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## DISCUSSION

### Session 2, part 2: Structural Modelling for Numerical Analysis

Introductory Report by Willam/Argyris, F.R.G.

Schnobrich (U.S.A.): In the beam model that you showed you smeared cracks and consequently you have infinite bond between the steel and concrete and yet with that model you have been able to reproduce strain conditions, even in the cracked zone that looked extremely good to me. So I am wondering what your comment is on the smeared crack approach.

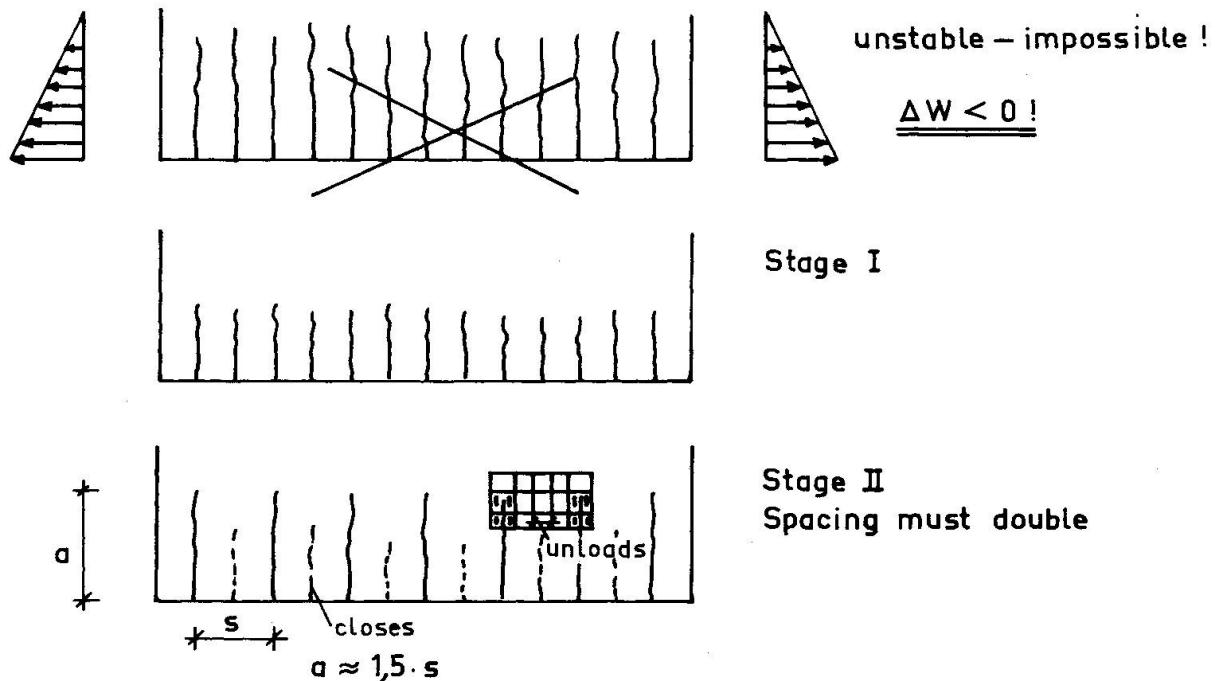
Willam: My comment is that you should use the simple tools first. Thus we assumed full bond between the steel and concrete in that first stage. We were also considering to introduce bond slip with spring elements. However, this was discussed after looking at material data from Dr. Eifel at the BAM, Berlin, who showed that if you account for plastification of the steel bar in the tensile flexural zone and if you introduce local cracking in heavily reinforced sections, i.e. for reinforcement ratios of higher than 0.6%, then there is virtually no difference between the perfect bond and the bond-slip formulations.

