Partially Prestressed Concrete Beams under Pure Bending

Poutres partiellement precontraintes en flexion pure

Teilweise vorgespannte Träger in reiner Biegung

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SUMMARY
Prestressing permits economical design of structures, particularly when the live loads attain a high level. Some lack of knowledge, however, still exists, especially about the permissible crack-width. A series of tests was carried out in order to obtain more information in this field. The beams were tested in pure bending; they had ractangular cross-sections and were prestressed by pretension at variable reates.

RÉSUMÉ
La précontrainte des structures en béton permet d'obtenir des éléments économiques, surtout dans le cas où les charges variables sont élevées. Cependant, cette technique n'est pas toujours utilisée au mieux de ses possibilités en raison des incertitudes qui subsistent sur son comportement, principalement de la fissuration que l'on doit accepter. Nous avons effectué des essais permettant de préciser ce comportement en flexion pure, dans le cas de poutres de section rectangulaire, précontraintes à des degrés divers à l'aide de fils adhérents.

ZUSAMMENFASSUNG
Die Vorspannung von Betonkonstruktionen wird nicht den Möglichkeiten entsprechend voll benutzt wegen der Unsicherheit, die über ihr Verhalten besteht, vor allem was die Rissbildung betrifft. Wir haben Versuche angestellt, bei denen wir das Verhalten bei reiner Biegung bestimmen, im Falle von Trägern mit rechteckigem Querschnitt bei gleichem Bruchmoment, aber verschieden stark vorgespannt.

INTRODUCTION

An experimental research program on the behaviour of partially prestressed beams with rectangular shaped cross section has been undertaken [1] [2]. This paper presents the results obtained under pure bending of such beams prestressed by pre-tension.

In the case of pre-stressing by post-tension, important economies can be made by replacing the wires with their casings and anchorages with ordinary reinforcing bars.

In the case of prestressing by pre-tension, the savings are essentially due to the simplification of the design because the prestress forces, when the beam is unloaded, are lower. Besides, the pre-tension technique permits to submit the prestressed beams to alternate bending and, consequently, it makes easier the design of continuous beams loaded with heavy mobile loads (rolling loads particularly). This process moreover seems to be properly fitted for seismic effects.

1. TESTING PROGRAM

1.1. Characteristics of the beams

The characteristics and dimensions of the tested beams are schematically shown in figure n° 1 and table n° 1.

Figure 1
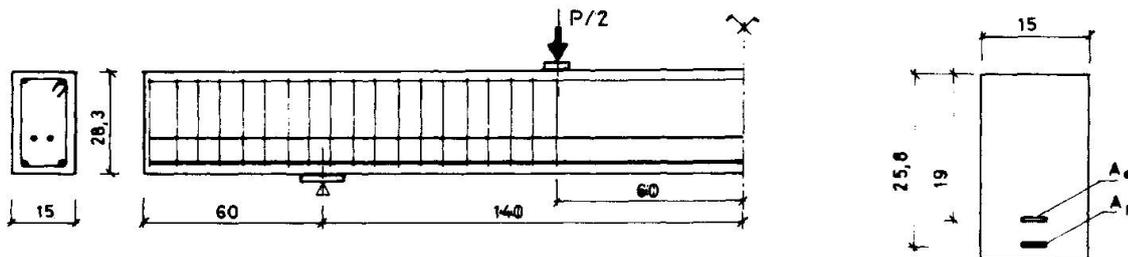
Nbr beam	prestressing rate P %	A_a cm^2 prestressed reinf.	A_p cm^2 ordinary reinf.
A	0	0	5,08
B	40	1,0	3,14
C	60	1,5	2,26
D	80	2,0	1,0
E	100	2,5	0

Table 1

Table n° 1 indicates for each beam :

- the cross-section of the prestressing steel, A_a,
- the cross-section of the ordinary reinforcing bars (hardened high-bond steel, type TOR), A_p,
- the rate of prestressing defined as the part of the failure bending moment equilibrated by the prestressing steel only.

The elastic conventional limit stress of ordinary reinforcing steel is equal to 412 MPa , the one of prestressing steel equals 1470 MPa. The corresponding actual mean values are, respectively, 460 MPa and 1520 MPa.

