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Plastic Rotations by Local Stress Analysis

Rotations plastiques etudiées par analyse locale

Lokale Berechnung von Fliessgelenken

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SUMMARY
A local finite element analysis applied to the zone between two adjacent cracks can provide information about the extension of the zone of concrete in which limit conditions are reached in a triaxial state of stress. Such results can help in the definition of the "plastic length" and can therefore suggest a model for the localized plastic rotations.

RÉSUMÉ
Un calcul local aux éléments finis appliqué à la zone entre deux fissures adjacentes peut donner des informations sur l'extension de la zone de béton qui peut atteindre des conditions-limites en régime triaxial. De tels résultats peuvent être utiles dans la définition de la "longueur plastique" et suggérer un modèle pour les rotations plastiques localisées.

ZUSAMMENFASSUNG
Mit Hilfe einer Finite-Elemente-Berechnung der Zone zwischen zwei benachbarten Rissen können Erkenntnisse gewonnen werden über die Ausdehnung des Bereiches, in welchem der dreiachsige Spannungszustand des Betons den Erschöpfungszustand erreicht. Die Ergebnisse können für die Definition einer "plastischen Länge" in einem Berechnungsmodell mit örtlich konzentrierten plastischen Rotationen verwendet werden.

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1. INTRODUCTION

Several cases of behaviour of r.c. structural members in the cracked stage have been successfully studied by means of finite element techniques. Most applications, nevertheless, concern cases in which the process of crack formation is studied |14| or the overall effect of cracking on the field of deformations or forces is considered |7| : the cracks are therefore considered to intervene at the edges of the elements or to be spread on the element itself.

A minor number of applications concerns the study of the stress field modifications caused in proximity of cracks by the presence of the cracks themselves; such studies require meshes having an higher degree of refinement, and their aim is limited to small areas, so that we can speak of a sort of a local analysis in comparison with the overall analysis of the structure.

The present paper aims to suggest the use of such local analyses by finite elements in the study of the rotation capacity of r.c. monodimensional members. This can be surprising: in fact, the research on the plastic rotations of flexural members has been basically performed on an experimental basis in the 60ies (under the urgent need of ductility conditions and of rotation values for non-linear analysis of frames); therefore, the results are very roughly expressed in simple empirical formulae, or based on a not well defined concept of "plastic length". But the authors treating the problem suggested since that time |1||10| - |4| that the phenomenon could find a satisfactory model only by a refined local analysis of the stress field near the cracks: the suggestion arose by the evidence of a concentrated nature of the local rotation, and by the belief that the formation of horizontal cracks may play an essential role in flexural failure |3|.

2. POSITION OF THE PROBLEM

Monodimensional flexural members can develop considerable plastic rotations θ_{pl} at the ultimate limit state. There are two ways of taking advantage of this property in current practice:

- by a nonlinear analysis of the structure;
- by taking account of the redistribution capacity that a ductile structure can develop, even if designed according to the usual linear assumptions.

An intensive experimental research provided safe values of the rotation capacity in function of several programmatas |1||2||4||5||6||9||11||12||23|, and empirical formulations of permissible values for θ_{pl} in function of the relative depth x/d of the compression zone were introduced in design Codes. For CEB Code the empirical curve of Fig.10 has been proposed |11|, and consequently the ductility condition for a degree δ of moment redistribution was formulated |11| in the following way:

$$\delta \geq 0.44 + 1.25 \frac{x}{d} \qquad ; \qquad 0.75 \leq \delta \leq 1.00$$

Several efforts were done in order to derive the plastic rotation by integration of curvatures, but the tests showed that it was caused by a concentrated rupture mechanism .

A whish of consistency with the assumptions made for the analysis of the section led to assume the conventional ultimate strain of concrete ε_{cu} as extended on a "plastic length" L_p; but the plastic length appeared soon to be another conventional concept, and the tentatives of its rational derivation involved the assumption of variable values for ε_{cu} |1|.

On the other hand, many researchers agreed on two important observations:

- the measured strains on concrete were frequently considerably higher than the values of ε_{cu} which were standardized through prism tests;
- the concrete failure appeared often in the ways schematically shown in Fig.1, with formation of horizontal or inclined cracks, so that vertical or inclined tensile stresses seemed to play an essential role.

