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Design of Reinforced Concrete Based on Mechanics

Calcul du béton armé basé sur la mécanique

Stahlbetonbemessung nach den Regeln der Mechanik

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SUMMARY

Design procedures for linear reinforced concrete members based on the principles of structural mechanics are presented. The general equations of the state of stress of a cracked reinforced concrete element and their solution at the cross section are given. Practical procedures and the consequences in usual design work are discussed. The section can be of general shape with a single symmetry axis. Axial force, bending moment and shear force can act on the concrete section. Numerical examples are presented and discussed.

RÉSUMÉ

Procédés pour le calcul des éléments linéaires en béton armé basés sur les principes de la mécanique technique sont présentés. Les équations générales de l'état de tensions d'un élément en béton armé fissuré sont données. Procédés pratiques et leurs conséquences dans le travail habituel de projet sont discutés. La section transversale peut avoir une forme géométrique quelconque avec un axe de symétrie. Force normale, moment fléchissant et effort tranchant peuvent exister dans la section transversale. Exemples numériques sont présentés et discutés.

ZUSAMMENFASSUNG

Gezeigt werden Bemessungsverfahren für Stahlbetonbauteile auf der Grundlage der technischen Mechanik. Die allgemeinen Gleichungen des Spannungszustandes in einem gerissenen Stahlbetonbauteil werden aufgestellt und die Lösung für einen Querschnitt angegeben. Die praktische Anwendung und die Konsequenzen für die Bemessung werden diskutiert. Die Form der Querschnitte ist beliebig, aber mit einer Symmetrieachse. Normalkräfte, Biegemomente und Querkräfte werden berücksichtigt. Rechenbeispiele werden gezeigt und diskutiert.



1. INTRODUCTION

Due to the possibility of the determination of stresses and strains of reinforced concrete members with the help of computers, taking into account many realistic features of the structural behaviour of reinforced concrete, it is possible nowadays to predict with good accuracy the load history of a reinforced concrete member. However, these calculations require a large amount of computer time and therefore they are not adequate to be applied in the everyday design work. To overcome these economic difficulties, a practical approach for design of reinforced concrete members will be presented herein. These design rules will be based on logical principles of the structural mechanics and at the same time they will demand small computer effort.

The criteria are based on the general equations of a cracked reinforced concrete element, however considering neither dowel nor aggregate interlock action between the cracks. The consideration of the load history is not taken into account in the development of the equations.

The procedures are valid essentially for linear reinforced concrete members presenting a single symmetry axis. No restrictions are made to the shape of the concrete section and the reinforcement distribution along the section.

It shall be shown that some classic considerations of reinforced concrete design are not consistent with the usual longitudinal design of reinforced concrete members, such as the use of the lever arm of the internal forces in the calculation of the shear stresses, as well as the staggering rule of the tensile reinforcement.

In this paper the variation of the longitudinal reinforcement in the direction of the longitudinal member axis is not considered. The reason for doing that was to simplify the equations of the problem, presenting them in a shorter form. However, for practical problems, this variation of the reinforcement does exist and must be taken into account (see DIAZ [2]).

The method of analysis presented handles the effects of the bending moment, axial force and shear force at the same time. Thus a full interaction of these effects is considered, requiring that the design in the longitudinal and transversal directions be made concurrently.

2. THE MATHEMATICAL MODEL FOR THE ANALYSIS

The structural member which will be investigated is a linear reinforced concrete member with a constant cross section presenting a single symmetry axis. A x -axis is defined along the longitudinal member axis. The cross section is symmetric in relation to the z -axis. The load effects at the given section are the axial force N , the bending moment M and the shear force V . It will be assumed that in the member element of length dx no variations of N and V exist. The positive directions of the load effects are shown in Figure 1. The origin of the z -axis does not need to be at the centroid of the section.

The analysis will be performed considering a plane state of stress of an assembly formed by cracked reinforced concrete elements. The concrete will be considered formed by curved struts, which are not able to resist tensile stresses. The compressive stresses in the struts will be considered parallel to the directions of the cracks. The transverse reinforcement placed in the