The mean strength of concrete in compression was equal to 43,5 MPa, the mean strength in pure tension was equal to 33 MPa and the ultimate compressive strain was $2,5 \cdot 10^{-3}$.

The beams were prestressed after 14 days hardening, the loading tests were performed on the 28 th day.

1.2. Loading program

The beams were submitted to a four points bending. The loads were applied in two successive steps :

- from zero to the service-load for flexure (P = 7 500 daN)
- from the service-load to the failure with a constant deflection-rate' (0,2 mm/mn).

1.3. Measurements

The strains on the upper fiber of the beam were measured with inductive transducers (active length 0,10 m ; total length of measurement area 0,40 m). The same process was used before cracking on the lower fiber.

After cracking, the mean deformation of the reinforcing unstressed bars, was obtained with transducers placed in the center of uncracked blocks. The width of the cracks was measured with other transducers placed over the crack-lines.

The deflection and edge-rotations were also recorded during the loading test.

The position of the cracks was detected as soon as they appeared by the aid of a microscope with magnifying power varying from 10 to 100.

1.4. Initial strains

The initial strains developing in the concrete and the bars during the hardening have been determined owing to the measurement of the deformation of the beams since the unmoulding. We have calculated the stresses in the concrete and in the reinforcement from the so obtained strains. The results are shown in table n° 2.

Beam P %	Lower fiber stress daN/cm^2	Upper fiber stress daN/cm^2	Ordinary bars stress daN/cm^2	Prestressing bars stress daN/cm^2
A (0 %)	− 6	+ 2	+ 147	−
B (40 %)	+ 38	+ 6	+ 647	− 11150
C (60 %)	+ 68	+ 5	+ 768	− 11190
D (80 %)	+ 96	+ 4	+ 1110	− 10910
E (100 %)	+ 130	0	−	− 10860

Table 2

(Compressive stresses are considered as positive)

2 - STRAIN STATE AT SERVICE LOAD STAGE

2.1. Theoretical calculation

The theoretical calculation of strains has been performed for a cracking cross-section. The results were then corrected to take into account the effect of the uncracked stretched concrete between the crack-lines. This first calculation assumed the Navier's hypothesis and a perfect bond between steel and concrete. We have also assumed that cracks appear when the tensile stress of concrete reaches its tensile strength. The elastic modulus in compression was choosen in correlation with the stress level, as the secant modulus.

The values of the strains were then corrected according to the CEB-FIP Model Code [3] which allows to take into account the decrease of the strain due to the existence of stretched concrete between the crack-lines.

2.2. Comparison between theoretical and experimental results

It appears that before cracking the experimental and theoretical values of strains are almost equal. Beyond, experimental strains are slightly higher than the calculated ones. It is essentially a creep-effect because we have waited ten minutes before measuring the strains at each step of loading, to let the deformation of concrete be stabilized. On the contrary the experimental strains in the rods are lower than the theoretical values, even after correction, as shown on figures n° 2, 3, 4 and 5. The difference between the experimental and theoretical values of strains in the rods tends to vanish when the load increases. This difference seems to be due to under-estimation of the effect of the stretched concrete at the casing of the rods and at the
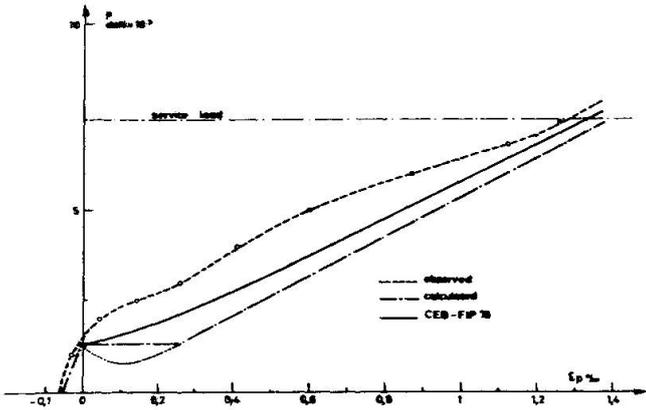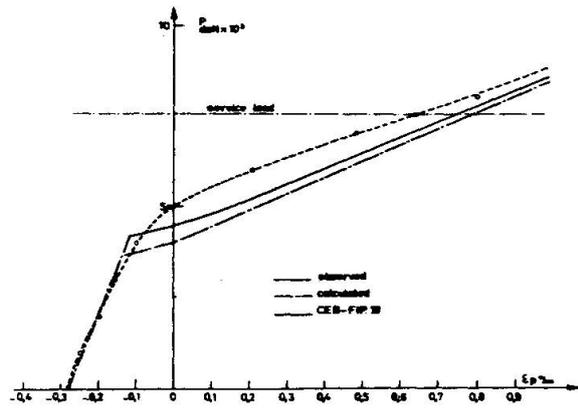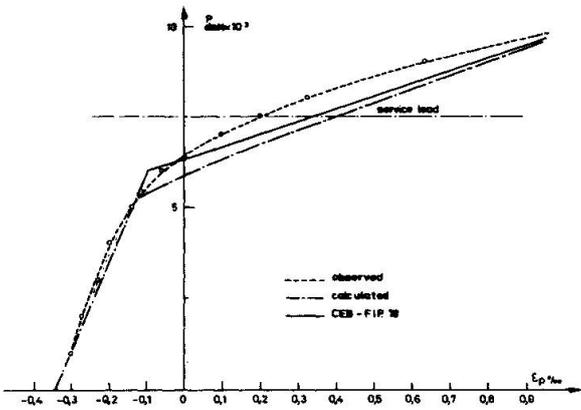
Figure n° 2
beam A