The present work is the beginning of a study intended to a better understanding of the phenomenon, taking into consideration the above experimental observations and the progress achieved in the meantime in the failure criteria in triaxial state of stress.

Preliminary studies done by Broms |3| and by the authors showed that (even in pure bending) the deviation of the neutral axis between adjacent cracks can cause sufficient tensile stresses in the zone of longitudinal compression to cause horizontal or inclined cracks.

The study is therefore performed along the following lines:

i) determination of the local state of stress in the region between two adjacent flexural cracks and particularly of the field of tensile stresses;

ii) determination of the region in which the failure is reached according to the triaxial criterion, and its interpretation as a physical plastic length;

iii) study of the variation of the plastic length as a function of the depth of the compression zone.

As a triaxial failure criterion, the formulation due to Ottosen |15| |16| has been adopted: it is stated as

$$f\ (I_1,\ J_2,\ J) = 0$$

where I_1, J_2 and J are expressed in function of the principal stresses and of the principal stress deviators.

3. LOCAL STRESS ANALYSIS

The problem of determining the stress state is three-dimensional.
The element of a rectangular flexural member was studied by means of a plane finite element model. The beam element is represented through a slice of member of unit thickness, taking proper account of the contribution of concrete and steel areas.
Two-dimensional rectangular finite elements were used and the model was studied in plane strain conditions, with the aim of considering the confinement due to the three-dimensional behavior and to the transversal reinforcement.

The F.E. mesh (fig.2) represents a part of the beam between two cracks.
The element of flexural member, subject to simple bending, was studied in conditions close to the ultimate state; the mesh includes a predetermined crack of a given height.

A non linear constitutive law has been assumed for concrete according to Fig.3; the uniaxial nature of this law (not consistent with the triaxial assumptions) will require in the future a check of its influence on the results.

The reinforcement was represented by truss elements. The bond linkage was performed by using suitable spring elements with non-linear bond-slip relationship (Fig.4), according to |7| and |8| .

Five cases, characterized by different steel area, and therefore by different crack heights were studied in order to evaluate the influence of the crack height on the plastic length.

4. RESULTS

The case with x/d = 0.56 is described in detail.
Fig. 5 shows the distribution of longitudinal stresses σ_y and of the transversal stresses σ_z in six cross sections in proximity to the flexural crack. The fluctuation of the neutral axis depth along the beam is clearly shown, and therefore the cause of the considerable values of the transveral tensile stress σ_z.

The law of variation of the longitudinal stress in the reinforcing steel is shown in Fig.6, and Fig.7 gives the longitudinal distribution of the bond stress at the interface between concrete and reinforcement.

The blackened zone in Fig.8 is formed by the elements where the centroidal stress state causes failure according to the triaxial criterion; in this localization the transversal tensile stress and the invariants of the stress deviator play an essential role.

Similar results are obtained (Fig.9) in the other four cases, within the range of x/d from 0.12 to 0.56, covering most of the possible cases in pure bending. The location of the critical elements and the main cause of failure (the transversal tensile stress, and therefore the formation of horizontal cracks) seems to be in good agreement with the observed failure patterns of Fig.1.

From the comparison of various cases in Fig.9 the critical zone (considered as an effective plastic length) appears essentially constant: this could be hardly accepted when reasoning in terms of diffusion, and therefore expecting wider plastic lengths for higher x values. Nevertheless, a constant plastic length might lead to a satisfatory relationship between $\frac{L_p}{x}$ and the ratio $\frac{x}{d}$, if compared with the experimental results and the current empirical laws (Fig.10).

Caution must be used in interpreting the results of these exercises, only presented here with the aim of indicating a possible approach to the problem.

The study should be extended to cases with closer flexural cracks and with a shear action, situations in which the described phenomena should be emphasized. On the contrary, it has been checked that a linear constitutive law for concrete does not sensibly affect the amplitude of the critical zone.

The effect of different assumptions on the concrete confinement should be examined.

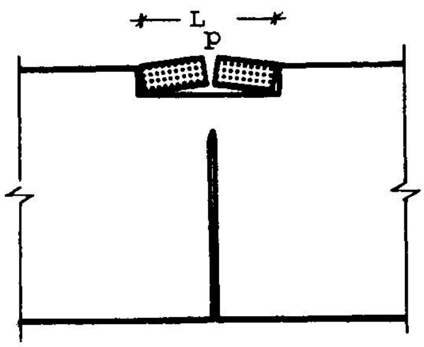


Fig. 1 - Typical failure patterns for different crack spacings.