Ingraffea (U.S.A.): I would like to object to your objections about objectivity. Your comments are perfectly correct on the basis of the examples you showed. Prof. Bažant is also correct on the basis of his examples. But there are fundamental differences among the examples. Of the two cases you showed, in the one with the thick-walled cylinder, you will have a classically globally stable crack propagation problem, that is the load must continually rise to push the crack. In the case of the reinforced concrete beam you will have a stable crack propagation problem because of the reinforcement acting as a crack arrester. The load must constantly increase to push the crack. In the case of Prof. Bazant's example of a crack in an unreinforced plate there is a globally unstable system. Once the load reaches the necessary level to start the crack, the system is destroyed. Objectivity in the sense of Prof. Bažant is correct in his example because in the limit, as the element gets smaller, the load necessary to get the crack to propagate goes to zero. But in your case since you have load redistribution capability because of the stable propagation, objectivity in the sense that he uses is irrelevant. The accuracy in the sense that you use it is accurate. So one has to be careful about whether you talk about unstable crack propagation or stable crack propagation when talking about objectivity in either sense.

Willam: My point is that we should not be talking about objectivity if we consider stable or unstable crack propagation. Objectivity in mechanics is referring to the invariance of the material formulation with regard to rigid body motion and in particular coordinate transformations. We are discussing here the question of accuracy of your finite element lay-out. We have a discrete formulation and the approximate solution is certainly dependent on the mesh refinement. In that sense I did not like the term objectivity, because objectivity refers to the invariance with regard to the coordinate rotations. However, Prof. Bazant observed correctly, that you have to use energy criteria instead of local stress criteria, since the local stress concentration goes to infinity because of the singularity at the crack tip. In my thinking the different sensitivity of the finite element results with regard to the failure criteria has nothing to do with objectivity.

Prof. Bazant made a discussion on the evolution of parallel cracks in concrete which may be summarized as follows. Formation of secondary cracks halves the crack spacing, but that is not the only possibility. The crack spacing can also double. Under certain loading, for example for bending, or cooling, or shrinkage stresses, one observes the closing of every other crack (see figure), with the effect that the opening of the open cracks doubles as the loading increases (In more detail, see Bazant, ASCE-EMD Journal, 1979, p.873 and IASS 1980, p.97).

### Fracture Mechanics – Instability of dense parallel cracks



Kotsos (U.K.): As the maximum load-carrying capacity of a beam under flexure is approached large tensile strain concentrations in the transverse direction occur in the compressive zone of the cross sections where flexural cracks appear. This causes a transverse deformation profile of the top surface along the length of the beam similar to that shown in fig. 1. It is apparent from this figure that such a shape of deformation profile will result in transverse internal forces (see fig. 1). Furthermore, the beam is subjected to internal forces acting in the loading direction due to its deflected shape (see fig. 2). The above forces combined with the longitudinal forces cause a complex triaxial, predominantly compressive state of stress in the compressive zone as indicated in fig. 3. In view of this triaxial state of stress which develops at load levels near the ultimate strength level, is it realistic to attempt to obtain, or indeed to expect, close predictions of ultimate load and corresponding deflexion of beams on the basis of a plane stress finite element analysis? (see page 496).

