Zeitschrift: IABSE reports of the working commissions = Rapports des commissions de travail AIPC = IVBH Berichte der Arbeitskommissionen

Band: 34 (1981)

Artikel: Rational analysis of shear in reinforced concrete columns

Autor: Minami, Koichi / Wakabayashi, Minoru

DOI: https://doi.org/10.5169/seals-26918

Rational Analysis of Shear in Reinforced Concrete Columns

Analyse rationnelle de "la contrainte de cisaillement" des poteaux en béton armé

Zweckmässige Berechnung der Querkraft in Stahlbetonstützen

KOICHI MINAMI
Lecturer
Department of Architecture
Osaka Institute of Technology
Osaka, Japan

MINORU WAKABAYASHI
Professor
Disaster Prevention Research Institute
Kyoto University
Uji, Japan

SUMMARY

Proposed in this paper is an evaluation method of the strength of reinforced concrete columns under combined axial, bending and shear stresses by an easy and convenient analysis. In the analysis, shear resistances of the beam- and the arch -mechanisms, which satisfy a statically admissible stress field, are first obtained and the ultimate shear strength of the column is computed applying a modified and extended additive strength theory.

RÉSUMÉ

On propose ici une méthode d'évaluation de la résistance des poteaux en béton armé sous l'effet des contraintes axiales, fléchissantes et de cisaillement, suivant une analyse simple et commode. Dans cette analyse, les résistances de "la contrainte de cisaillement" du mécanisme de la poutre et de l'arc, qui satisfont un champ de force statiquement admissible, sont obtenues en premier, et la résistance limite de "la contrainte de cisaillement" du pilier est estimée, en appliquant une théorie des forces d'addition modifiées et étendues.

ZUSAMMENFASSUNG

In dieser Abhandlung wird eine Methode vorgeschlagen, die durch eine einfache und passende Analyse die Tragfähigkeit von Stahlbetonstützen unter der kombinierten Axial-, Biege- und Querkraft evaluiert. In der Analyse werden zuerst die Querwiderstände der Balken- und Bogenmechanismen, die einem statisch zulässigen Spannungsfeld Genüge leisten, festgestellt; dann wird der äusserste Schubwiderstand der Stützen berechnet, indem man eine modifizierte und erweiterte Widerstandsadditionstheorie anwendet.

1. INTRODUCTION

Extensive studies have been made to estimate shear strength of concrete members, including ordinary reinforced concrete, prestressed concrete, mixed steel and concrete members, by means of plastic analysis[1-9]. There already is an established method of determining the flexural strength of reinforced concrete members when subjected to compression and bending. Also, it was proved that an extended additive strength theory, which is analytically very simple and clear, is practically applicable to the same problem.

Described in this paper is a trial to provide analytical approach to obtain the strength of reinforced concrete members with ordinary or diagonal main reinforcement under combined state of compression, bending and shear by applying the extended additive strength theory, and to systematically know the correlation among compressive, flexural and shear strengths of reinforced concrete members, which has often been discussed individually. Following are the method of analysis and the feature of the analytical solutions. Finally, the comparisons of theoretical prediction with experimental results are shown.

2. ORDINARY REINFORCED CONCRETE COLUMNS

2.1 Basic Assumptions in the Analysis

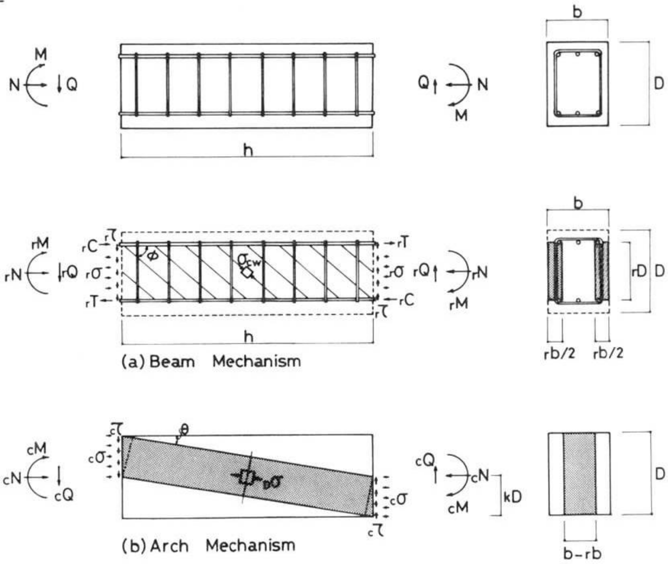


Fig.1 Resistance Mechanism of Reinforced Concrete Members

Generally, there are two types of shear transmitting mechanisms in reinforced concrete members ; beam mechanism and arch mechanism. To solve the shear problem, it is important to determine the shear resistance of each mechanism corresponding to elastic state to ultimate state. In this paper, however, two types of shear transmitting mechanisms are assumed on the basis of ultimate state of stress, as illustrated in Fig.1 ; the beam mechanism consists of longitudinal and shear reinforcements, and concrete with the width of ${}_rb$, and the arch mechanism consists of reinforcement-less concrete with the width of $(b-{}_rb)$. Once a set of strength, $({}_rM, {}_rN, {}_rQ)$ and $({}_cM, {}_cN, {}_cQ)$, is determined, which satisfies the static admissible stress field for each resistant mechanism, the member strength, (M,N,Q), is assumed to be given by,

$$M = {}_rM + {}_cM \ , \ N = {}_rN + {}_cN \ , \ Q = {}_rQ + {}_cQ \tag{1}$$

based upon the extended additive strength theory. Here, the stresses at the member ends are assumed to be in anti-symmetric flexural-shear state and the following relation is assumed to hold.

$$M / Q = {}_rM / {}_rQ = {}_cM / {}_cQ = h / 2 \tag{2}$$

Also, it is assumed for both beam and arch mechanisms, that plastic deformation is allowed to the extent where the component steel and concrete strengths can be added. Concrete material causes compressive deformation at and maintaining the stress equal to the cylinder compressive strength, F_c, and steel material causes either tensile or compressive plastic deformation at and maintaining the steel yield stress, ${}_r\sigma_y$ or ${}_{rw}\sigma_y$.