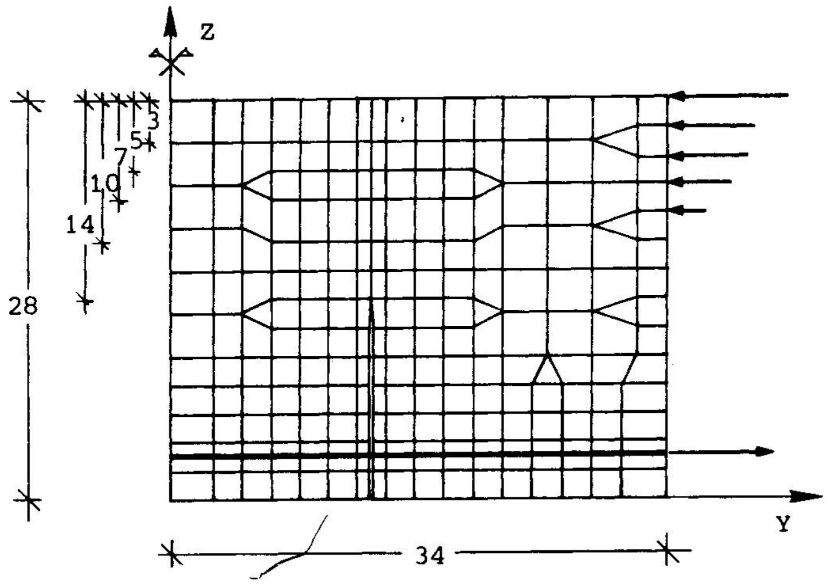


Fig. 2 - F.E. mesh for the five cases with different crack heights

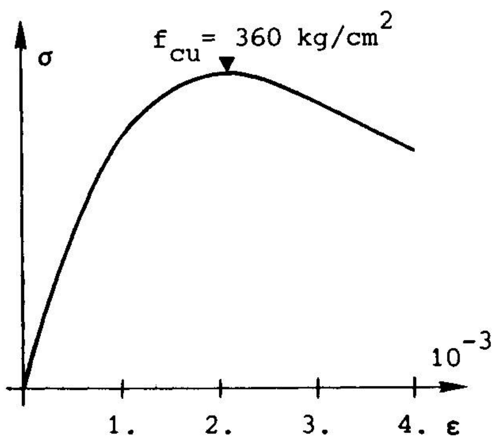