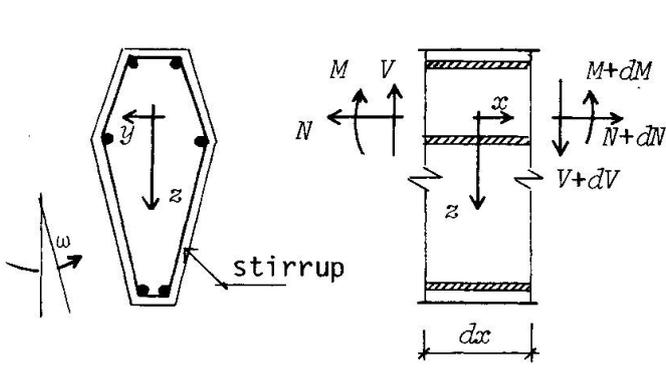


Fig. 1 Coordinate axis

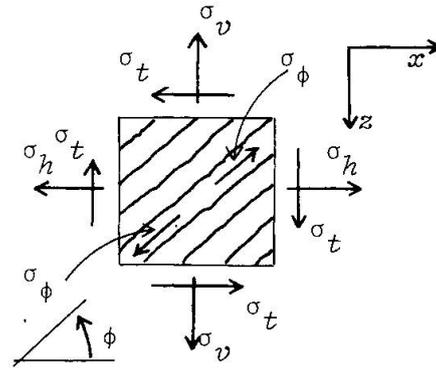


Fig. 2 Concrete stresses

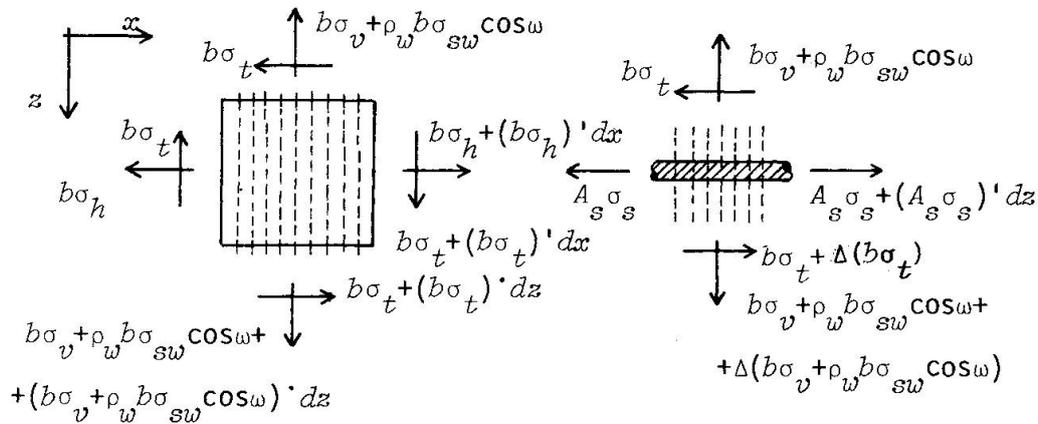


Fig. 3 Equilibrium conditions

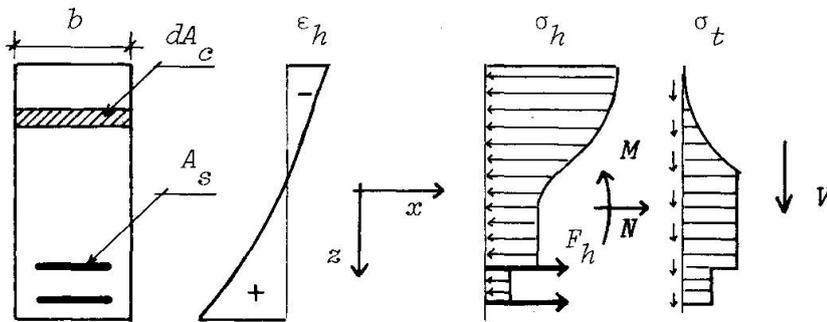


Fig. 4 Distribution of concrete stresses

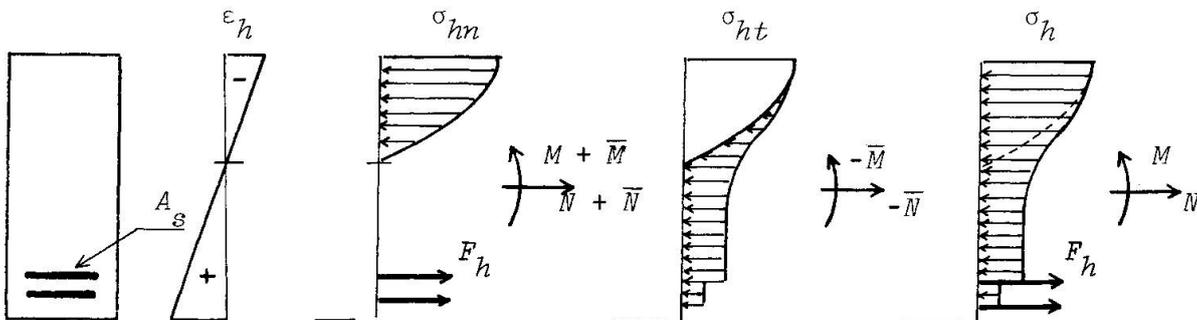


Fig. 5 Increments of loads effects

direction of the z -axis is supposed continuously distributed and variable along the x -axis. This reinforcement is actually formed by the stirrups. The longitudinal reinforcement will be assumed to be distributed in discrete quantities along the height of the member. This representation is required due to the usual practice of concentrating the longitudinal reinforcement at the top and bottom edges of the member. No shear transfer will be considered along the cracks due to aggregate interlock action of the cracked concrete. The dowell effects of the reinforcement going through the cracks will also be disregarded. In order to simplify the equations, the concrete corresponding to the volume occupied by the reinforcement will be considered as existing.