The analytical technique adopted in this paper is similar to those described in Ref.[3]. The difference is that this paper considers concrete without any reinforcements as arch mechanism, hence the advantage of this technique over others lies in its simplicity in treating truss nodal points in application.

2.2 ${}_rn$-${}_rq$ Equations for Beam Mechanism

As shown in Fig.1(a), the beam mechanism is assumed to consist of the three elements; longitudinal reinforcements that resist tension or compression, shear reinforcements that resist tension, and concrete blocks with the width of ${}_rb$, whose axis is twisted by the angle of ϕ from the member axis. The condition that the maximum shear resistance is attained for the beam mechanism gives $\phi=\pi/4$, then the non-dimensional shear strength of the beam mechanism, ${}_rq$ can be obtained by Eq.(3) when the shear resistance is determined by tensile yielding of the shear reinforcements, and by Eqs.(4) and (5) when determined by either tension or compression of the longitudinal reinforcements, respectively.

$$-2{}_r\mu_t + {}_r\mu_w\cdot({}_rD_1 + \eta) \leqq n \leqq 2{}_r\mu_t + {}_r\mu_w\cdot({}_rD_1 - \eta)$$
$${}_rq = {}_r\mu_w\cdot{}_rD_1 \tag{3}$$

$$-2{}_r\mu_t \leqq {}_rn \leqq -2{}_r\mu_t + {}_r\mu_w\cdot({}_rD_1 + \eta)$$
$$n = -2{}_r\mu_t + (1 + \eta / {}_rD_1)\cdot{}_rq \tag{4}$$

$$2{}_r\mu_t + {}_r\mu_w\cdot({}_rD_1 - \eta) \leqq {}_rn \leqq 2{}_r\mu_t$$
$$n = 2{}_r\mu_t + (1 - \eta / {}_rD_1)\cdot{}_rq \tag{5}$$

The maximum shear resistance of the beam mechanism is attained when both the top and the bottom longitudinal reinforcements yield simultaneously, and the resistant shear, ${}_rq_o$, is given by,

$${}_rq_o = 2{}_r\mu_t\cdot{}_rD_1 / \eta \tag{6}$$

and the required amount of shear reinforcements for the above shear resistance, ${}_r\mu_{wo}$, is given by,

$${}_r\mu_{wo} = 2{}_r\mu_t / \eta \tag{7}$$

This is the amount of shear reinforcements necessary to produce flexural-compression failure of the member. The concrete width required to constitute the beam mechanism, ${}_rb_1(={}_rb/b)$, varies depending on the magnitude of resistant shear of the same mechanism, that is, ${}_rb_1=2{}_r\mu_w$ when the capacity is determined by the tensile yielding of shear reinforcements (Eq.(3)), ${}_rb_1=4{}_r\mu_t/\eta$ when determined by the simultaneous yielding in tension and compression of the longitudinal reinforcements (Eq.(6)), and ${}_rb_1=2{}_rq/{}_rD_1$ when determined by Eqs.(4) and (5). Therefore, the width of resisting concrete for the arch mechanism, ${}_cb_1(={}_cb/b$, ${}_cb=b-{}_rb)$ can be expressed as,

$${}_cb_1 = 1 - 2{}_r\mu_w$$
$${}_cb_1 = 1 - 4{}_r\mu_t / \eta \tag{8}$$
and
$${}_cb_1 = 1 - 2{}_rq / {}_rD_1$$

depending, in the same order, on the above three cases.

2.3 ${}_cn$-${}_cq$ Equations for Arch Mechanism

It is assumed that the arch mechanism is constituted when the resultant uniaxial compressive stress, D^{σ}, of the nomal stress, ${}_c\sigma$, and the shear stress, ${}_c\tau$, both uniformly distributed over the compression region at both ends of the reinforcement-less concrete with the width ${}_cb$ which is the remaining width used for the beam mechanism, are produced in the direction that is off from the member axis by the angle of θ. Further, the maximum shear resistance of the arch mechanism is assumed to take place when the above-mentioned resultant stress, D^{σ}, reaches F_c, when the shear strength, ${}_cq$, can be expressed by Eq.(9).

$$ {}_cq = {}_cb_1 \{ \sqrt{4 \cdot {}_cn / {}_cb \cdot (1 - {}_cn / {}_cb_1) + \eta^2} - \eta \} / 2 \tag{9}$$

in which, $\quad 0 \leqq {}_cn \leqq {}_cb_1$

Therefore, by substituting the corresponding values of ${}_cb_1$ given above into Eq. (9), the shear strength of the arch mechanism can be computed depending on those values of the beam mechanism. Note that Eq.(9) can also be written in the form shown below.

$$ ({}_cq + {}_cb_1 \cdot \eta / 2)^2 + ({}_cn - {}_cb_1 / 2)^2 = {}_cb_1^2 \cdot (1 + \eta^2) / 4 \tag{10}$$

2.4 Application of Extended Additive Strength Theory

Next step of the analysis is computation of the strength of reinforced concrete member as a whole (n-q equation) by applying the additive strength theory using the above-obtained strengths of the beam mechanism (${}_rn$-${}_rq$ equation) and the arch mechanism (${}_cn$-${}_cq$ equation). However, the conventional theory of adding component strengths is not directly applicable to the shear problem under consideration. Illustrated in Fig.2 is the scheme of the strength addition for the current problem, where ${}_rI$ represents ${}_rn$-${}_rq$ interaction curve of the beam mechanism, and ${}_cI_a$, ${}_cI_b$ and ${}_cI_c$ represent ${}_cn$-${}_cq$ interaction curves of the arch mechanism associated with the change in concrete width, ${}_cb_1$. In the current analysis, the member strengths are to be calculated under the condition of, ${}_rb_1 + {}_cb_1 = 1$. Therefore, the additive strength can be obtained considering that the arch mechanism varies with the condition of the beam mechanism strength. For instance, in Fig. 2, the strength of the beam mechanism represented by the point "a" (Eq(3))