Fig. 3 - Assumed uniaxial stress-strain relationship for concrete

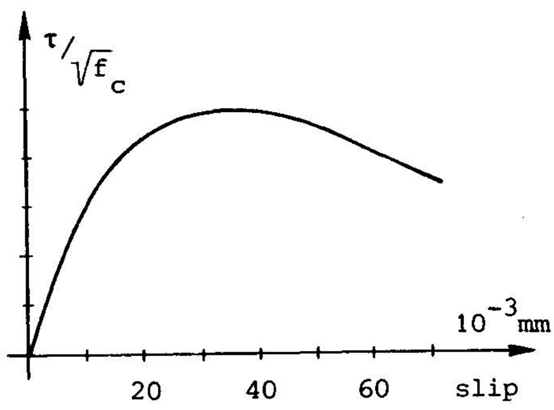
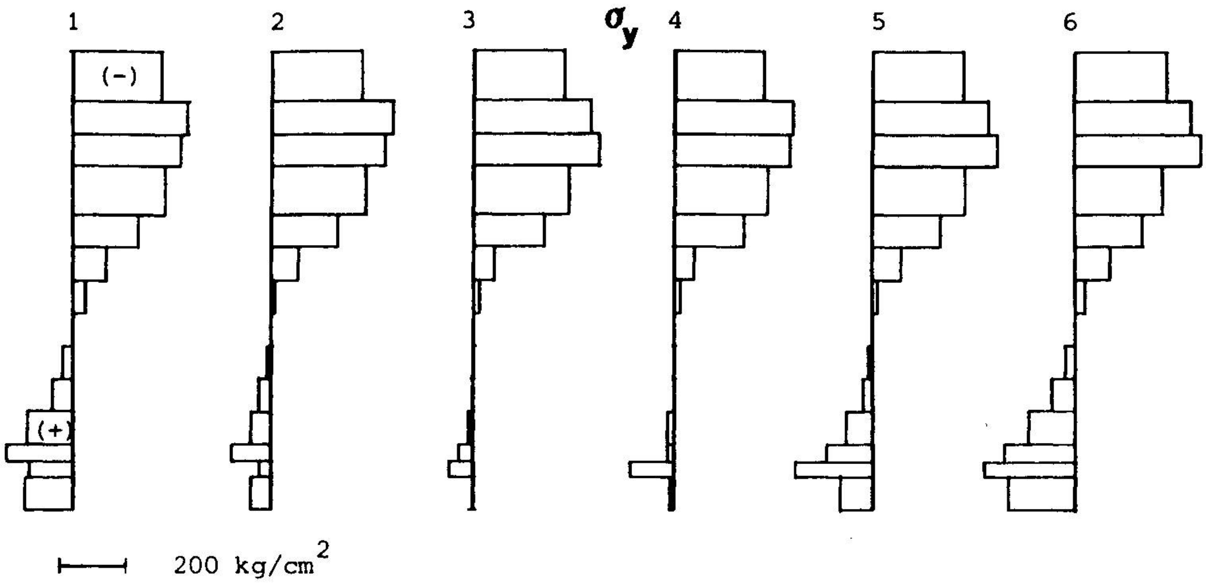
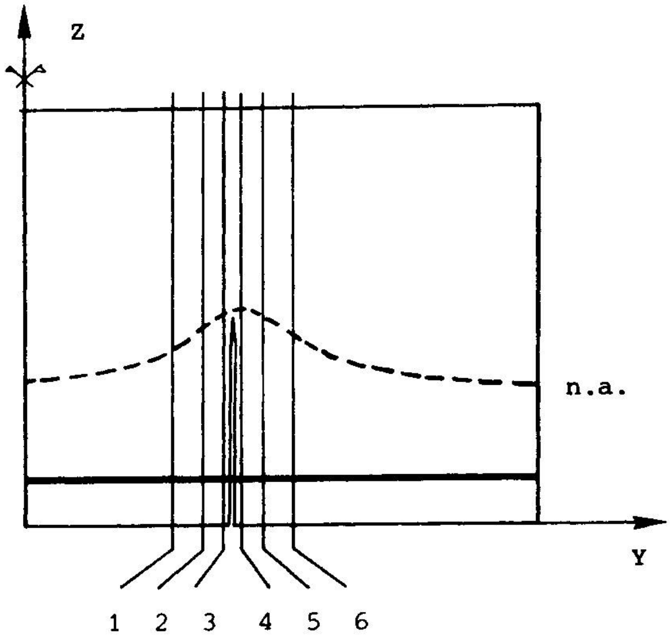


