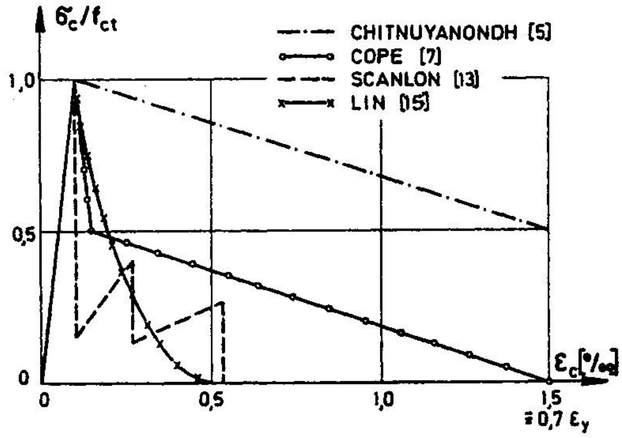

Fig.2: Comparison of fictitious stress-strain relationships for concrete in tension

2.5 Fictitious additional reinforcement

The average tensile force in the concrete between cracks can also be represented by a fictitious reinforcement to be added to the real reinforcement. The German concrete code (DIN 1o45) recommends to enlarge the tensile reinforcement by 1o %. Another approach is followed by CAUVIN [4] who has derived the equation

$$\Delta A_s = c \frac{f_{tc} \cdot b\ (d - x)}{\sigma_s} \tag{6}$$

where: ΔA_s = area of fictitious reinforcement
c = factor, can be taken as 1/6
σ_s = stress in reinforcement ($A_s + \Delta A_s$)

This proposal can be regarded as a special case of the methods discussed in chapter 2.2.

2.6 Determination of effective stiffness

The stiffness of reinforced concrete beam elements can also be determined directly without explicit evaluation of the average stress and strain state in the tension zone by means of moment-curvature relations.

KRAEMER, THIELEN, GRASSER [14] have used piecewise linear moment-curvature functions which are characterized by the points:
- cracking moment and corresponding stiffness of uncracked section,
- yield moments of tension and compression reinforcement and corresponding stiffness of cracked section
- ultimate limit state.

The fact that the formation of cracks in reinforced concrete beams is influenced by the random nature of the concrete tensile strength is considered by RAUE, TUNG [17].

For the calculation of deflections, the ACI Building Code (318-71 and 318-77) gives a simple expression for the effective moment of inertia to be used (originally developed by BRANSON):

$$I_{eff} = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr} \qquad (7)$$

where: I_{eff} = effective moment of inertia
I_g = gross moment of inertia
I_{cr} = moment of inertia of cracked transformed section
M_{cr} = cracking moment
M_{max} = maximum moment

3. COMPARISON WITH TEST RESULTS

The results which are obtained using some of the methods described in chapter 2 are compared with test results. For the calculations, a parabolic-rectangular stress-strain relationship for concrete in compression is used as defined in the CEB-FIP Model Code [12] . The maximum concrete stress is taken as 85 % of the concrete cube strength f_c. The tensile strength f_{tc} of concrete is evaluated from

$$f_{tc} = 0{,}30\ f_c^{2/3} \qquad (8)$$

It must be noted that the behaviour of reinforced concrete beams is subject to random variations even if they are produced under laboratory conditions. Therefore, small deviations between test results and calculated values cannot be regarded to be essential.

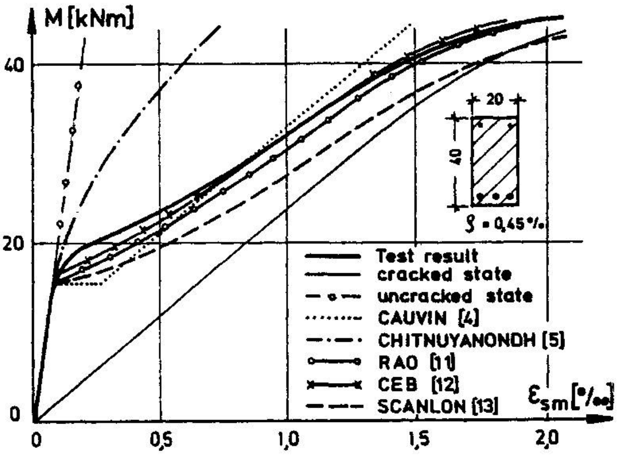

Fig.3: Mean strain of tension reinforcement as a function of bending moment for beam 4 of CLARK, SPEIRS [1]