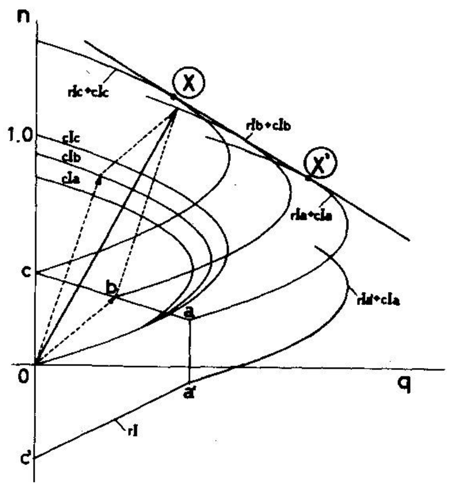


Fig.2 Extended Concept of Superposition

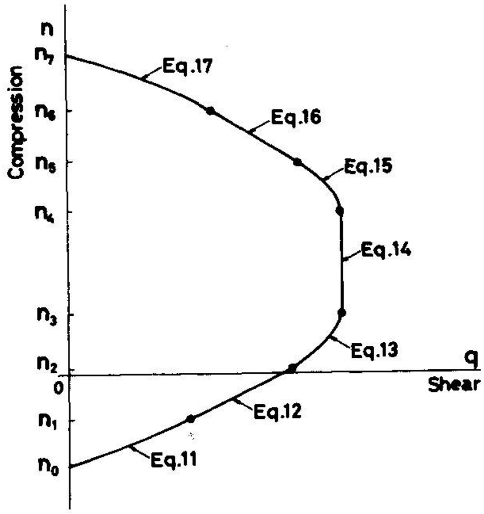


Fig.3 Interaction Cruves between Compression and Shear

should be added to that of the arch mechanism given by the curve ${}_cI_a$. Similarly, the strength of the point "b" (Eq.(5)) should be added to the curve ${}_cI_b$ and the point "c" to ${}_cI_c$. Therefore, the n-q interaction curve of the entire reinforced concrete member is given by the envelope of the curves obtained by the above-mentioned technique. The envelope is then expressed by the n-q equations, in which the shear strength, q, is given by Eqs.(11) through (17) depending on the magnitude of working compression, n(see Fig.3). In the current analysis, it is assumed that there exists linear relationship between the reduction ratio of the concrete width of the beam mechanism and the increase ratio of that of the arch mechanism. Therefore, the interaction curve between X and X' of the envelope in Fig.2 is represented by a straight line.

(a) $n_0 \leqq n < n_1$ $$q = \{\sqrt{4(n + 2{}_r\mu_t)(1 - n - 2{}_r\mu_t) + \eta^2} - \eta\} / 2 \quad (11)$$

(b) $n_1 \leqq n < n_2$ $$q = \lambda_1(n - n_1) + q_1 \quad (12)$$

(c) $n_2 \leqq n < n_3$ $$q = \delta\{\sqrt{4 n_a(1 - n_a) + \eta^2} - \eta\} + {}_r\mu_w \cdot {}_rD_1 \quad (13)$$

(d) $n_3 \leqq n < n_4$ $$q = \delta(\sqrt{1 + \eta^2} - \eta) + {}_r\mu_w \cdot {}_rD_1 \quad (14)$$

(e) $n_4 \leqq n < n_5$ $$q = \delta\{\sqrt{4 n_b(1 - n_b) + \eta^2} - \eta\} + {}_r\mu_w \cdot {}_rD_1 \quad (15)$$

(f) $n_5 \leqq n < n_6$ $$q = \lambda_2(n - n_6) + q_6 \quad (16)$$

(g) $n_6 \leqq n \leqq n_7$ $$q = \{\sqrt{4(n - 2{}_r\mu_t)(1 - n + 2{}_r\mu_t) + \eta^2} - \eta\} / 2 \quad (17)$$

in which,
$$n_0 = -2{}_r\mu_t$$
$$n_1 = \{\beta_1 \cdot \gamma - \sqrt{(\beta_1 \cdot \gamma)^2 - \omega_1 \cdot \rho}\}\gamma / \omega_1 + (1 - 4{}_r\mu_t)/2$$
$$n_2 = 2\delta \cdot (n_1 + 2{}_r\mu_t) - 2{}_r\mu_t + {}_r\mu_w \cdot ({}_rD_1 + \eta)$$
$$n_3 = \delta - 2{}_r\mu_t + {}_r\mu_w \cdot ({}_rD_1 + \eta)$$
$$n_4 = \delta + 2{}_r\mu_t + {}_r\mu_w \cdot ({}_rD_1 - \eta)$$
$$n_5 = 2\delta \cdot (n_6 - 2{}_r\mu_t) + 2{}_r\mu_t + {}_r\mu_w \cdot ({}_rD_1 - \eta)$$
$$n_6 = \{\beta_2 \cdot \gamma + \sqrt{(\beta_2 \cdot \gamma)^2 - \omega_2 \cdot \rho}\}\gamma / \omega_2 + (1 + 4{}_r\mu_t)/2$$
$$n_7 = 1 + 2{}_r\mu_t$$
$$q_1 = \{\alpha \cdot \gamma + \sqrt{(\alpha \cdot \gamma)^2 - \omega_1 \cdot \rho_1}\}\gamma / \omega_1 - \eta / 2$$
$$q_6 = \{\alpha \cdot \gamma + \sqrt{(\alpha \cdot \gamma)^2 - \omega_2 \cdot \rho_2}\}\gamma / \omega_2 - \eta / 2$$

where,
$$\lambda_1 = \frac{\alpha \cdot \omega_1 - \{\alpha \cdot \gamma + \sqrt{(\alpha \cdot \gamma)^2 - \omega_1 \cdot \rho_1}\}\gamma}{\beta_1 \cdot \omega_1 - \{\beta_1 \cdot \gamma - \sqrt{(\beta_1 \cdot \gamma)^2 - \omega_1 \cdot \rho}\}\gamma}$$
$$\lambda_2 = \frac{\alpha \cdot \omega_2 - \{\alpha \cdot \gamma + \sqrt{(\alpha \cdot \gamma)^2 - \omega_2 \cdot \rho_2}\}\gamma}{\beta_2 \cdot \omega_2 - \{\beta_2 \cdot \gamma + \sqrt{(\beta_2 \cdot \gamma)^2 - \omega_2 \cdot \rho}\}\gamma}$$
$$n_a = \{n + 2{}_r\mu_t - {}_r\mu_w \cdot ({}_rD_1 + \eta)\} / 2\delta$$
$$n_b = \{n - 2{}_r\mu_t - {}_r\mu_w \cdot ({}_rD_1 - \eta)\} / 2\delta$$