Fig. 4 - Assumed bond-slip relationship





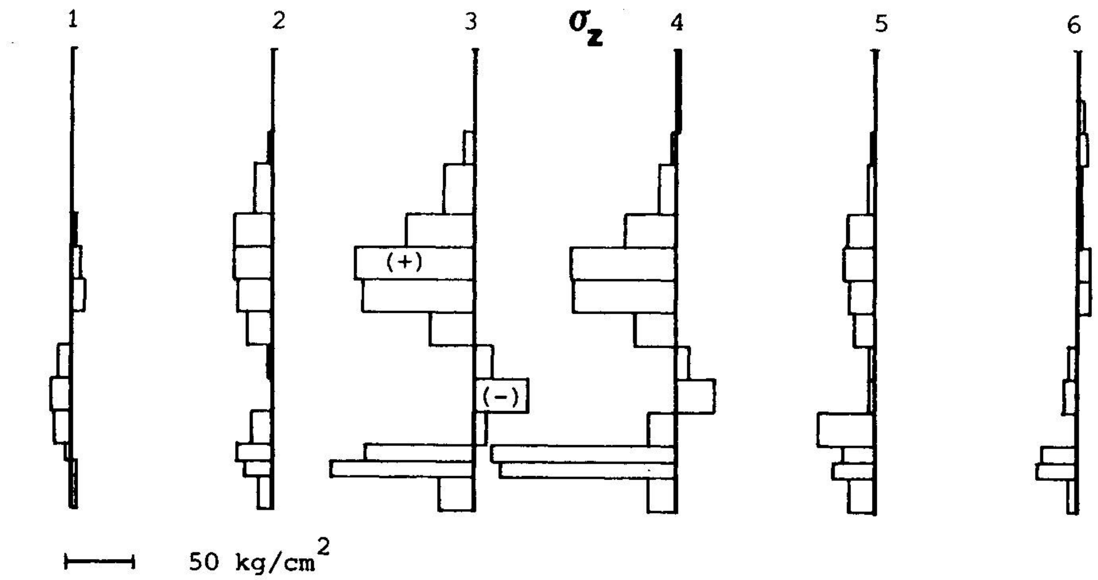


Fig. 5 - Longitudinal and transversal stress distributions close to the cracked section (case x/d = 0.56)

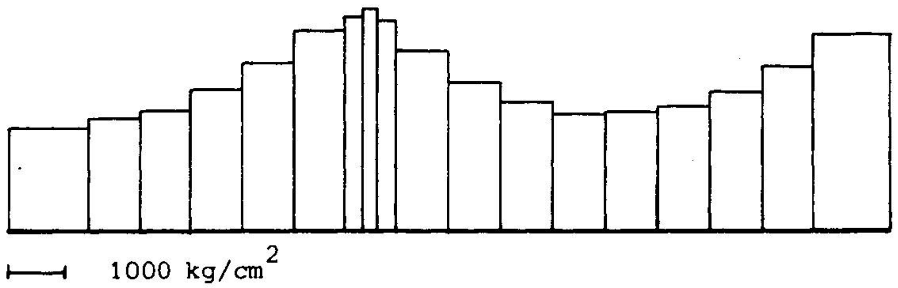


Fig. 6 - Longitudinal stresses along the reinforcement (case x/d = 0.56)

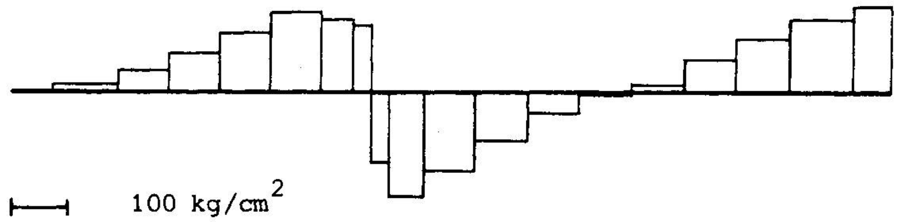


Fig. 7 - Bond stresses at the interface between concrete and steel (case x/d=0.56)

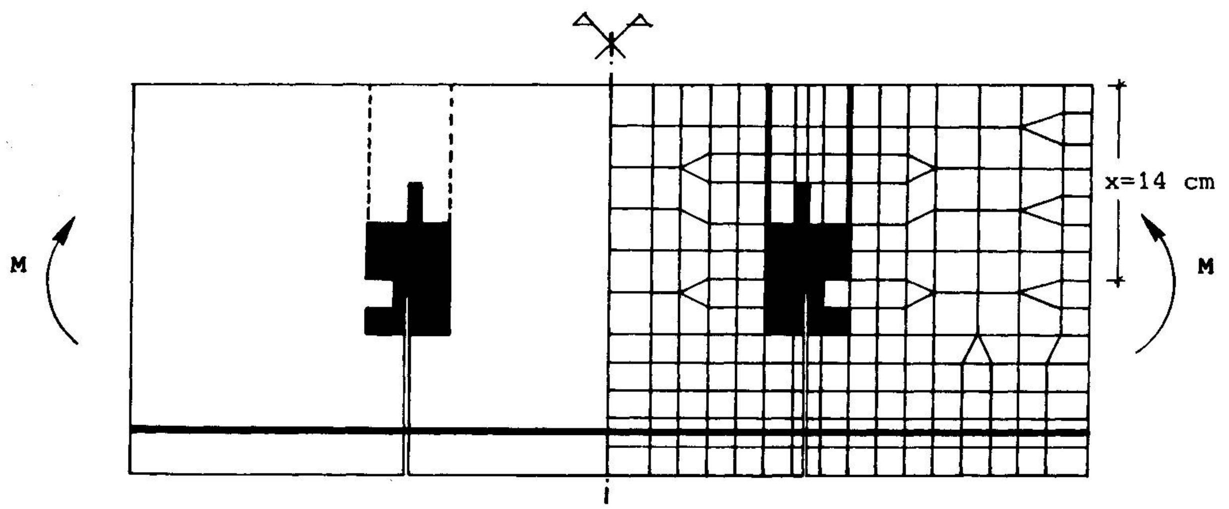


Fig. 8 - Failure zone found by applying the triaxial failure criterion (case x/d = 0.56)