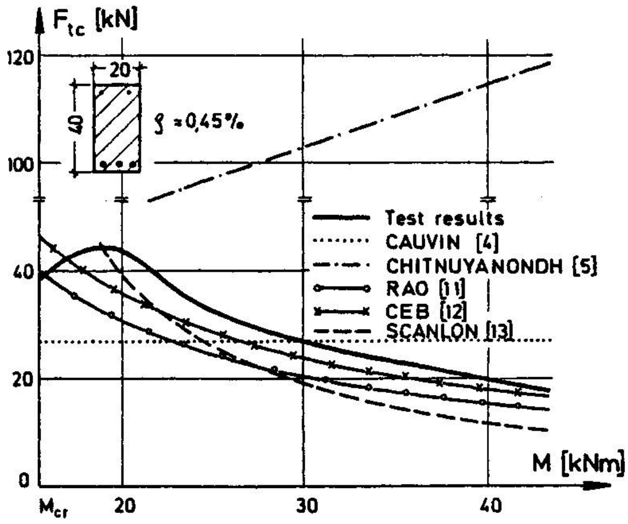

Fig.4: Tension stiffening force as a function of bending moment for beam 4 of CLARK, SPEIRS [1]

In fig. 3, the average strains ε_{sm} in the tensile reinforcement calculated with different procedures are compared with test results obtained by CLARK, SPEIRS [1] . The strains in an uncracked section and in a cracked section without any contribution of concrete in tension are also shown. The fact that the reduction of average tensile strains with respect to the cracked state is diminishing when the applied bending moment is increased, is well reflected by most methods. Only the fictitious stress-strain relationship for concrete in tension proposed for prestressed concrete wall segments by CHITYUNDANONDH et al 5 leads to unrealistic results in this case. The results of the procedures of RAO [11] and the CEB-FIP Model Code [12] show excellent agreement with the test results.

The tension stiffening effect can also be described by a tensile force F_{tc} in the concrete, which is shown in fig. 4 as a function of the applied moment for the same beam of CLARK, SPEIRS. For the test results and the methods which do not explicitly define the average tensile stress state of concrete, the concrete tensile force is assumed to act at the level of the reinforcement and can be defined by

$$F_{tc} = (\sigma_m - \sigma_{II}) \cdot A_s$$

where: σ_m = average stress in the reinforcement
σ_{II} = stress in a cracked section under the same bending moment
A_s = area of reinforcement cross-section

It can be stated that the tension stiffening force is well approximated by most methods.

For the evaluation of deformations, it is necessary to determine the stiffness of beam elements. The stiffness is in most cases defined by means of moment-curvature relations. In fig. 5, curves which are derived theoretically are compared with the values determined by CLARK, SPEIRS from average strain measurements. For the methods which do not define the tensile stress state of concrete, it is assumed that the maximum concrete strain can be taken as the value calculated for a cracked section. It can be seen that the results of different procedures agree quite well and that the stiffness is, for the example considered, somewhat underestimated.

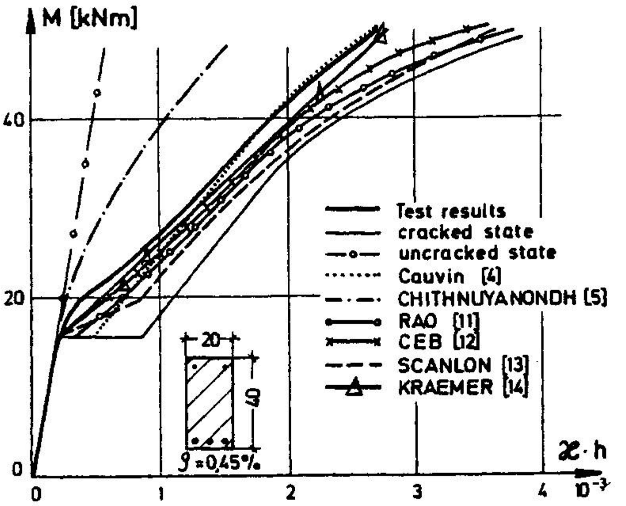

Fig.5: Moment-curvature relationship for beam 4 of CLARK, SPEIRS [1]