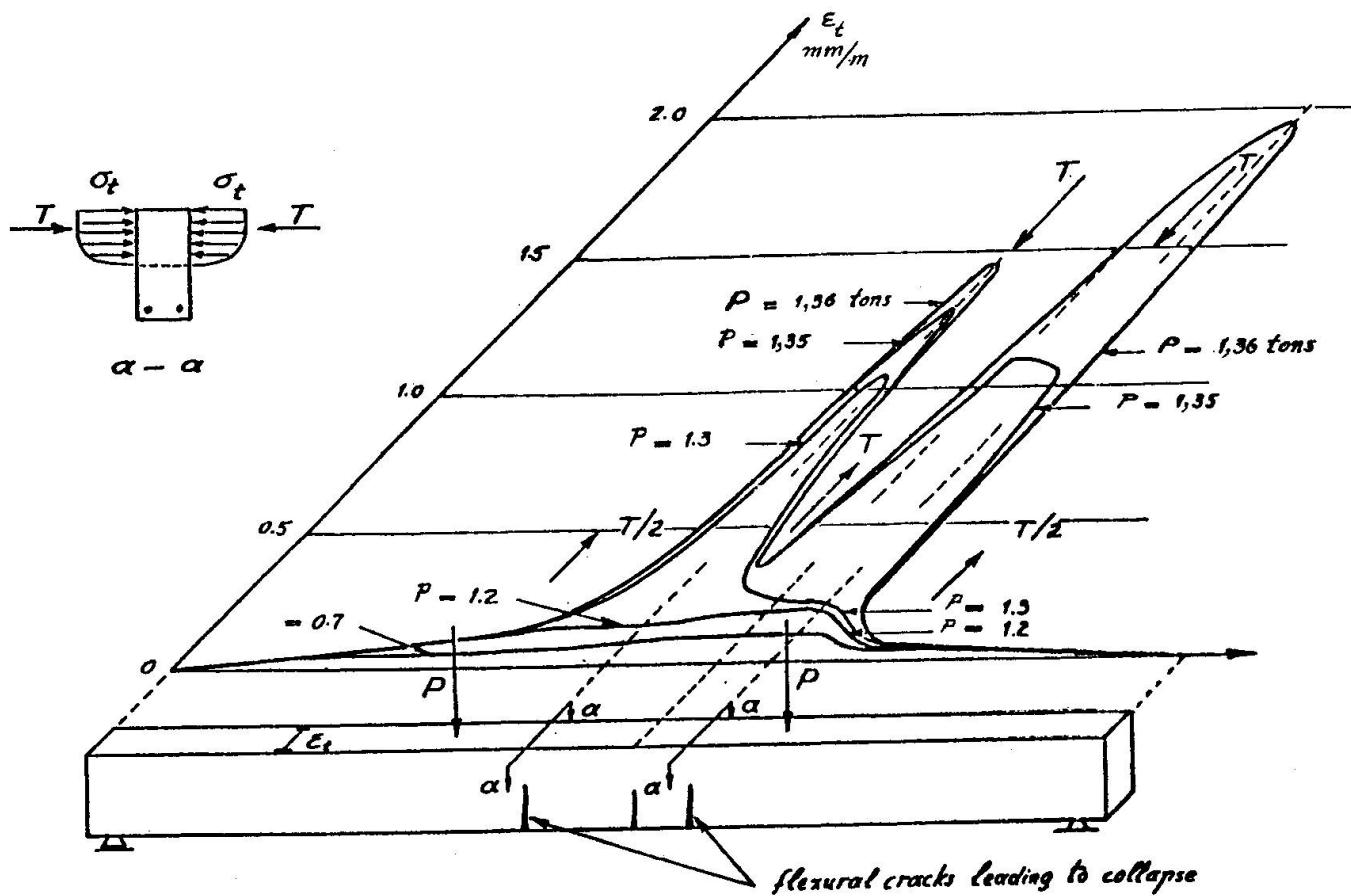


Fig. 1

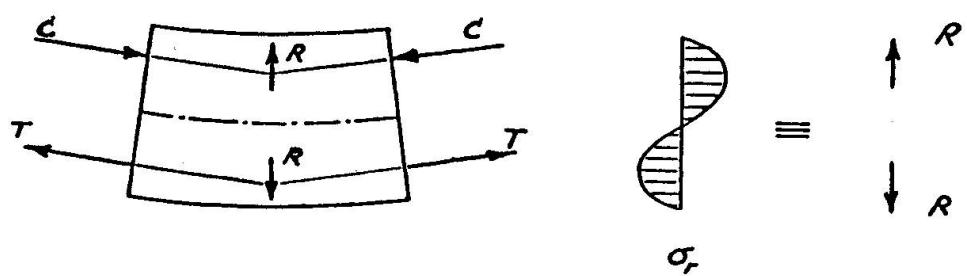


Fig. 2

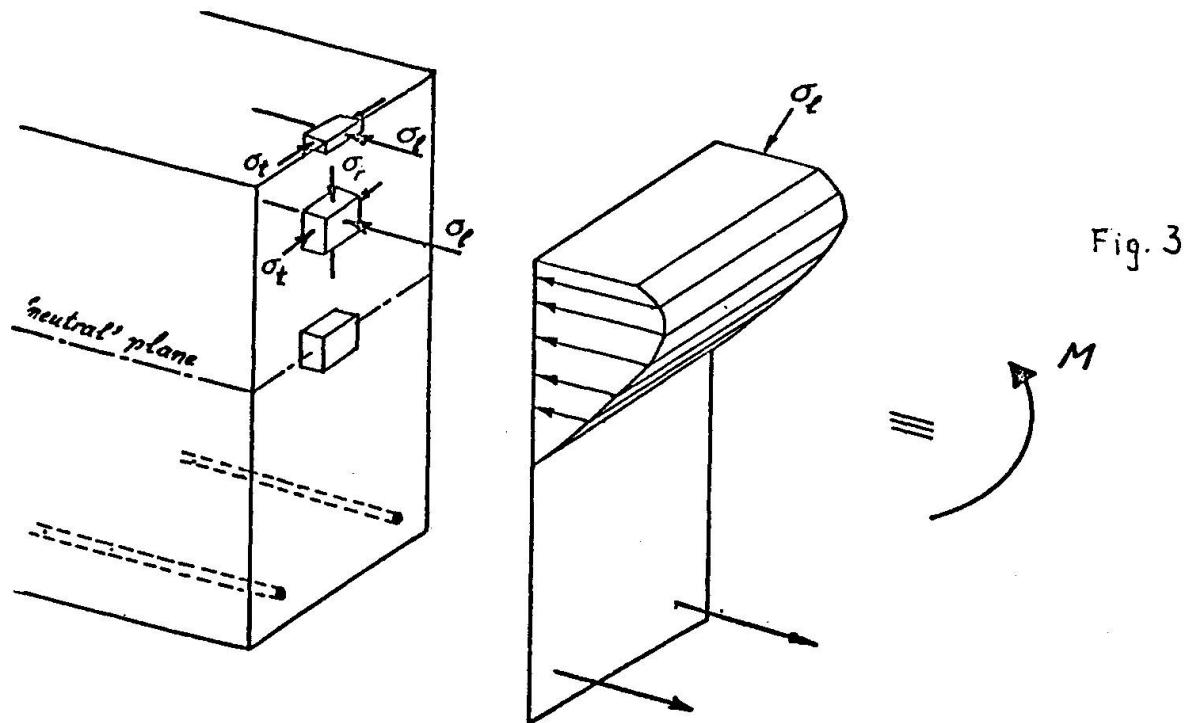


Fig. 3

Willam: Let me repeat, we performed a plane stress analysis where the stresses perpendicular to the plane of bending are assumed to be zero. You could of course think of a three-dimensional analysis and our programs are perfectly capable of performing a fully three-dimensional analysis, but we had already 1624 degrees of freedom in the case of the fine mesh with computer times in the range of several hours. So you can imagine what is going to happen if you would adopt a three-dimensional idealisation of the beam problem which is "normally" solved as a one-dimensional flexural problem. Perhaps our Japanese colleagues have sufficient computing power to tackle this problem, but we are at ISD somewhat limited by the capacity of our University computer center.

Braestrup (Denmark):

1. Does the tensile strength affect the ultimate load and why did you use such a high tensile strength?
2. Concerning the compressive strength why didn't you use a kind of effective strength block instead of the cylinder strength in order to get a better agreement with test results?

Willam: To your first question: The tensile strength does not have any influence on the ultimate load behaviour. It has an influence on the cracking load; this occurs on a lower level. In order to get a good agreement with the test data, we had to use a high tensile strength value from the rupture test since we used no tension softening, i.e. brittle post-cracking behaviour.

To your second question: Within the framework of the Mohr-Coulomb formulation, the program predicts a compressive strength block automatically if sufficient stress redistribution takes place. However, the form of the compressive stress block is also a question of mesh refinement; how many elements are available in order to pick up the non-linear stress distribution in the compressive zone? Anyway, this was the main reason for the considerable differences in the ultimate load of all reinforced concrete beams when predicted by the coarse and fine mesh lay-out.

Paper by Saouma/Ingraffea, U.S.A.

Cervenka (Tsjechoslovakia): In your analysis the spacing of cracks was dependent on the size of the mesh. So the analysis is not suitable for determination of crack location, but only shows crack propagation.

Saouma: The initiation of the crack is based on the point with the highest tensile stress. This tensile stress is compared with the tensile strength; when this is exceeded the crack is initiated.