Figure n° 3
beam B

Figure n° 4
beam C
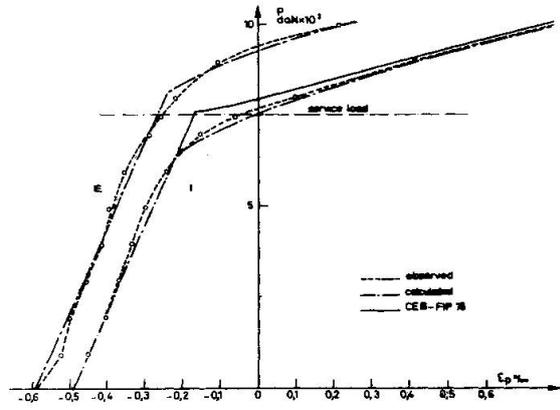
Figure n° 5
beam D and beam E

Figures 2, 3, 4, 5   —   Ordinary renforcement stresses

crack-tips. Other authors [4] assume indeed that in that last case the stretched concrete could be submitted to high unelastic strains without cracking.

In conclusion, the calculation of curvatures from theoretical values of strains in a cracking cross-section, after the CEB-FIP Model Code correction, gives a very satisfactory approximation of the experimental actual values. In the case of totally reinforced beams and under low loads, there still exists a higher gap.

2.3. Comparison of the behaviour of beams with different prestressing rates.

As shown on figure n° 6 the curvatures of the beams decrease strongly when the prestressing rate increases. So, at the service load, the curvature of the beam totally reinforced is almost 2,5 higher than the one of the totally prestressed beam. For a prestressing rate of 40 % there is no very appreciable improvement. For 60 % and especially 80 % the beams get an interesting stiffness. In this last case, indeed, we can see that the curvatures of beams D (80 %) and E (100 %) are almost equal till a load level in the neighbourhood of 90 % of the service load, without noticeable change beyond this load level. As the service load P is scarcely reached and as the area in which the bending moment would be higher than 0,9 x service moment is probably very small, it does not seem necessary to prestress the beams beyond a rate equal to 80 %. A rate lower than 60 %, on the contrary, does not seem to be sufficient.
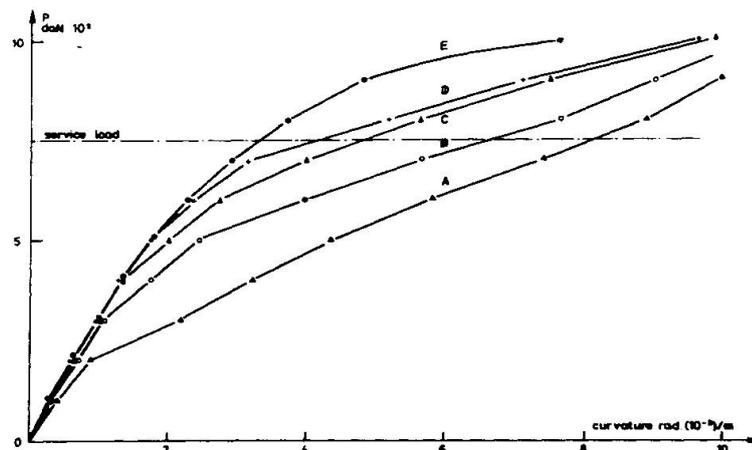
Figure n° 6
Curvature of beams
versus prestressing rate