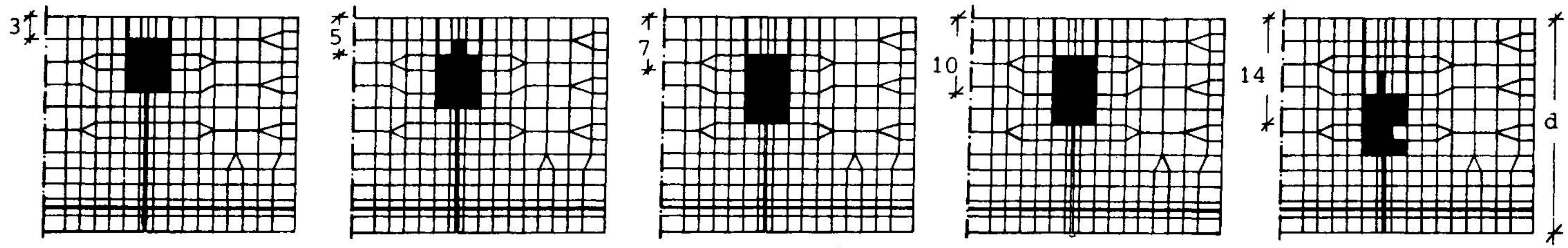


Fig. 9 - Failure zones for the five cases (x/d = 0.12; 0.20; 0.28; 0.40; 0.56)

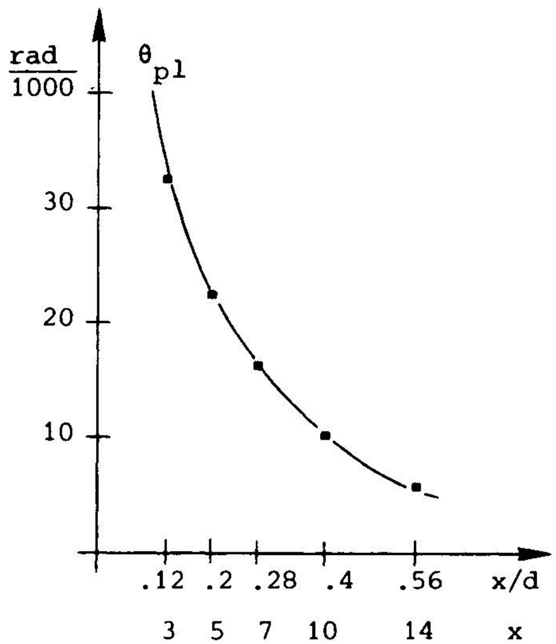


Fig. 10 a) - Plastic rotations versus x/d (empirical relationship)

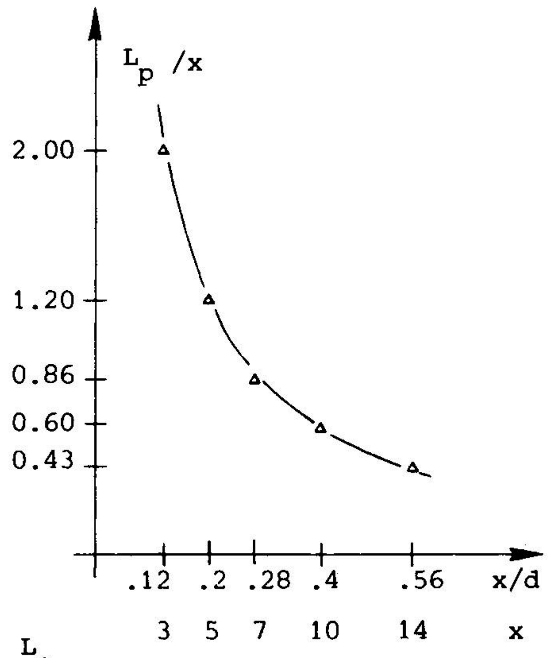


Fig. 10 b) - $\frac{L_p}{x}$ ratio vs x/d (from finite element analysis)

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