Bazant comments on this that in the case of a homogeneous stress distribution you do not know where the crack is initiated when you use a strength criterion. The right answer comes from fracture mechanics. You have a certain minimum spacing, you can calculate energy contained in the structure before cracking and then you take a finite jump to the cracked state. That jump requires a certain consumption of energy for the formation of the crack. From this one can calculate the minimum crack spacing. That is how you have to start in my opinion. That is the final conclusion of prof. Bažant.

Cervenka (Tsjechoslovakia): If the mesh size is sufficiently fine in order to describe the stress concentration, then the analysis can properly determine the crack location.

Paper by Blaauwendraad/Grotenboer, The Netherlands

Ingraffea (U.S.A.): First a comment because you referred to the previous paper which involves my work and dr. Saouma's work. In fact our cracks are not at all constrained by the mesh. The mesh does exactly what the cracks want it to do.

Now the question:

What criterium or what method do you use to predict the angle at which the crack is going to propagate at each increment and how do you predict how far the crack will go in each load increment?

Blaauwendraad: I understood that your mesh is not dominating the position and the propagation of your cracks. But you have to make a new mesh. I hesitate to believe that we have to go this way, because it has enormous consequences regarding renumbering of elements and nodes, rearrangements of matrices etc. We succeeded in avoiding that. Concerning your question:

The direction in which the crack propagates is determined by the main principal tensile stress and when the crack enters into an element it immediately splits up the total element. It means that in practice we have to use a sufficiently fine element mesh to allow for this.

Bazant (U.S.A.): What is your opinion on the assumption that the crack goes in the principal stress direction or do you believe that there are also other criteria for crack propagation?

Blaauwendraad: Until now I have no reason to take another criterium. But it is open for better proposals. We use an aggregate interlock model which in fact is of a rigid plastic nature. The stress level at which slip occurs is dependent on the crack opening. This model can be changed in a model in which shift occurs first when a certain amount of crack opening is reached. We have to study if this means that we have to change the criterium for cracking. I dare not say it at this moment.

Saouma (U.S.A.): What is the reason of taking a reduction factor of 20% to 30% in your tensile strength, and do you use the same factor if you are analyzing another type of structure?

Blaauwendraad: The amount of reduction is a result of tuning the model. First we did not use the reduction and found erroneous results of crack propagation. After lowering the tensile strength to approximately 20% to 30%, we found in a number of situations, that this leads to satisfying results in practice. We have no information at this moment, whether or not this value is dependent on the stress situation or type of structure we examine. We did not find a typical correlation.

Paper by Dotreppe, Belgium

Fawzi (The Netherlands): Have you taken the modulus of elasticity as a constant? (see also written comment Fawzi on p.p. 501-506).

Dotreppe: I have not spoken about the material properties; this would have been too long for the paper. Of course you have to take into account the decrease of all material properties, such as compressive strength, tensile strength, modulus of elasticity and so on.

Paper by Hinton/Abdel Rahman/Zienkiewics, U.K.

Willam (F.R.G.): The quasi Newton-Raphson method is an approximation of the actual Newton-Raphson method, so you would expect that the quasi Newton-method needs more iterations than the Newton-Raphson method. You found the contrary. Why?

Hinton: The Newton-Raphson method does indeed require fewer iterations than the Quasi-Newton method. However, in the example quoted in the paper, when the Newton-Raphson method was adopted, on the penultimate load increment premature failure occurred. This was probably due to ill-conditioned stiffness matrices.

Crisfield (U.K.) (answer to question of Willam): The Newton-Raphson used is a method without line searches; the quasi Newton method is a method with line searches. If in both methods the line search is used you will find that the Newton-Raphson method will use less iterations than the quasi Newton method. This is the explanation.

Bazant (U.S.A.): Mathies and Strang developed an efficient method for elastic-plastic analysis with a sudden change of incremental stiffness. This method has been tried with the ADINA program with a great improvement in efficiency. In your problem, you have a gradual change in stiffness, so I cannot see how you can expect an improvement there.

Hinton: Our experience to date shows that for the non-linear analysis of reinforced concrete plates the best overall method is the Quasi-Newton method with line searches. However, it should be mentioned that our program PLASAN includes a large number of alternative solution strategies and we are still experimenting.