3. CRACKING CONDITIONS

3.1. Cracking bending moment

The formation of cracks appears as a sudden change in the response of the transducers which are placed, on the Lower fiber, over a crack-line. The microscope allows, with a magnifying power equal to 50, to locate the crack more precisely. At this moment the crack cannot be seen with the naked eye : it becomes possible for a noticeably higher load.

Table n° 3 shows the comparison between the values of the cracking bending moment, actual, determined with the naked eye and theoretical. The latter has been calculated according to the previous assumptions, about the limit stress cracking criterion.

Moment daNxm / Beam	Actual value	Naked eye determined value	Theoretical value	σ_{br} actual (°)	σ_{br} apparent (°)
A	910	1110	630	39	53
B	1710	2110	1580	39	57
C	2110	2510	2200	29	47
D	2510	2910	2680	25	44
E	3110	3310	3270	25	35

Table 3   (°) σ_{br} daN/cm^2

It appears clearly that the experimental values are very close to the theoretical ones, calculated under the previous hypothesis. The σ_{br} corresponding stresses (1st crack) are very close to the mean value calculated for the failure under pure tension (33 daN/cm^2).

The differences that can be observed, in comparison with this mean value are of the same importance than the summation of the incertitudes about the strength of the concrete in tension and about the initial values of the strains.

The lower values observed for the beams D and E, however, let us to think that the concrete which has been highly compressed and submitted to creep-effects could be less resistant in tension.

The value of the bending moment determined with the naked eye is, on the contrary, always very higher than the actual value. The corresponding stress, calculated assuming the cross-section uncracked, should be 50 % higher than the tensile strength.

For these reasons, we think that the cracking of stretched concrete has not followed some measurable unelastic strains.

3.2. Decompression bending moment (craked beams)

This value of the bending moment plays an important part because it corresponds to the re-opening of the craks. If, indeed, cracking is permitted in partially prestressed concrete, the cracks must not open, generally, before a certain loading level defined as a part of the live loads. In such conditions the risks of corrosion of the prestressed wires remain low enough.

The decompression bending moment has been theoretically calculated assuming that the stresses when un loaded are the same that before the first loading and, besides, that the concrete behaves elastically even after cracking as long as the cracked cross-sections are compressed.

The experimental value of this bending moment has been calculated from the recording of the transducers placed on the stretched fiber.

Table n° 4 allows the comparaison between the experimental and theoretical values of the decompression bending moment M_{dec}.

Beam \ Moment daNxm	Exp.	Theor.
B	810	844
C	1230	1486
D	1710	1996
E	2110	2612

Table 4

One can see that the experimental value of the decompression bending moment is always lower than the theoretical one ; the difference increases with the prestressing rate and is equal to 20 % of the theoretical value. Hence it seems necessary to take this difference into account in the design, if the condition of closing of the cracks has to be respected.

The difference we have observed, seems to be due to the role of the bonding wires. Indeed, when the cracks open, a sliding appears near the crack-lines, between the concrete and the wire, although prevented by the bonding forces (friction). During unloading, the bonding forces, acting in the opposite way, compress the prestressed wire. The consequence will be a loss of prestress for the wires or an increase of compressive stress in the ordinary reinforcing bars, which implies a decrease of the value of the decompression bending moment.

In our case, this phenomenon seems to appear for the two kinds of bars, since for the beam E totally prestressed the decrease is important.

3.3. Progression of cracking.

In totally reinforced or slightly prestressed concrete, when a crack appears in the lower fiber, it reaches immediatly the upper part of beam. MALDAGUE [5] showed that there occurs, under constant or increasing loading, a momentary instability, because the tensile strength in the reinforcement increases at a lower rate than the one relaxed by the cracking concrete.