With the help of Figure 2 the following equilibrium conditions at a point of the concrete web of the member are obtained:

$$\sigma_t \tan\phi = -\sigma_v \quad (1)$$

$$\sigma_\phi \sin\phi \cos\phi = -\sigma_t \quad (2)$$

$$\sigma_t = -\tan\phi \sigma_h \quad (3)$$

in which σ_ϕ is the stress in the concrete strut, σ_h , σ_v , σ_t the stress components of σ_ϕ , and ϕ the slope angle of the cracks. It must be pointed out that the tensile stresses are denoted by positive signs.

The differential equations of equilibrium of a reinforced concrete element with transverse reinforcement as well as of a longitudinal reinforcement element (see Figure 3) are:

$$(b \sigma_t)' + (b \sigma_h)' = 0 \quad (4)$$

$$(b \sigma_t)' + (b \sigma_v + \rho_w b \sigma_{sw} \cos\omega)' = 0 \quad (5)$$

$$\Delta(b \sigma_t) + (A_s \sigma_s)' = 0 \quad (6)$$

$$\Delta(b \sigma_v + \rho_w b \sigma_{sw} \cos\omega) = 0 \quad (7)$$

in which b is the variable member thickness, ρ_w the relative amount of transverse reinforcement, σ_{sw} the stress in the transverse reinforcement, A_s the steel area of the longitudinal reinforcement at a certain height and σ_s the steel stress of the longitudinal reinforcement. The angle ω defines the slope of the stirrup in relation to the x -axis, as shown in Figure 1. In this paper f' will denote the partial derivative $\partial f/\partial x$ and f'' the partial derivative $\partial f/\partial z$. The symbol Δ will represent a discrete increment of the assigned function in the z -direction.

The overall equilibrium conditions over the entire height (see Figure 4) are:

$$N = \int \sigma_h dA_c + \Sigma \sigma_s A_s \quad (8)$$

$$M = \int z \sigma_h dA_c + \Sigma z \sigma_s A_s \quad (9)$$

$$V = \int \sigma_t dA_c \quad (10)$$

$$M' - V = 0 \quad (11)$$

in which dA_c is the elementary concrete area, calculated as $dA_c = b dz$. The force $A_s \sigma_s$ in Figure 4 is designated by F_h .

The state of deformation of the reinforced concrete plate is defined by the strains ϵ_h , ϵ_v in the longitudinal and transversal directions and by the distortion γ . These strains are to be considered as average values of the strains and must encompass the crack openings between the concrete struts (see [5]).



Through geometric considerations and assuming that the strains in the y -direction are null, the following relationships are obtained:

$$\epsilon_{sw} - \epsilon_v \cos^2 \omega = 0 \quad (12)$$

$$\epsilon_\phi - \epsilon_h \cos^2 \phi - \epsilon_v \sin^2 \phi + \gamma \sin \phi \cos \phi = 0 \quad (13)$$

The principle of the complementary work yields the condition:

$$(\epsilon_h - \epsilon_v) \sin 2\phi + \gamma \cos 2\phi = 0 \quad (14)$$

The usual compatibility conditions must hold:

$$\epsilon_h'' + \epsilon_v'' - \gamma'' = 0 \quad (15)$$

At each reinforcement height the strains of concrete and steel must be equal:

$$\epsilon_h = \epsilon_s \quad (16)$$

The constitutive equations of concrete and steel are represented by:

$$\sigma_s = \sigma_s (\epsilon_s) \quad (17)$$

$$\sigma_\phi = \sigma_\phi (\epsilon_\phi) \quad (18)$$

$$\sigma_{sw} = \sigma_{sw} (\epsilon_{sw}) \quad (19)$$

The set of 19 equations described above together with the prescribed boundary conditions define the problem of a stress state of a plane reinforced concrete member in a cracked condition.

The expression (14) was derived without consideration of the member load history. It is also assumed that the concrete is cracked everywhere, because no tensile stresses are to be supported by the concrete struts in their transversal direction.

3. THE SOLUTION OF THE ENTIRE SET OF EQUATIONS

The entire set of equations was solved numerically by SCHULZ [9] in the neighbourhood of a section, assuming for the strain in the x -direction the approximation:

$$\epsilon_h = \sum_{p=1}^n \alpha_p z^{p-1} \quad (20)$$

The assumption (20) takes into account that the section warps after deformation. This solution will be named herein "the warping section solution". The method proposed by SCHULZ assumes variations for the stresses and strains in the x -direction in the form of 3rd grade polynomials. Four sections are examined in the neighbourhood of the given section by numerical methods. Due to the approximations involved in expression (20), the conditions (8), (9) and (15) are treated by numerical methods based on the method of weighted residuals (see [12]). A summary of the thesis [9], is presented in the paper [4] by SCHULZ and DIAZ.