Crisfield (U.K.): I would like to make a brief comment on the arc length method. It seemed the best method in your limited examples. I have tackled the same problem with the arc length method. Indeed, for a reasonably high value of tension-stiffening it is effective. I have found that if the strain softening is coming down fast, you are then in trouble with the standard version of the arc length method. I think that this relates to the whole issue of line searches. My conclusion has been that the really important thing with concrete cracking is the line search business. It is difficult to introduce the line search into the standard versions of the arc length methods. But one certainly can take even a standard modified Newton-method and include the line search. For concrete analysis this will give a great improvement in performance.

Hinton: From subsequent analyses it was found that for problems with no tension stiffening, a slight ripple occurred in the load-displacement curve at the onset of cracking. However, after this happened, the solution progressed to the correct failure load. Further investigations are being carried out to determine the reasons for this unusual behaviour.

Proposal Collins (Canada): Issues an invitation to a blind test. There are several people here with models for the behaviour of reinforced concrete. All these models have many open parameters which have to be given values in order to get the model fit with the results of experiments. What we suggest now is that we ask for predictions before you know the answer of the tests. We are currently conducting a series of these panel tests that I explained yesterday. At the end of this summer we come to the more general stress situations, having a general plane stress situation in the reinforced concrete panel (see figure).

We ask for doing predictions for the behaviour of these panels. Write to us in Toronto what kind of input parameters you need to conduct your predictions.

In august 1981 we can provide you with all the material characteristics and the data on the reinforcement of the panels. We can tell how we plan to load the panel. Probably it will be a combination of shear stress and compression in one direction and tension the other way. What we are asking for is make a prediction for this experiment on what will be the maximum value of the load as expressed for instance by the shear; it will be monotonically increased to failure and the ratio's between the shears and the normal stresses will be maintained constant throughout the test. We want to receive your predictions in december 1981, so that we can publish the test results together with the various predictions in january 1982.

Bažant (U.S.A.): suggests a more complex loading path but Collins rejects this suggestion. His philosophy is "simple loading paths first".

## INVITATION TO A BLIND TEST

M.Collins, University of Toronto

Given:

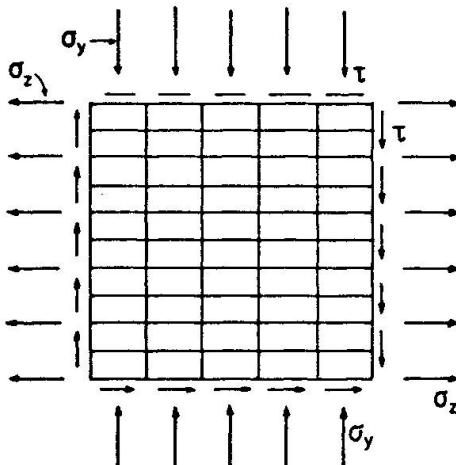
$\rho_x, \rho_y, \dots$  etc.

$f_{yx}, f_{yy}, \dots$  etc.

$f'_c, \epsilon_0, \dots$  etc.

$\sigma_x = k_x \cdot \tau$

$\sigma_y = k_y \cdot \tau$

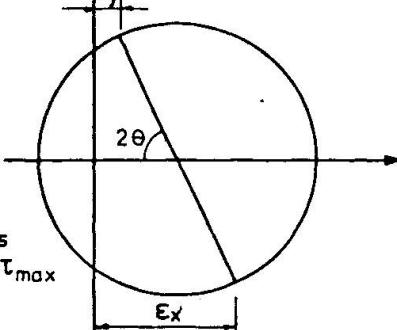


Predict:

$\tau_{max}$  and strains in panel at

$0,25 \tau_{max}, 0,50 \tau_{max},$

$0,75 \tau_{max}, 1,00 \tau_{max}.$



Timing:

Submit names + list of desired data : July 1981

Give out data : Aug. 1981

Receive predictions : Dec. 1981

Publish results : Jan. 1982

COMMENTS ON DR. DOTREPPE PAPER  
ADVANCED MECHANICS OF REINFORCED CONCRETE  
IN STRUCTURAL FIRE ANALYSIS

Prepared by  
Dipl.-Ing. A.E.R. Fawzi  
Consulting Engineer  
Rotterdam, The Netherlands

In the Paper of Dr. J. Dotreppe "Advanced mechanics of reinforced concrete-in Structural Fire Analysis" a numerical model for the evaluation of the fire resistance of reinforced concrete structures at elevated temperature is presented.