In partially prestressed beams, on the contrary, this phenomenon does not exist beyond a prestressing rate equal to 40 %. The cracking progression is then very regular for the tensile strength progressively relaxed is balanced by a modification of the compressive stresses diagram (in the concrete).

This characteristic behaviour favours the use of partially prestressed concrete : one can be sure that the prestressed wires remain in an uncracked area even after cracking.

3.4. Opening of cracks

a) Comparison between experimental and theoretical values

The theoretical values have been calculated from the code model CEB-FIP 1978 [3].

The mean, experimental and theoretical (1978 CEB-FIP Model Code) values are nearly coincident, although the calculation from CEB under-estimates the cracking bending moment. Theoretical value of the maxima width presents an important safety allowance for 0 %, 40 %, 60 % rate beams. This safety allowance is lower for important loads but remains satisfactory for loads close to the service-level. (Figure n° 7)
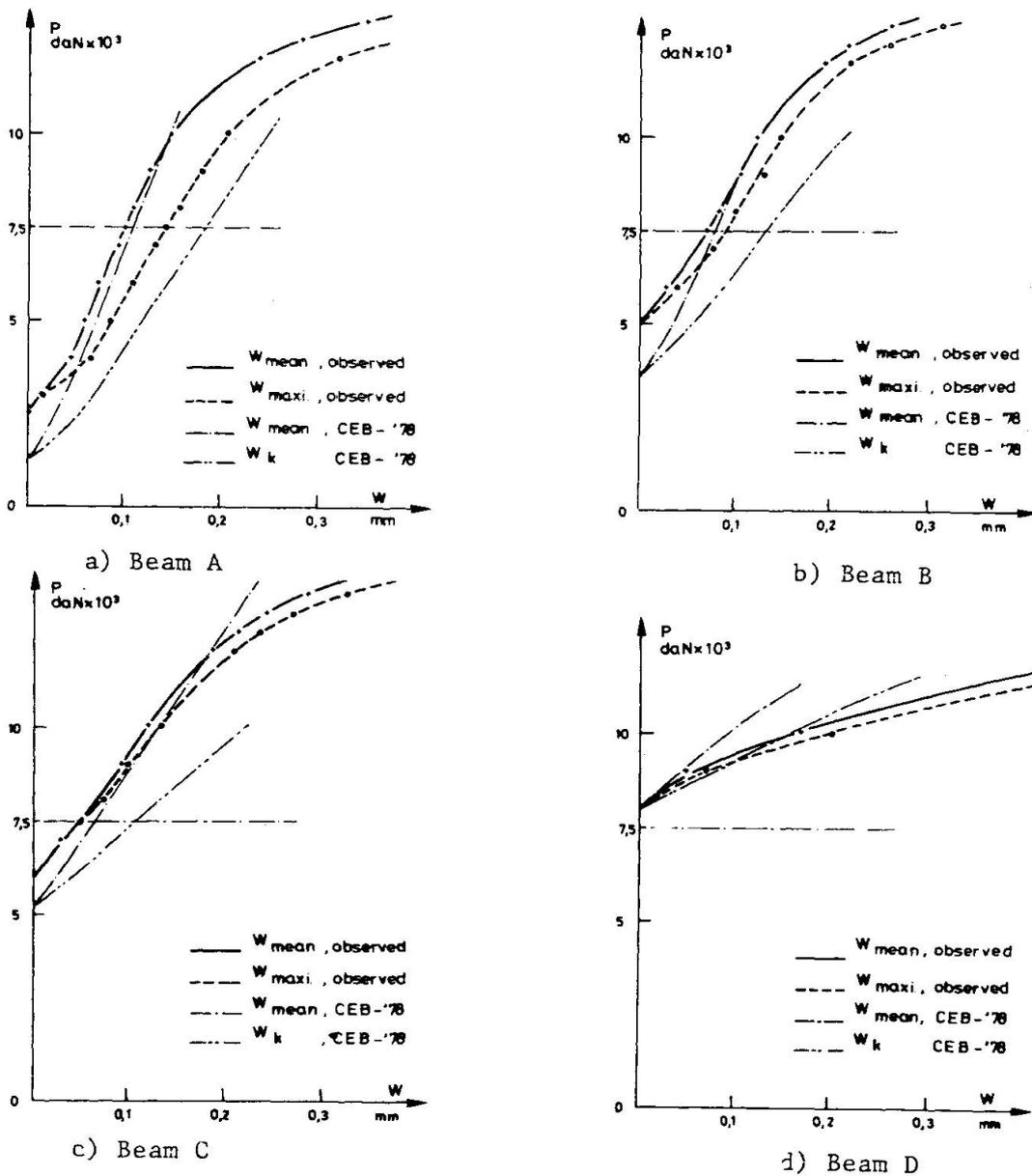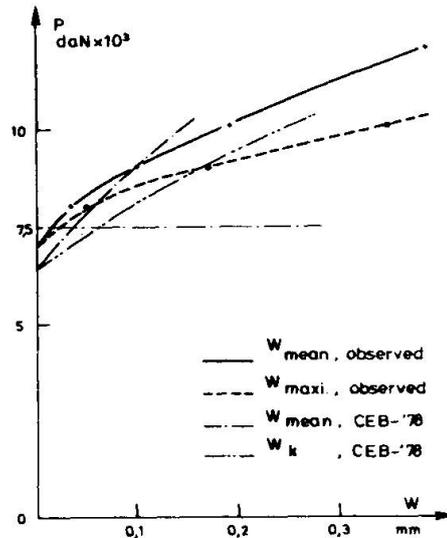