Another set of solutions is possible to be made as long as the linear function for the longitudinal strain in a section is defined:

$$\epsilon_h \cong \epsilon_0 + \kappa z \quad (21)$$

in which ϵ_0 is the longitudinal strain ϵ_h for $z=0$ and κ the curvature of the deformed section.

COLLINS [6], as well as GUEDES and PRÉ [8] have chosen this way to give a rational approach to the problem. However due to the approximation inherently



present in expression (21) the complete set of equations can not be employed in numerical methods. Usually the conditions (13), (14) and (15) are eliminated from the problem and substituted by the condition:

$$\tan^2 \phi = (\epsilon_\phi - \epsilon_h) / (\epsilon_\phi - \epsilon_v) \quad (22)$$

which is obtained from (13) and (14).

Both numerical methods based on expression (20) or (21) need time consuming computer runs which turn the methods uneconomical for normal design work.

4. THE THEORY OF THE CRACKED REINFORCED CONCRETE PLATE AND THE USUAL DESIGN OF REINFORCED CONCRETE MEMBERS

In Figure 4 it can be seen that the distributions of the σ_h , σ_t stresses are different from those usually considered in the design of reinforced concrete members. Actually the design procedures in the design codes of reinforced concrete structures are based on considerations which are not logical in respect to structural mechanics, but produce adequate results for normal design work. The approximations are necessary in order to allow the manual calculations in normal practice. It will be shown herein how far the discrepancies have gone.

In Figure 5 a reinforced concrete section is represented together with the distribution of the longitudinal deformations ϵ_h , which shall be assumed to be linear as given by the expression (21). It shall now be defined, as an approximation, that the longitudinal stresses denoted by σ_{hn} are determined as a function of the strain ϵ_h :

$$\sigma_{hn} \cong \sigma_\phi(\epsilon_h) \quad (23)$$

in which the stress-strain relationship σ_ϕ shall be the same as used before in the constitutive law (18). The stress resultants are now:

$$N_\alpha = N + \bar{N} = \int \sigma_{hn} dA_c + \sum \sigma_s A_s \quad (24)$$

$$M_\alpha = M + \bar{M} = \int z \sigma_{hn} dA_c + \sum z \sigma_s A_s \quad (25)$$

The acting load effects must now be incremented by \bar{N} and \bar{M} in order to reproduce the same longitudinal strains. It is important to note that the steel stresses of the longitudinal reinforcement are dependent upon the longitudinal strains, therefore the consideration of the force increments is extremely important to the design procedures of reinforced concrete members. Indirectly, the force increments are considered in the usual design practice through a procedure named staggering rule of the tensile force in the longitudinal reinforcement.

As shown in Figure 5 the stresses σ_h and σ_{hn} can define a new stress σ_{ht} with the help of:

$$\sigma_h = \sigma_{hn} + \sigma_{ht} \quad (26)$$

Comparing expression (26) with the equilibrium conditions (24) and (25), together with the conditions (8) and (9), it can be stated that the stress resultants of σ_{ht} are $-\bar{N}$ and $-\bar{M}$.

5. THE EQUIVALENT SECTION METHOD

As proposed by DIAZ in [1], [2] and [3] it is possible to use the equivalent section method for the determination of the tangential stresses σ_t as long as the approximations (21) and (23) are taken together with the additional considerations:

$$\sigma_{ht}' \cong 0 \quad (27)$$

$$\sigma_t' \cong 0 \quad (28)$$

The conditions (27) and (28) state that the variations of σ_{ht} and σ_t can also be considered as null for the cases, in which there is no variation of the shear force. These approximations have been shown to be adequate in the thesis [9] prepared by SCHULZ. Therefore their use in the design practice is possible. The discrepancies between the warping section solution, described in item 3, and the method based on the equivalent section method are very small.

The derivative of expression (21) yields:

$$\varepsilon_h' = k_1 + k_2 z \quad (29)$$

Based on assumption (27) and with the help of the expressions (8), (9), (11), (24), (25) and (26) together with the assumption $N'=0$, the following equilibrium equations are obtained:

$$(N + \bar{N})' = 0 \quad (30)$$

$$(M + \bar{M})' = V \quad (31)$$

Through successive derivation of (17) and (23) the following expressions are obtained:

$$\sigma_{hn}' = E_c \varepsilon_h' \quad (32)$$

$$\sigma_s' = E_s \varepsilon_s' \quad (33)$$

in which E_c and E_s are the derivatives $\partial\sigma/\partial\varepsilon$ at the points of the stress-strain curves for the deformations ε_h and ε_s . The values of E_c and E_s can be null as long as the curves are levelled, i.e. the yield point has been attained.