Under point 2 of the paper "Material properties" which is subdivided into: 1) Thermo-physical- and 2) Mechanical properties of materials. These material properties will be significantly affected due to the high temperatures, therefore, they have to be considered in the numerical model.

In a step by step numerical analysis, in addition to the classical mechanical properties, such as ultimate strength in tension and compression, yield stress and modulus of elasticity, the instantaneous stress-strain relationship as well as the thermal creep of concrete and steel have to be considered.

The stress-strain relationship for concrete at high temperature is indeed the basic topic for numerical analysis.

The heat function relation for the elevated heat propagation through the structural element

$$f(\sigma, \bar{\sigma}, \varepsilon, E, \theta, t) = 0 \text{ ----- (1)}$$

is characterizing the material behaviour under varying loading and temperature conditions.



The modulus of elasticity  $E$ , is not included in equation (1) of Dr. Dotreppe's paper, i.e. he assumed it as constant.

As a matter of fact the modulus of elasticity has to be determined for the test, and must be considered for the numerical analysis as varying with the time  $t$  in respect of the elevated temperature. The stress-strain curve in our problem is distinctly non-linear and may be represented by the following equation of P.H. Thomas

$$\frac{\sigma}{\sigma_{\max}} = \frac{\varepsilon}{\varepsilon_{\max}} \exp \left( \frac{1 - \varepsilon}{\varepsilon_{\max}} \right)$$

Over the whole range of  $\varepsilon/\varepsilon_{\max} > 0$

for the small values of  $\varepsilon/\varepsilon_{\max}$  the curve is approximately linear so that:

$$\frac{\sigma}{\sigma_{\max}} = C \frac{\varepsilon}{\varepsilon_{\max}}$$

where  $C$  is constant, i.e.  $\sigma = (C \frac{\sigma_{\max}}{\varepsilon_{\max}}) \varepsilon$

but  $\sigma = E\varepsilon$  (elastic range)

then  $E = C \cdot \frac{\sigma_{\max}}{\varepsilon_{\max}}$

The constant  $C$  can be determined from the values of  $E$  and  $\frac{\sigma}{\sigma_{\max}}$  at ambient temperature.

It is well known that  $E$  is the tangent of the stress-strain curve in elastic range which gives

$$C = 2.18 \text{ and } E = 2.18 \frac{\sigma_{\max}}{\varepsilon_{\max}} \text{ ----- (2)}$$

The variation of the modulus of elasticity  $E$  with temperature can be determined by using equation (2) by which  $\sigma_{\max}$  and  $\varepsilon_{\max}$  can be derived from stress-strain curves.

Actually the modulus of elasticity of concrete is influenced directly by the vaporization of the capillary water and the associated shrinkage phenomena.

At 200°C the reduction of the modulus of elasticity is about 25% of its original value at room temperature

$$\text{i.e. } \frac{E_{200^\circ\text{C}}}{E_{20^\circ\text{C}}} \approx 75\%$$

By increasing the temperature up to 500°C a significant reduction occurs in the range of 70%

$$\text{i.e. } \frac{E_{500^\circ\text{C}}}{E_{20^\circ\text{C}}} = 30\%$$

Since the range of our investigation is up to 800°C, there is a tremendous reduction factor in modulus of elasticity. The additional thermal stress equ. (9) in the paper.

$$\begin{aligned} \Delta \sigma = E \Delta \epsilon \text{ must be } \Delta \sigma = E_t \Delta \epsilon \\ = E_t (-\alpha \Delta \theta + \Delta \epsilon_0 + Z \Delta \chi) \end{aligned}$$

This is valid for Equ. (10) through (13).

The restrain stresses  $E \alpha \Delta \theta$  on page 299, must be  $E_t \alpha \Delta \theta$

The stiffness matrix ( $E_t I$ ) is directly related to the modulus of elasticity,  $E$ , as time, temperature dependent. This means that the stiffness of a concrete structural member diminishes at high temperatures and plastic hinges are capable of large rotational capacities, the ductility of concrete will be higher at elevated temperatures.