a) Beam A

b) Beam B

c) Beam C

d) Beam D

Figure n° 7 - Observed and calculated width of cracks.

Figure n° 7 (e)

Observed and calculated width
of cracks

beam E

**b) Comparison between beams with different prestressing rates**

The cracking-load increases with the prestressing rate as well as the cracks
progression.

Figure n° 8 shows that till 0,5 M_R, the width of the cracks decreases when the
prestressing rate increases; on the contrary, beyond 0,6 M_R the cracks of the
most prestressed beams are the widest. We think that this phenomenon is due to
the decrease of the total steel cross section, the increase of the bonded con-
crete section and the decrease of the bonding properties, when the prestressing
rate increases.

In our case, for the service-load, the improvment obtained becomes perceptible
beyond a prestressing rate equal to 60 %. Below, the decrease of the crack-width
in comparison with the totally reinforced beam is not sensitive. This remark does
not, however, make us forget that the decompression bending moment increases
noticeably with the prestressing rate. So, a prestressing rate can be choosen
only according to the duration of live-loading and the surrounding aggressiveness.
Particularly, if the service-load is scarcely applied, 80 % prestressing rate
is sufficient enough in aggressive surroundings, for the cracks will almost
never open.

We can notice, besides, that the measured openings of the cracks under service-
load are lower than the limit value (0,1 mm) proposed by Code Model CEB-FIP for
frequent cases.

Figure n° 8
Openning of cracks
Versus prestressing rate
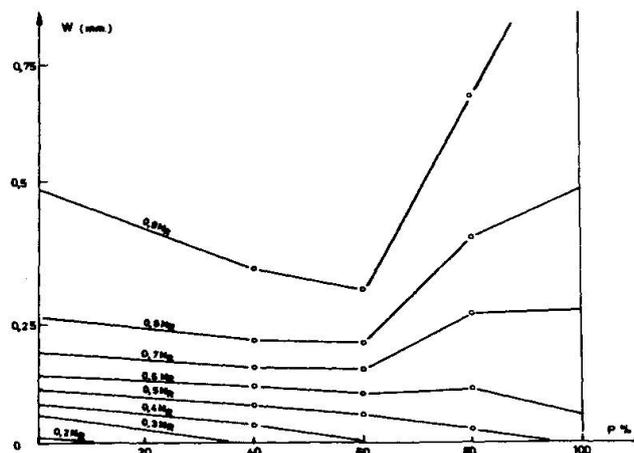
4. FAILURE CONDITIONS

4.1. Comparison between the experimental and theoretical values of the failure bending moment and the ultimate limit moment

The theoretical calculation has been performed with the common hypothesis, assuming therefore that failure of compressed concrete occurs when its compressive strain reaches $5 \cdot 10^{-3}$ (experimental value obtained by preliminary tests) for the determination of the failure bending moment.