With the help of (24), (25), (29), (30), (31), (32) and (33) the following equations are derived:

$$k_1 A^e + k_2 S^e = 0 \quad (34)$$

$$k_1 S^e + k_2 I^e - V = 0 \quad (35)$$

in which:

$$A^e = \int E_c dA_c + \sum E_s A_s \quad (36)$$

$$S^e = \int z E_c dA_c + \sum z E_s A_s \quad (37)$$

$$I^e = \int z^2 E_c dA_c + \sum z^2 E_s A_s \quad (38)$$

The equations (34) and (35) allow the determination of the coefficients k_1 and k_2

The shear flow $v = b \sigma_t$ is determined with the help of the conditions (4) and (6), taking expression (27) into account:

$$v = \int_{z_s}^z E_c (k_1 + k_2 z) dA_c + \sum_{z_s}^z E_s (k_1 + k_2 z) A_s \quad (39)$$

in which z_s denotes the value of the variable z for the upper edge of the section.



The equivalent section is defined as the section formed by multiplication of the actual concrete and steel areas by the moduli E_c and E_s . Figure 6 shows how the equivalent section is obtained.

If the origin of the z -coordinate is chosen so that S_c^e is null, i.e. the origin is placed over the centroid of the equivalent section, the value of k_1 is also null, as can be seen from expression (34).

From (35) and (39) the following expression is derived:

$$v = (VS) / I \quad (40)$$

in which I is the moment of inertia of the equivalent section with respect to its centroid and S is the static moment of the elements of the equivalent section between z_s and z with respect to the centroid of the equivalent section.

Expression (40) allows the determination of the shear flow along the height of the member as long as the strains ϵ_h are known. Figures (7), (8) and (9) show some examples of this determination. It can be seen from these examples that, in the cases in which the yield stresses in the concrete and in the steel are attained concurrently, the shear stress at the edges must be null. This fact can be proved from expression (40), since the values of E_c and E_s are also null in these cases.

6. THE DESIGN PROCEDURES FOR THE TRANSVERSE REINFORCEMENT

If the shear flow is known, the design of the transverse reinforcement can be made using the rules of the codes as shown by DIAZ in [2].

Due to expression (28) the equilibrium condition (5) assumes the form:

$$\sigma_t \tan\phi = \rho_w \sigma_{sw} \cos\omega \quad (41)$$

In the modern design codes for reinforced concrete structures it is possible to obtain design rules which assume the following form:

$$\rho_w \sigma_{sw} \cos\omega = \text{function} (\sigma_t, \sigma_{hn}) \quad (42)$$

in which the function of the stresses σ_t , σ_{hn} is defined in the code itself.

For the calculation of the concrete stresses of the struts a simplified procedure can be derived from the expressions (41) and (42):

$$\sigma_t \tan\phi = \text{function} (\sigma_t, \sigma_{hn}) \quad (43)$$

Expression (43) allows the determination of the angle ϕ , which is consistent with the design rule (42). The stresses σ_ϕ are determined with the help of expression (2).

More details about these simplified design procedures are given in [1], [2] and [3].

7. AN ITERATIVE PROCEDURE TO DETERMINE THE SLOPE OF THE CRACKS

The determination of the angle ϕ in item 6 was based on an approach which has to be consistent with the design rules of the codes. However if these rules are disregarded a rational procedure can be found for the determination the angle ϕ (see SCHULZ [4] and [9]).

With the help of the expressions (1), (2), (7), (18), (19), (22), (23) and (41) the following equation can be obtained:

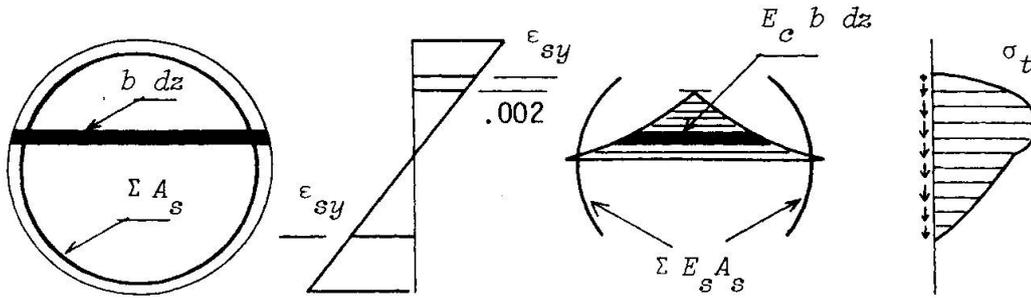


Fig. 6 The equivalent section

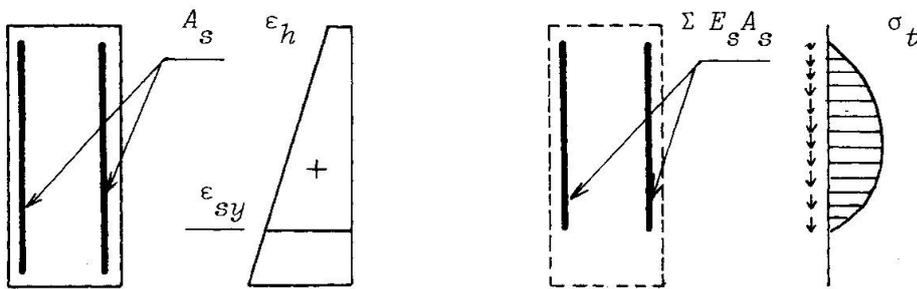


Fig. 7 Section subjected to M , V and positive N

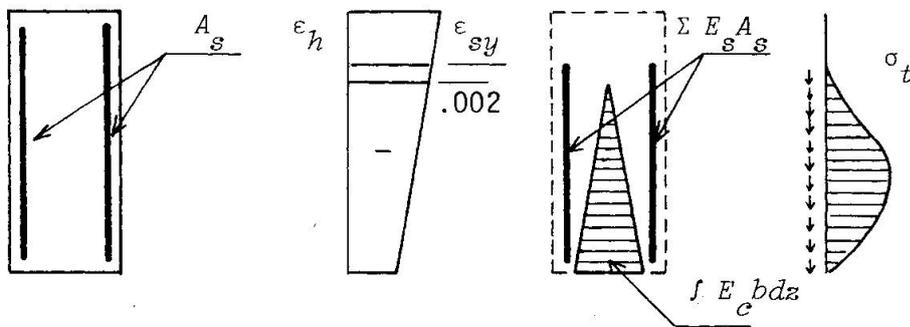


Fig. 8 Section subjected to M , V and negative N

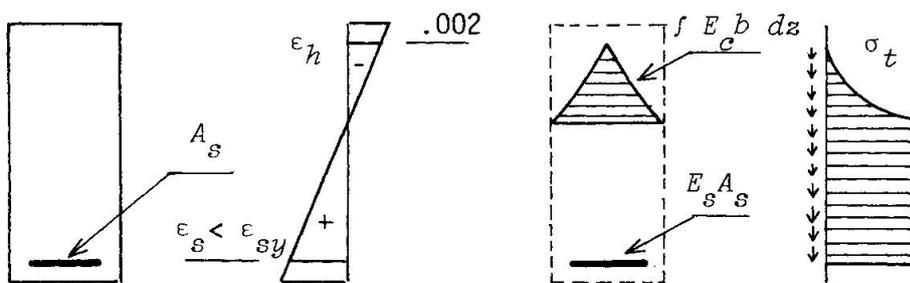


Fig. 9 Section subjected to M without yield of the reinforcement



$$\tan^2 \phi = \frac{\varepsilon_{\phi}(\sigma_t, \phi) - \varepsilon_h}{\varepsilon_{\phi}(\sigma_t, \phi) - \varepsilon_v(\sigma_t, \phi)} \quad (44)$$

The equation (44) allows the determination of the angle ϕ as long as the values of the functions σ_t and ε_h are known along the height of the member.

Before the iterative procedure begins a first set of approximate values of the deformation ε_0 and the curvature κ from expression (21) must be determined, with the help of the formulae of the structural mechanics, considering a linear behaviour of the materials. Afterwards the following steps are performed:

- a- Based on the values of ε_h the characteristic values of the equivalent section are determined with the help of the expressions (36), (37) and (38).
- b- The values of the coefficients k_1 and k_2 are determined through (34) and (35).
- c- The values of σ_t along the height of the member are calculated with the help of expression (39).
- d- The values of the angle ϕ are determined iteratively by means of equation (44). A Newton-Raphson procedure accelerates the convergence.
- e- The values of σ_h are determined along the height with expression (3).
- f- The stress resultants N^i and M^i are computed using the expressions (8) and (9). The stress resultants are compared with the given acting load effects N and M . If the difference between the approximate values of the stress resultants and the given load effects is small, the computation is finished. Otherwise a new set of values ε_0 and κ must be determined.
- g- The estimate of the new values of ε_0 and κ is based on the calculation of the increments $\Delta\varepsilon_0$ and $\Delta\kappa$ which are obtained from the equations:

$$\Delta N = (\partial N / \partial \varepsilon_0) \Delta \varepsilon_0 + (\partial N / \partial \kappa) \Delta \kappa \quad (45)$$

$$\Delta M = (\partial M / \partial \varepsilon_0) \Delta \varepsilon_0 + (\partial M / \partial \kappa) \Delta \kappa \quad (46)$$

in which the partial derivatives are computed through expressions which are presented by SCHULZ in [9]. The increments ΔN and ΔM correspond to the difference between the given load effects N , M and the approximate values N^i , M^i determined in step f. After a new estimate of ε_0 and κ is made the whole set of steps must be repeated.

This numeric method represents a Newton-Raphson procedure due to the nature of the expressions (45) and (46). The iterative procedure converges quickly requiring small computer effort.