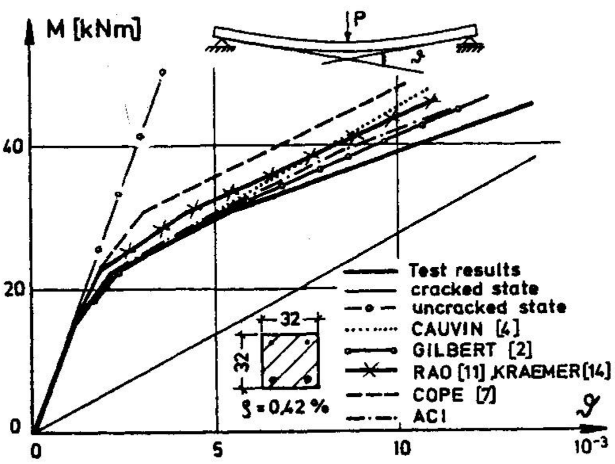

Fig.6: Rotation of support sections as a function of maximal moment for beam 1.13 of EIFLER, PLAUK [16]

Since there are no deformations recorded in [1], measured and calculated deformations are compared in fig.6 for a beam tested by EIFLER, PLAUK [16]. To determine the deformations, moment-curvature relations have been derived in an analogous way as for fig.5. Also, the effective stiffness method of the ACI code (eq.7) is included in the comparison. For all calculations, shear deformations have been disregarded. It can again be stated that the calculated deformations agree reasonably well with the test results for most methods, but are in this case somewhat smaller than the recorded values indicating an overestimation of stiffness.

The distribution of internal forces in reinforced concrete beams is in general not very much affected by tension stiffening effects. But this influence can be large for internal forces due to actions which depend directly on stiffness such as imposed deformations. If imposed deformations are dominant, it must be considered that the crack pattern may not be complete (NOAKOWSKI [3], MACCHI, SANGALLI [6])

Since there are very few experiments with imposed deformations, a theoretical example is considered. A fixed end beam is acted upon by a constant temperature gradient $\Delta t = 20^{\circ}$, which produces negative moments almost of the magnitude of the cracking moment. Fig.7 shows the maximum support and span moments of the beam as a function of an applied distributed load p. It can be seen that there is a large difference between the moments calculated for the uncracked and cracked state, the latter being determined without taking into account concrete tensile stresses. The tension stiffening effect of concrete influences the distribution of bending moments up to a load level where the maximum support moment reaches half of the yield moment M_y. For larger loads, the distribution of bending moments is very close to the fully cracked state.

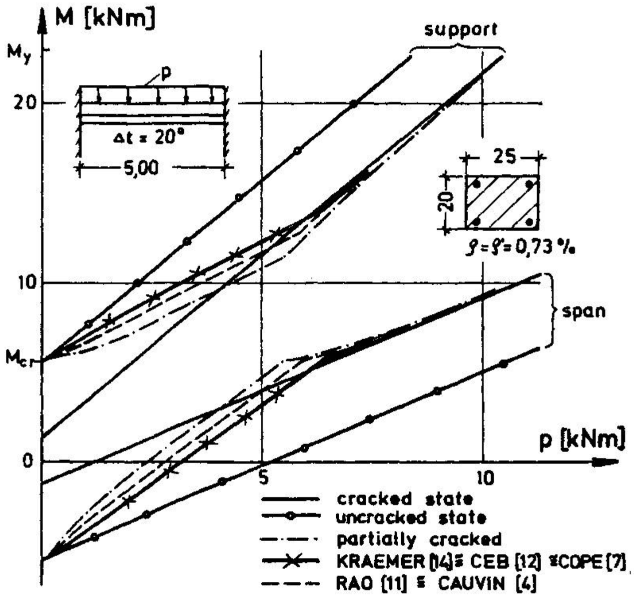

Fig.7: Calculated maximum support and span moments for distributed load p and temperature gradient Δt

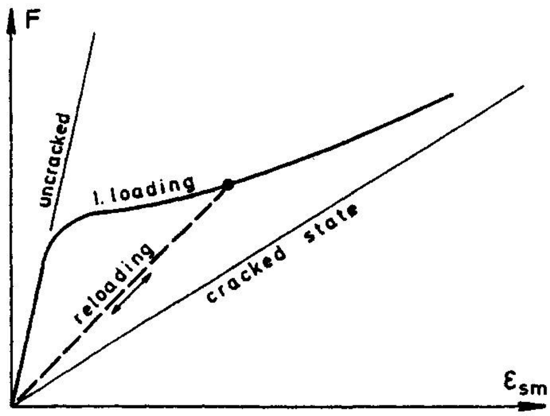

Fig.8: Mean steel stress at reloading according to SCHLAICH et al [18]