The ultimate limit moment was calculated with a failure compressive strain value equal to $3.5 \cdot 10^{-3}$ (CEB–FIP Model Code).

Table n° 5 shows the obtained results and allows the comparison. (The experimental value of the failure moment corresponds to the maximum of loading).

Beam	A	B	C	D	E
Prestressing rate	0	40	60	80	100
Experimental value	5,91	6,15	6,35	6,07	5,99
Theoretical value	5,93	6,20	6,46	6,34	6,18
Ultimate value	4,65	4,55	4,57	4,35	4,27
Exp./Ult.	1,27	1,35	1,39	1,40	1,40

Table 5

We can see that theoretical calculation gives a good approximation but is shightly higher. We think that this difference is due to the slidings between concrete and steel.

Besides, the CEB–FIP calculation offers a safety allowance independent from the prestressing rate (but slightly lower for the totally reinforced beam). The safety of these structures can however be different according for instance to the scatter of mechanical characteristics of prestressed or ordinary reinforcements.

4.2. Failure strains

Table n° 6 shows the experimental and theoretical values of the failure strains.

| Beam	prestressing rate (%)	experimental				Theoretical strain variation 10^{-3}	
		mean strain (1) variation 10^{-3}		max. strain (2) variation 10^{-3}			
		$\Delta\varepsilon_b$	$\Delta\varepsilon_s$	$\Delta\varepsilon_b$	$\Delta\varepsilon_s$	$\Delta\varepsilon_b$ (hypoth.)	$\Delta\varepsilon_s$ (cal.)
A	0	4,07	13,37	4,21	15,4	5	20,1
B	40	4,25	12,07	4,63	14,9	5	16,1
C	60	4,48	10,75	4,54	13,8	5	14,2
D	80	4,73	9,44	5,11	12,5	5	13,6
E	100	5,15	9,52	5,26	10,3	5	11,9

Table 6

(1) mean value calculated from the recording of four transducers (measurement area length $\simeq$ 40 to 50 cm)

(2) maxima value of the recordings.

The slight differences which appear between theoretical and experimental strain values correspond to those previously noticed about the bending moments. They are probably due to the non-planearity of the cross-section and to the measurement method.

We have then calculated the ultimate curvatures plotted on the figure n° 9, versus the prestressing rate. The curvature decreases slightly with the prestressing rate but the variation is not very important which shows the low influence of the prestressing rate upon the unelastic deformations of the beams. This result is essentially valid for partially prestressed beams by pre-tension. In the case of post-tension it won't be so favourable.

Figure n° 9
Ultimature curvature
versus prestressing rate
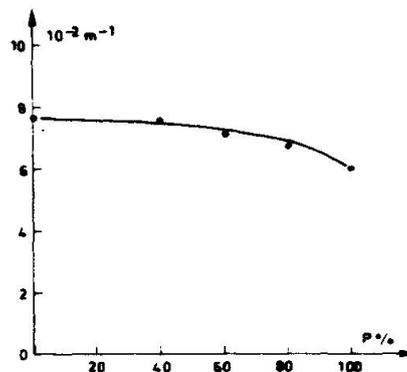
CONCLUSIONS

The obtained results show that the behaviour of partially prestressed by pre-
tension beams can be foreseen owing to the theoretical calculation in a satis-
factory way on condition that the estimation of the initial stage stresses
takes into account the mechanical effects of the ordinary reinforcement.

The cracking load seems to decrease when the lower fiber has previously been
highly compressed (beams with 80 % os 100 % prestressing rate). The width or
the cracks obeys to the 1978 CEB-FIP Model Code previsions. The prestressing
rate near of 70 % – 80 % seems necessary to limit efficiently the width of the
cracks at the service-load, but if the design conditions allow a maximum width
equal to 0,1 mm the optimum prestressing rate may be lower than 70 %.

There is no fondamental difference in the failure behaviour of the beams with
respect to the prestressing rates. The safety allowance in relation with the
CEB-FIP Model Code does not seem dependent on the prestressing rate, nor the
failure strains values.

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