$$\omega_1 = \alpha^2 + \beta_1^2 , \quad \omega_2 = \alpha^2 + \beta_2^2$$

$$\rho = \gamma^2 - \alpha^2 , \quad \rho_1 = \gamma^2 - \beta_1^2 , \quad \rho_2 = \gamma^2 - \beta_2^2$$

$$\alpha = ({}_rD_1 + \eta) / 2 , \quad \beta_1 = ({}_rD_1 + \eta - 1) / 2$$

$$\beta_2 = ({}_rD_1 - \eta - 1) / 2, \ \gamma = \sqrt{1 + \eta^2} / 2, \ \delta = (1 - 2 {}_r\mu_w) / 2$$

$$m = M / bD^2F_c , \quad n = N / bDF_c , \quad q = Q / bDF_c$$

2.5 n-q Interaction Curves for Reinforced Concrete Columns

Schematic example of the n-q interaction curves of reinforced concrete members obtained by the forementioned analysis are illustrated in Fig.4 for various amount of shear reinforcements, ${}_r\mu_w$. In the figure, the interaction curves for the case of ${}_r\mu_w=0$ and for ${}_r\mu_{wo}$(Eq.(7)) are shown in dotted lines, and those for the case of $\eta=\infty$, which corresponds to the case where the shear strength is computed from end moments of the member that fails in flexure under compression and bending, are shown in chained lines. When a reinforced concrete member is subjected to combined compression, bending and shear, it is learned that the strength of the member does not reach flexural failure strength under compression and bending because of the effect of shear, even if the member is provided with the amount of shear reinforcements enough to cause simultaneous yielding of the longitudinal reinforcements both in compression and tension. It is also noted that the region where the shear strength maintains the constant ratio to the compression increases with the decrease in the amount of shear reinforcements. This proves that the influence of compression upon the shear strength reduces as the amount of shear reinforcements decreases.

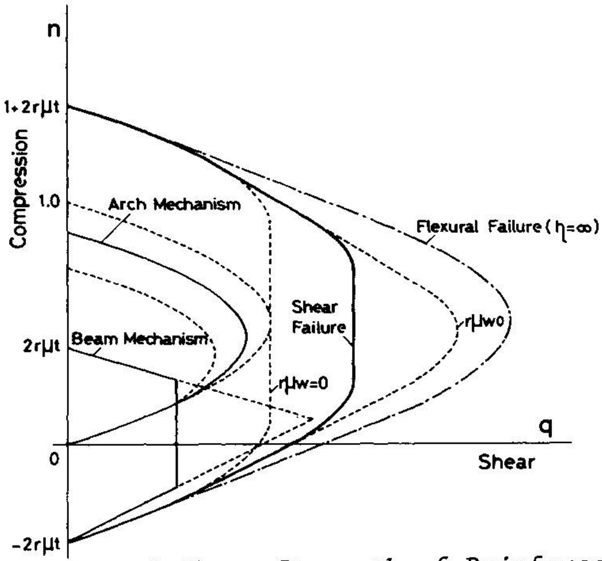


Fig.4 Shear Strength of Reinforced Concrete Column

Shown in Fig.5 are the n-q interaction curves of the reinforced concrete members with ${}_r\mu_t=0.2$, ${}_r\mu_w=0.05$ and ${}_rD_1=0.8$, computed for variable η . It is indicated that, for a constant value of ${}_r\mu_w$, the effect of compression on shear strength tends to become smaller as η becomes smaller. Hence, it is supposed that the compression tends to affect less in reinforced concrete members that fails in flexure than those that fails in shear.

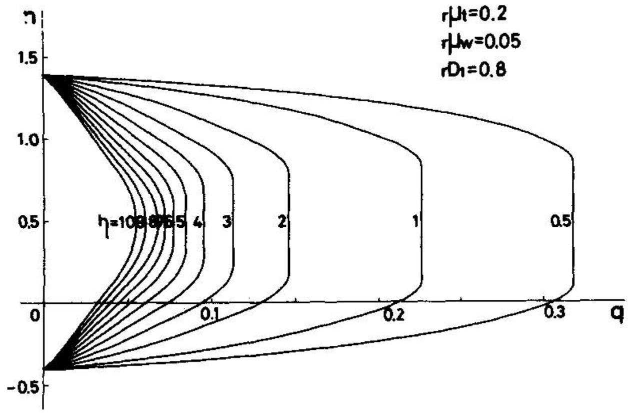


Fig.5 Nondimensional Interaction Curves between Compression and Shear

2.6 m-n-q Relationship of Reinforced Concrete Columns

Shown in Fig.6 are the m-q interaction relationship, transformed from the interaction curves given in Fig. 5, with n and η as variables. It is learned that, in the vicinity of the compression n=0.5 where either flexural or shear strength is the maximum, the m-q interaction curves are of concave configuration. In other words, the m-n-q interaction surface that represents the strength of a reinforced concrete member which predominantly fails in shear under combined compression, bending and shear does not show convex configuration.

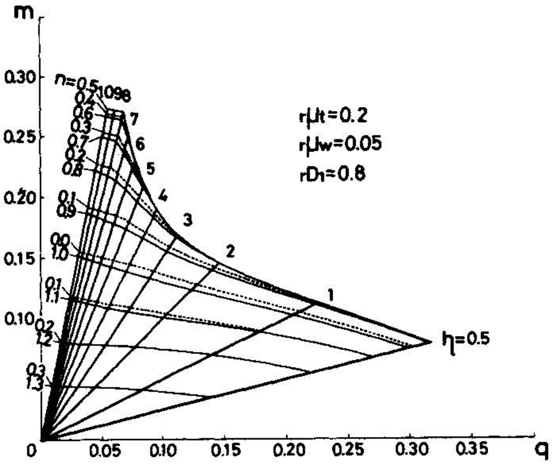


Fig.6 Nondimensional Interaction Curves between Bending and Shear

As an example of m-n-q interaction surface which contains a portion showing convex configuration, the interaction surface of the reinforced concrete members with $_r\mu_t$=0.2, $_r\mu_w$= 0.05 and $_rD_1$=0.8 are shown in Fig.7.