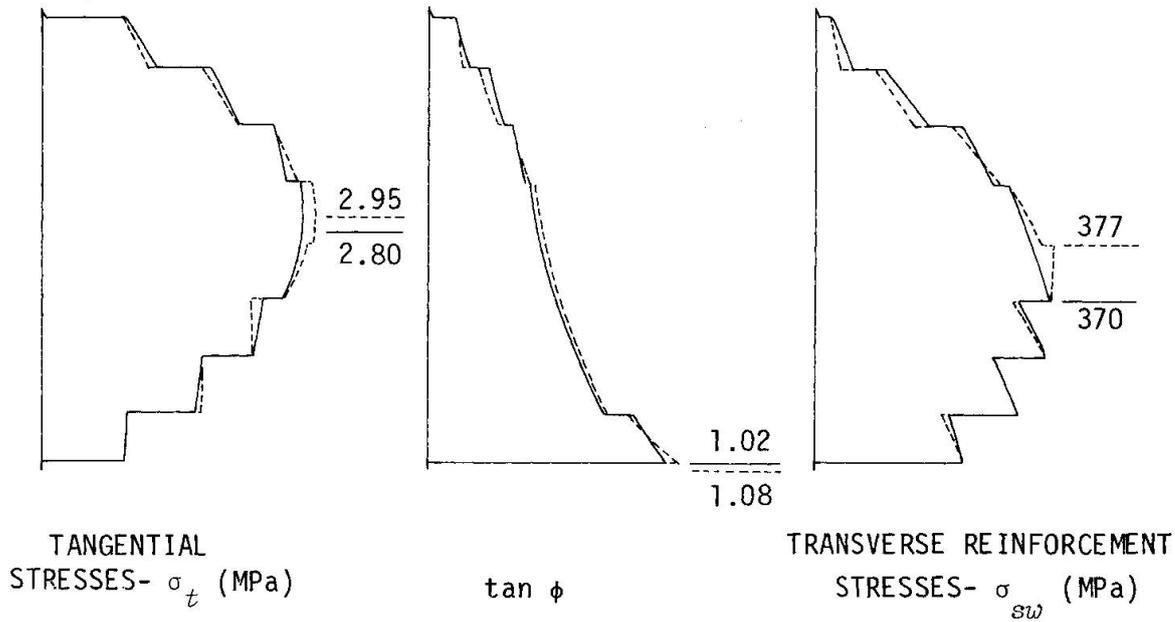
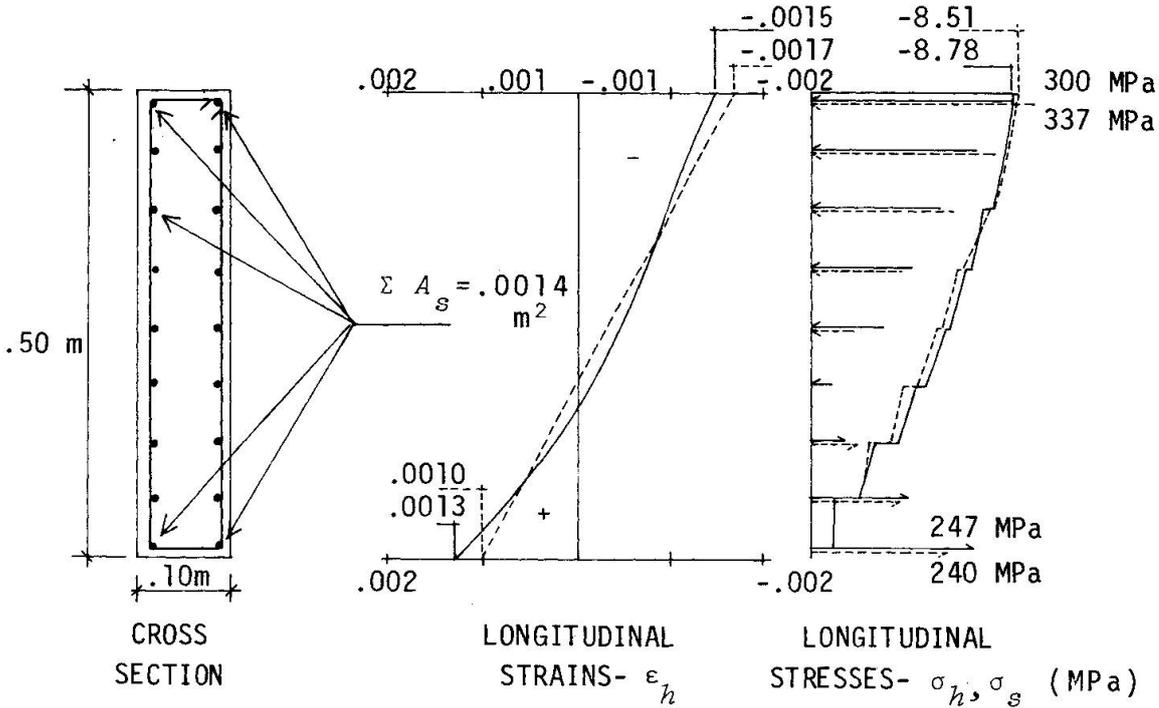
In Figure 10 a comparison between the results of the warping section method and the approximate method described herein is given. The agreement between the two methods has been excellent in all the examples which were calculated.