4. INFLUECNE OF LOAD REPETITIONS

It is a well known fact, that the influence of concrete tensile stresses on the stiffness of reinforced concrete beams is reduced by load repetitions, but there have been much less efforts to evaluate this reduction. KRIPANARAYANAN, BRANSON [2o] have modified the ACI formula (eq.7) to account for the influence of load repetitions

$$I_{rep} = \psi\, I_{eff} + (1 - \psi)\, I_g \qquad (1o)$$

with: $\psi = (P_{ult}-P_{rep})/(P_{ult}-P_{cr})$

I_{rep} = effective moment of inertia for load repetions
I_{eff} = effective moment of inertia without load repetions (eq.7)
I_g = gross moment of inertia
P_{ult} = ultimate load estimated with ACI code procedures
P_{rep} = maximum load of load repetition
P_{cr} = load at initial cracking

SCHLAICH, SCHOBER, KOCH [18] have performed tests on reinforced concrete tubes under axial load and bending. They have studied the stiffening effect of concrete in tension also during 3o load cycles of the transverse force with the upper load limit being the working load according to the German concrete code (DIN 1o45). From their experiments they deduce that the mean steel strain in the tensile reinforcement can be determined from a straight line connecting the strain at the maximum preload (fig.8) with the origin.

The tension stiffening effect is only considerable in beams with a small amount of reinforcement. In such beams the maximum steel strain is much larger than the maximum concrete strain in the compression zone. Considering these facts, it can be assumed that an analogous approximation can also be made for the moment-curvature relationship (fig.9). It must be noted that the reloading branch can only be assumed to go back to the origin, if the reinforcement has not exhibited plastic deformations. In the latter case, the curvature $\varkappa_{pl}$ due to plastic strains must remain after unloading.

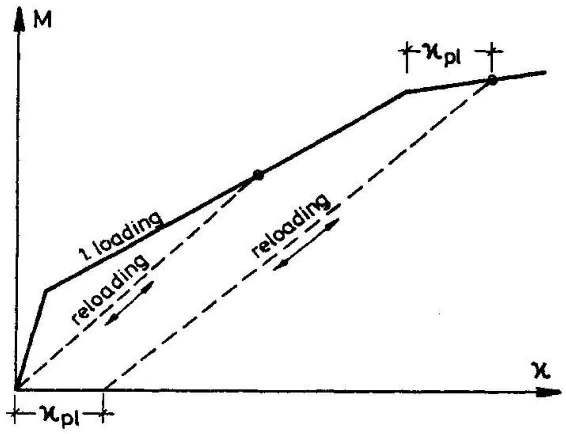

Fig.9: Moment-curvature relations for reloading

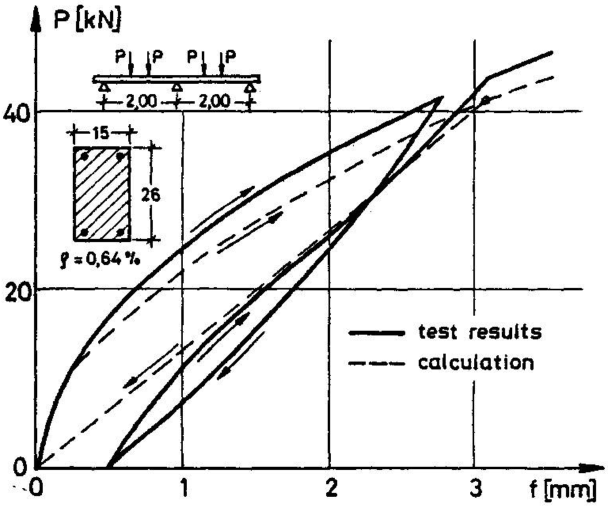

Fig.1o: Calculated and measured deflections at first loading and reloading of beam B 2 of MONNIER [19]

This approximation of the curvature at unloading together with a trilinear moment-curvature relationship for the first loading are used to calculate the maximum deflection of a test beams of MONNIER [19] . It can be seen, that the deflections are also reasonably well determined during unloading and reloading except for very small loads (fig.1o).

5. CONCLUSIONS

Methods to evaluate the stiffening effect of concrete tensile stresses between cracks for first loading and reloading have been described and their results have been compared with test results. It can be concluded that the influence of concrete tension at first loading on mean steel stresses, moment-curvature relations and deformations can be well approximated with most methods. Some of the proposed fictitious relationships between average concrete tensile strains and stresses cannot be used for reinforced concrete beams. It has been found that tension stiffening effects influence considerably the distribution of internal forces in statically indeterminate beams due to imposed deformations when additional external loadings are not dominant.

The stiffening effect of concrete in tension is reduced at reloading and can be evaluated approximately using a modified moment-curvature relationship.

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