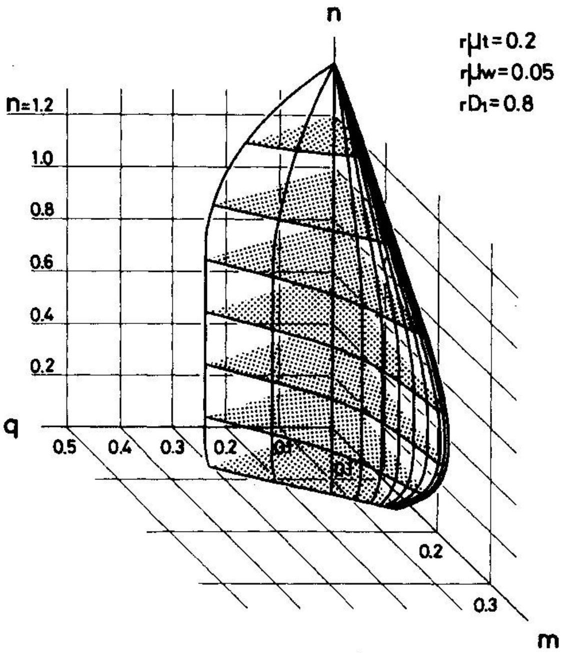


Fig.7 Nondimensional Failure Interaction Surface

Shown in Fig.8 is the m-q interaction curves for n=0.5, $_rD_1$=0.8 and $_r\mu_t$=0.25 with η and $_r\mu_w$ as variables. The outermost curve among these corresponds to the case where n=0.5 and the longitudinal reinforcements yield simultaneously in both tension and compression and shows that the m-n-q interaction surface is convex in all region when flexural failure predominates. Also, it is implied that the ratio of the flexural failure strength to the shear failure strength is greatly affected by the values of η and $_r\mu_w$, and the contribution of shear reinforcements to the strength capacity increase is smaller when η is smaller.

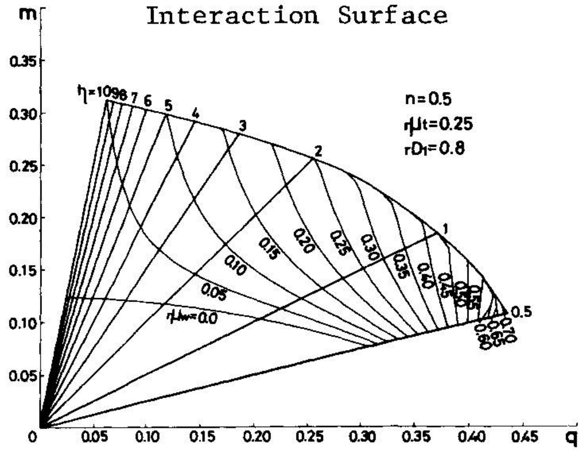


Fig.8 Nondimensional Interaction Curves between Bending and Shear

3. Diagonally Reinforced Concrete Columns

3.1 Failure Mechanism

Illustrated in Fig.9 is the ultimate state of columns with diagonal reinforcements resisting compression, bending and shear. The strength of a reinforced concrete column can be given by the sum of concrete strength and the strength of the system of diagonal reinforcements. Concrete, without reinforcement, only works against diagonally introduced compression and prevents compressed reinforcements from buckling. Therefore, no shear failure takes place in concrete although eccentric compressive failure may occur. One set of main reinforcements is subjected to uniform compression, and the other set to uniform tension. When they reach their yield stress, plastic deformation progresses. At this state, bondage between concrete and main reinforcements is not necessary to resist the external load, and no bond failure should take place.

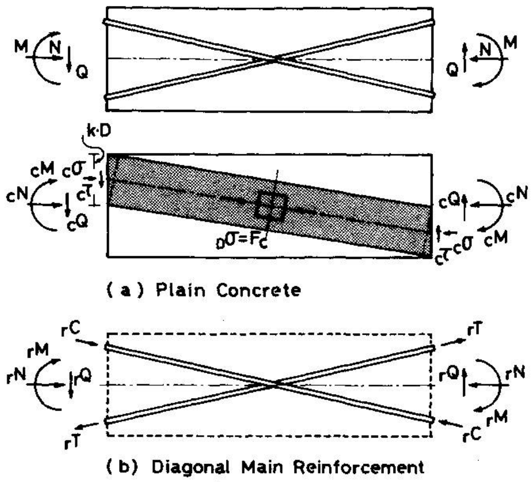


Fig.9 Equilibrium Analysis.

3.2 Equilibrium Analysis

Using a simple model of statical equilibrium of concrete and system of reinforcements as shown in Figs.8(a) and 8 (b), their strengths are obtained. The strength of a reinforced concrete member is then calculated by the extended concept of adding component strength of concrete and reinforcements. Computed shear strength of reinforced concrete columns with diagonal reinforcements in a non-dimensional expression are given below, depending on the non-dimensional magnitude of working compression, n as indicated in Fig.10.

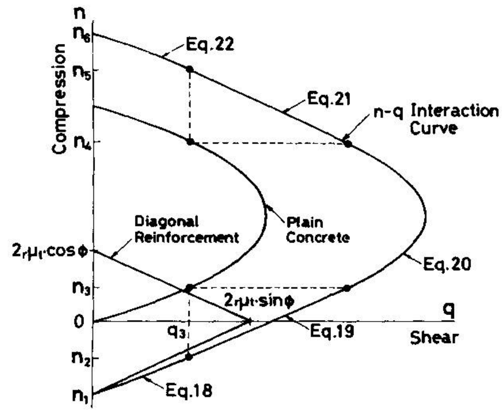


Fig.10 Method of Superposition.