8. THE CONSEQUENCES OF THE PROPOSED PROCEDURES IN THE DESIGN OF REINFORCED CONCRETE MEMBERS

The design of reinforced concrete members has been made in the longitudinal direction with the values of the load effects N and M . The longitudinal tensile reinforcement is increased afterwards due to the staggering rule of the tensile force in the longitudinal reinforcement. It was shown in item 4 that a more adequate procedure is obtained when the acting load effects are incremented by the forces \bar{N} and \bar{M} . These increments can be determined with the help of the expressions (24) and (25) as long as the values of ε_h are



$M = 40 \text{ kN m}$; $N = -250 \text{ kN}$; $V = 70 \text{ kN}$; $A_{sw} = .0004 \text{ m}^2/\text{m}$



————— Warping Section Method
 - - - - - Equivalent Section Method

Fig. 10 Comparison between the warping section and the equivalent section methods

known. Consequently a computer procedure must be performed in the case of a section of general shape. It is possible to present approximate values for \bar{N} and \bar{M} which are adequate for normal design work (see [3]). TERRA has shown in [13] that the computer calculations can be performed in a programmable calculator such as the HP-41 C.

For members subjected to axial loads the staggering rule does not give the exact representation of what is really happening. It can happen in the cases of members subjected to tensile axial force, bending moment and shear force, that the usual design procedure fails to determine the necessary longitudinal tensile reinforcement at the "compressed" edge of the member, which actually can present positive strains. As shown by THUERLIMANN in [11], in those cases, it is necessary to increment the axial force N by $\bar{N} = |V| \cot \phi$, which causes tensile stresses in the "compressed" edge of the member.

Another important point deals with the use of the lever arm in the calculation of shear stresses in reinforced concrete members. It has been shown in item 5 that the calculation of the tangential stresses can be performed with the usual formula $v = (V S) / I$. However when the materials are yielding at the edges the values of σ_t must be null. This fact is not really new, for in structural steel design it has always been considered that in the plastic state, the shear stresses must be absorbed in the central part of the member height. Therefore the lever arm of the internal forces can not give a correct evaluation of the actual shear stresses existent along the height. Only in the cases in which the compressive zone of the section is small the usual method for the determination of the shear stresses gives good results.

All the procedures presented herein are adequate to be used in almost all the cases encountered in practice. This includes the consideration of sections of general shape with a general distribution of longitudinal reinforcement. The set of acting forces at the section can be given in any combination of value and sign as long as the section has not attained the ultimate limit state. Only vertical stirrups are handled and the section must have at least a single symmetry axis. Due to the limitations in the development of the equations, the longitudinal reinforcement could not vary along the member axis. This limitation can be overcome as shown by DIAZ in [2].

The calculation of the problem variables in a single section, as shown in item 7, presents an additional advantage. The design procedures are entirely performed in a single section and the longitudinal reinforcement does not need to be further increased. For automatic design of a member along its axis this design procedure has enormous advantages because no use of the staggering rule is necessary. The automatic design of a reinforced concrete member along its axis until now requires the creation of a virtual tensile force diagram of the tensile reinforcement, which must be virtually staggered by the computer program. This cumbersome procedure is no longer necessary.

9. CONCLUSIONS

The equations for the plane stress of a cracked reinforced concrete plate have been given. These equations can be employed in the solution of the problem of a reinforced concrete member subjected to axial force, bending moment and shear force.

The basic ideas for the solution of these equations with the assumption of the

warping of the section after deformation have been presented.

For practical calculations a new method was discussed for the determination of stresses and strains of a reinforced concrete member. This method is based on the concept of the equivalent section, which is obtained by multiplying the actual areas by the tangent deformation moduli of concrete and steel. This method needs little computational effort, which can be performed in programmable pocket calculators.

The method is valid for application to concrete sections of general shape presenting a single symmetry axis. The distribution of the longitudinal reinforcement can also be general. Only vertical stirrups are treated.

Some comments were made in connection with the shortcomings of the use of the lever arm of the internal forces in the determination of the shear stresses as well as to the staggering rule of the tensile force diagram. The latter is unable to predict the need of tensile reinforcement in some cases of members subjected to tensile axial force, bending moment and shear force.

The solution presented are consistent with the principles of structural mechanics, therefore they are entirely logical. The equilibrium equations are satisfied as well as the compatibility conditions for the strains.

Some examples have been presented showing the accuracy of the proposed approximate solutions.

The design of reinforced concrete members for shear force using the presented method does not require the design rules of the codes of reinforced concrete structures since the crack slopes are determined in the analysis.

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