(a) $n_1 \leqq n < n_2$ $$q = \frac{1}{2}\cdot\{\sqrt{\eta^2 + 4\cdot(n - n_1) - 4\cdot(n - n_1)^2} - \eta\} \quad (18)$$

(b) $n_2 \leqq n < n_3$ $$q = (n - n_3)\cdot\tan\phi + 2\cdot\mu_t\cdot\sin\phi + q_3 \quad (19)$$

(c) $n_3 \leqq n < n_4$ $$q = 2\cdot\mu_t\cdot\sin\phi + \frac{1}{2}\{\sqrt{\eta^2 + 4\cdot n - 4\cdot n^2} - \eta\} \quad (20)$$

(d) $n_4 \leqq n < n_5$ $$q = -(n - n_4)\cdot\tan\phi + 2\mu_t\cdot\sin\phi + q_3 \quad (21)$$

(e) $n_5 \leqq n \leqq n_6$ $$q = \frac{1}{2}\cdot\{\sqrt{\eta^2 + 4\cdot(n + n_1) - 4\cdot(n + n_1)^2} - \eta\} \quad (22)$$

in which $n_1 = -2\cdot\mu_t\cdot\cos\phi$, $n_2 = -2\cdot\mu_t\cdot\cos\phi + (1 - \lambda)/2$,

$n_3 = (1 - \lambda)/2$ and $\geqq 0$, $n_4 = (1 + \lambda)/2$ and $\leqq 1$,

$n_5 = 2\cdot\mu_t\cdot\cos\phi + (1 + \lambda)/2$, $n_6 = 1 + 2\cdot\mu_t\cdot\cos\phi = 1 - n_1$,

and $q_3 = \frac{1}{2}\cdot\{\sqrt{\eta^2 + 4\cdot n_3 - 4\cdot n_3^2} - \eta\}$

where $n = N / bDF_c$ (positive for compression) , $q = Q / bDF_c$,

$\mu_t = T_o / bDF_c$, $\lambda = \sqrt{\tan^2\phi\cdot(1 + \eta^2)/(1 + \tan^2\phi)}$, $\eta = h / D$

b : column width , D : column depth , h : column length ,

F_c : compressive strength of concrete ,

T_o : tensile yield strength of diagonal reinforcement ,

ϕ : angle between column axis and diagonal reinforcement ,

N : applied axial compression , Q : shear strength of column .

4. COMPARISON OF ANALYSIS WITH EXPERIMENTS

Followings are the comparisons of analytical prediction to the experimental results. As shown in Fig.11, the experiments of 184 column specimens with ordinary main reinforcements were performed by the authers and these tests were composed of four series, Series I to IV, and refered to references [10][11][12] and [13] respectively.

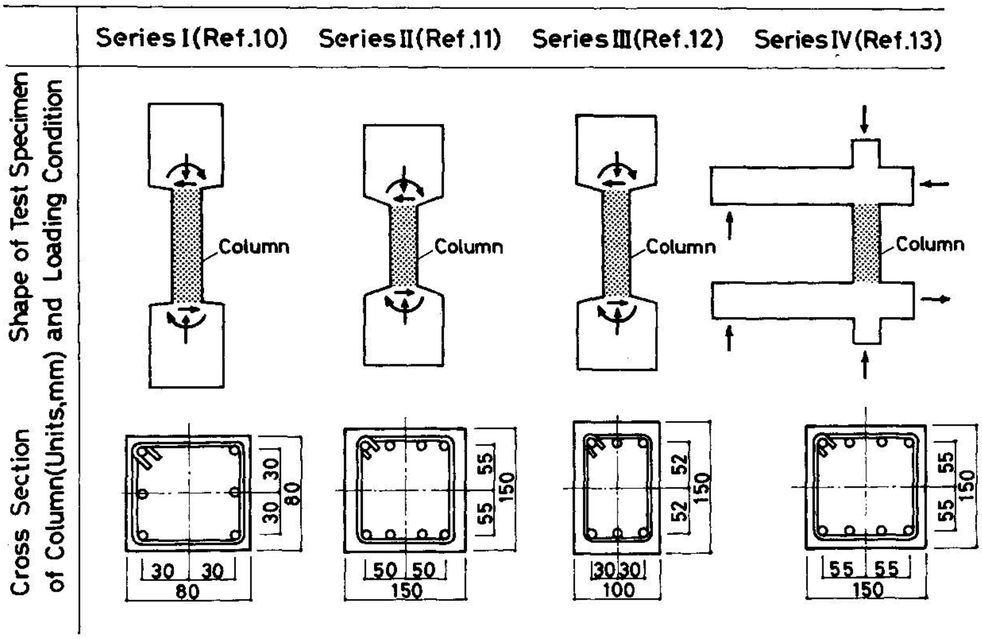


Fig.11 Shear Tests of Ordinary Reinforced Concrete Columns and Frames

Experimentally obtained failure interaction surface and that obtained by the analysis are compared in Fig.12, where the former was systematically made up from the experimental results on 100 test specimens of reinforced concrete members both with and without reinforcements of Series I[10]. In each of Fig. 12, three axes represent non-dimensional axial force (n), bending moment (m) and shear (q), and the figures on the left are drawn from the experimental results while those on the right show the analytical prediction. Series I-1(Fig.11(a)) is for the member without any reinforcement, that is, ${}_r\mu_t$=0 and ${}_r\mu_w$=0, and Series I-2 to I-4(Fig.12(b) to (d)) are for those of ${}_r\mu_t$=0.1 with ${}_r\mu_w$ being equal to 0, 0.05 and 0.1 respectively.

General trend common in all the series is that the analytically obtained failure interaction surfaces are slightly smaller than those obtained by the experiments. However, the configurational characteristics of both surfaces are alike each other. Here again, the failure interaction surface shows concave shape in the region where shear failure is predominant over flexural failure while the surface is convex for members with much shear reinforcements and larger values of η, hence the shear failure is unlikely to take place. Also, it is indicated that the influence of working axial force upon the shear capacity is less in those members with less amount of shear reinforcements and smaller value of η.

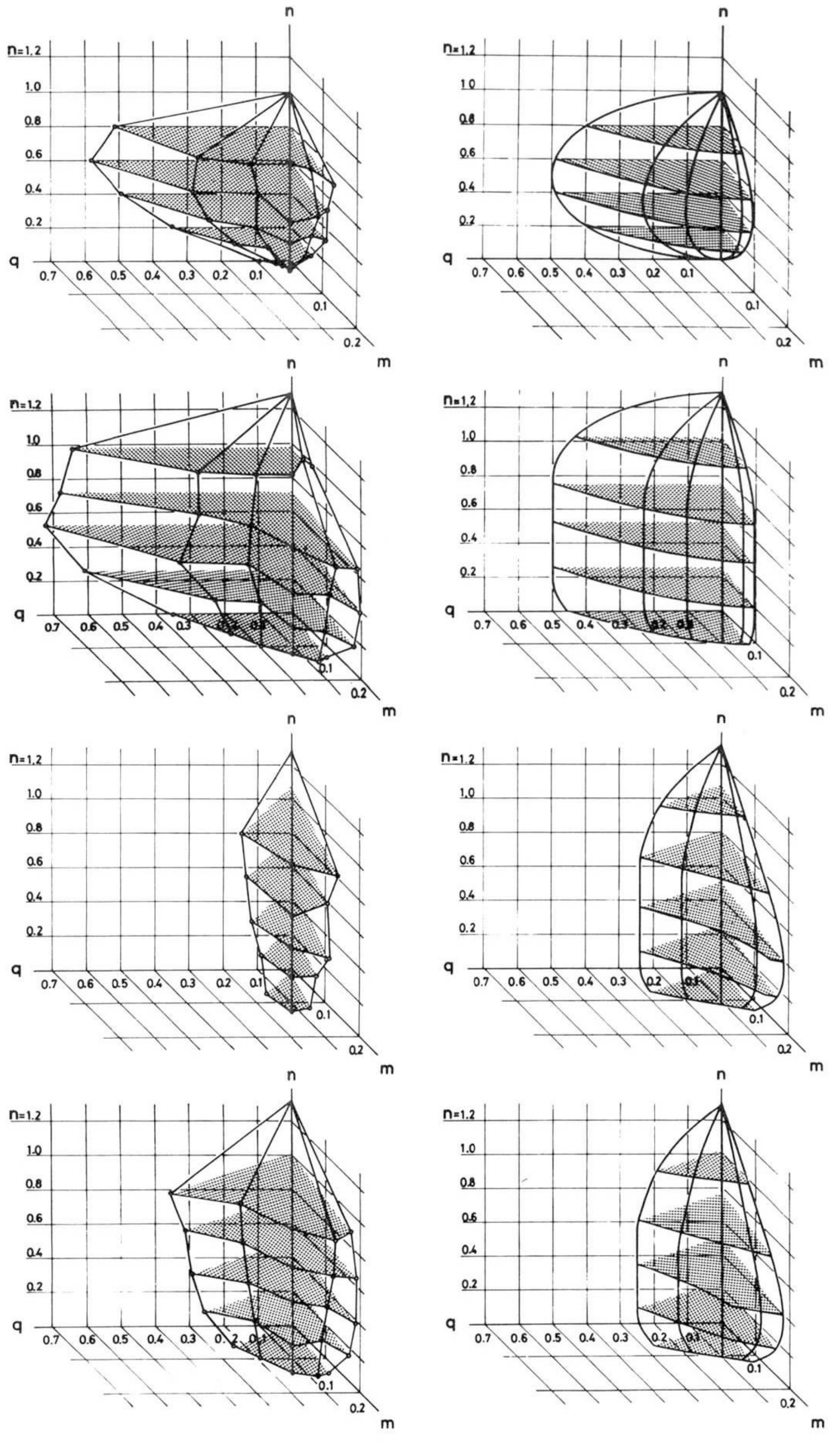
Figure 13 shows the histogram of the ratios of experimental results to analytical prediction obtained from Series II to IV. The mean value of the ratio is 1.08 and the standard deviation is 0.228.

(a) Series I-1
Plain
Concrete
$r^{\mu}{}_{t} = 0$
$r^{\mu}{}_{w} = 0$

(b) Series I-2
Reinforced
Concrete
$r^{\mu}{}_{t} = 0.1$
$r^{\mu}{}_{w} = 0$

(C) Series I-3
Reinforced
Concrete
$r^{\mu}{}_{t} = 0.1$
$r^{\mu}{}_{w} = 0.05$

(d) Series I-4
Reinforced
Concrete
$r^{\mu}{}_{t} = 0.1$
$r^{\mu}{}_{w} = 0.10$



Experimental Results Analytical Prediction

Fig.12 Comparison of Analytical Prediction to Experimental Results

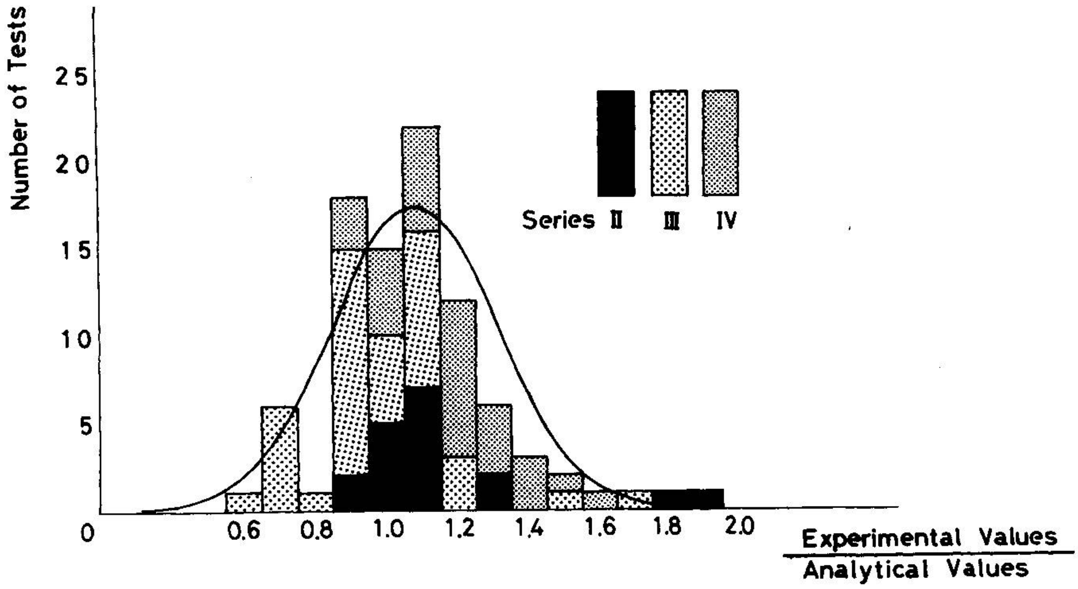


Fig.13 Histogram of the Ratios of Experimental Values to Analytical ones

5 CONCLUSION

Proposed in this paper was an analytical approach to determine the shear capacity of reinforced concrete members subjected to combined compression, bending and shear. The analysis made use of the so-called extended additive strength theory based upon beam and arch mechanism concept. In the case of diagonally reinforced concrete columns, the additive strength theory is based on diagonal reinforcement and arch mechanism.

The resulting analytical solution for ordinary reinforced concrete columns were expressed in terms of the magnitude of compression, the length-to-depth ratio of column, the amount of longitudinal and shear reinforcements, material strength, cross-sectional dimensions and others. For reinforced concrete columns, m-n-q failure interaction surface was obtained which enables one to know the effect of axial force and bending upon shear strength of columns. It is indicated that failure surface is concave in the region where shear failure is predominant, and that the ultimate strength of reinforced concrete member that is supposed to fail in shear is hardly affected by axial force.

The configurational characteristics of theoretically and experimentally obtained interaction surfaces are alike each other and the mean value of the ratios of the experimental values to analytical ones are 1.08.

NOTATION

b = width of rectangular section
D = depth of rectangular section
${}_rD$ = distance between top and bottom longitudinal reinforcement
${}_rD_1$ = distance ratio of longitudinal reinforcement (${}_rD/D$)
F_c = concrete cylinder strength
h = column length
M = bending moment in general
m = non-dimensional bending moment (M/bD^2F_c)
N = axial force in general

n = non-dimensional axial force (N/bDF_c)
Q = shear force in general
q = non-dimensional shear force (Q/bDF_c)
$_rp_t$ = longitudinal reinforcement ratio
$_rp_w$ = shear reinforcement ratio
T_o = tensile yield strength of diagonal reinforcement
$_r\sigma_y$ = yield stress of longitudinal reinforcement
$_{rw}\sigma_y$ = yield stress of shear reinforcemtnt
η = column length-to-depth ratio (h/D)
μ_t = diagonal reinforcement parameter (T_o/bDF_c)
$_r\mu_t$ = longitudinal reinforcement parameter ($_rp_t \cdot {_r\sigma_y}/F_c$)
$_r\mu_w$ = shear reinforcement parameter ($_rp_w \cdot {_{rw}\sigma_y}/F_c$)
ϕ = angle between column axis and diagonal reinforcement

REFERENCES

1. HATTORI, T., T. SHIBATA and K. ONO, "On the Mechanism of Shear Failure of Reinforced Concrete Members", Trans. Architectural Institute of Japan, No. 200, Oct., 1972, pp.35-44 (in Japanese).
2. SUENAGA, Y. and R, ISHIMARU, "Kinematic Analysis of Concrete Members under Combined Stresses-Part 4 — A Theoretical Study on the Ultimate Shear Strength of Reinforced Concrete Beams and Columns", Trans. Architectural Institute of Japan, No.221, July, 1974, pp.9-16 (in Japanese).
3. KATO, B, and R. SHOHARA, "Strength of Steel-Reinforced Concrete Members", Trans. Architectural Institute of Japan, No.266, Apr., 1978, pp.19-29 (in Japanese).
4. BRAESTRUP, M. W., "Plastic Analysis of Shear in Reinforced Concrete", Magazine of Concrete Research, Vol.26, No.89, Dec., 1974, pp.222-228.
5. GROB, J. and B. THURLIMANN , "Ultimate Strength and Design of Reinforced Concrete Beams under Bending and Shear", Publication, IABSE, 1976, pp.105-120.
6. NIELSEN, M. P., M. W. BRAESTRUP and F. BACH, "Rational Analysis of Shear in Reinforced Concrete Beams", IABSE Periodical, No.2, May, 1978, IABSE Proceedings P-15/78.
7. NIELSEN, M. P. and M. W. BRAESTRUP, "Shear Strength of Prestressed Concrete Beams without Web Reinforcement", Magazine of Concrete Research, Vol.30, No. 104, Sept., 1978, pp.119-128.
8. THURLIMANN, B., "Plastic Analysis of Reinforced Concrete Beams", Introductory Report of IABSE Colloquium KOPENHAGEN 1979, Plasticity in Reinforced Concrete, 1979, pp.71-90.
9. CAMPBELL, T. I., L. CHITNUYANONDH and B. dev BATCHELOR, "Rigid-Plastic Theory B. Truss Analogy Method for Calculating the Shear Strength of Reinforced Concrete Beams", Magazine of Concrete Reseach, Vol.32, No.110, Mar., 1980, pp. 39-44.
10. WAKABAYASHI, M., K. MINAMI and S. TOKURA, "Experimental Studies on Failure Surface of Reinforced Concrete Columns under Axial Force, Bending and Shear", Annuals, Disaster Prevention Research Institute, Kyoto University, No.18-B, April 1975, pp.125-141 (in Japanese).
11. WAKABAYASHI, M., K. MINAMI, T. NAKAMURA, R. SASAKI and S. MORINO, "Some Tests on Elastic- Plastic Behavior of Reinforced Concrete Frames with Emphasis on Shear Failure of Column", Annuals, Disaster Prevention Research Institute, Kyoto University, No.17-B, Oct. 1974, pp.171-189 (in Japanese).
12. WAKABAYASHI, M. and K. MINAMI, "An Experimental Study on Hysteretic Characteristics of Reinforced Concrete Columns Failing in Shear", Proc. of the Symposium on Resistance and Deformation of Structures and Their Components, 18th National Symposium on Bridge and Structural Engineering, Japan Society for the Promotion of Science, Tokyo, 1972, pp.97-112.
13. MINAMI, K. and M. WAKABAYASHI, "Seismic Resistance of Reinforced Concrete Beam-and-Column Assemblages with Emphasis on Shear Failure of Column", Proc. of 6th WCEE, New Delhi, Jan. 1977, Vol.III, pp.3101-3106.