My last contribution point is about the thermal creep of concrete and steel. Dr. Dotreppe left this important topic open because of the complexity of the creep models for both materials. In the course of the last years, there are many different tests, studies and investigations about thermal creep under steady state, non steady state, isothermal creep and transitional thermal creep.

The transitional thermal creep "according to Illston and Sanders" which can fit in our model of analysis is defined as "the creep strains appearing during very short heating periods, which are far beyond those values according to creep measurements at constant temperatures. In my opinion, it is possible nowadays to predict with good accuracy the thermal creep laws.

One of the phenomenal studies on this subject, is the work of Dr. U. Schneider, "Ein Beitrag zur Klärung des Kriechens und der Relaxation von Beton unter instationärer Temperatureinwirkung".

Creep and transitional creep laws:

$$\left(\frac{\partial \varepsilon}{\partial t}\right)_\sigma = \sigma \cdot \left(\frac{\partial J(\bar{\sigma}, t)}{\partial t}\right)_\sigma \quad (3)$$

$$\varepsilon - \varepsilon_0 = \int_{\theta_0}^{\theta} \left[ \frac{\bar{\sigma} \left( \frac{\theta - \theta_0}{W} \right)}{E \left( \frac{\theta - \theta_0}{W} \right)} + \sigma \left( \frac{\theta - \theta_0}{W} \right) \cdot \left( \frac{\partial J}{\partial \theta} \right)_\sigma \right] d\theta \quad (4)$$

$$\varepsilon = \frac{\bar{\sigma}}{E(\theta)} + \bar{\sigma} \cdot J(\sigma, \theta) \quad (5)$$

$$J = \frac{1}{E(\theta)} \cdot \Phi(\bar{\sigma}, \theta) \quad (6)$$

$$\varepsilon = \frac{\bar{\sigma}}{E(\theta)} [1 + \Phi(\bar{\sigma}, \theta)] \quad (7)$$

Where  $\Phi(\bar{\sigma}, \theta)$  as creep function of concrete

$$\Phi = C_1 \tanh \gamma_1 (\theta - \theta_0) + C_2 \tanh \gamma_2 (\theta - \theta_1) + C_3$$

The parameters can be seen in Table 1.

See also fig. 1.

The general creep law for Reinforcement (Steel)

$$\dot{\varepsilon}_{cr}(\sigma) = C_1 \sigma^{C2} + t B \sigma^n$$

Where  $B$  = creep rate intercept at log stress 1.0, a material constant for a given temperature.

$C_1$  and  $C_2$  = Const. (material depend.)

The rupture time is:

$$\log T_r(\sigma) = C_3 + C_4\sigma \text{ (at a temperature)}$$

The time interval  $\Delta T_i$  with a stress  $\sigma_i$  the damage fraction equals

$$F_i = \frac{\Delta T_i}{T_{ri}(\sigma)}$$

See elevated temperature properties of carbon steel ASTM special tech. Publication No. 180.

Table 1  
Parameter for  $\phi$  function

Parameter	Dimension	Wert
$c_1$	1	2,51
$\gamma_1$	$^{\circ}\text{C}^{-1}$	$2,72 \cdot 10^{-3}$
$\vartheta_0$	$^{\circ}\text{C}$	$2,0 \cdot 10^1$
$c_2$	1	3,0
$\gamma_2$	$^{\circ}\text{C}^{-1}$	$7,5 \cdot 10^{-3}$
$\vartheta_1$	$^{\circ}\text{C}$	$6,0 \cdot 10^2$
$c_3$	1	2,9

Conclusion:

The influence of the Modulus of Elasticity and Thermal Creep on the numerical model for the analysis can not be smaller than the possible accuracy of measurements in such kind of tests.

I wonder how we could achieve such good agreement between theoretical analysis and experimental results.

Finally, I look forward to seeing further results along the lines of Dr. Dotreppe's work especially when the topics of my comments are included. A particularly valuable feature of this work is the development of the numerical analysis methods in parallel with the test programme on the structures he is analysing.

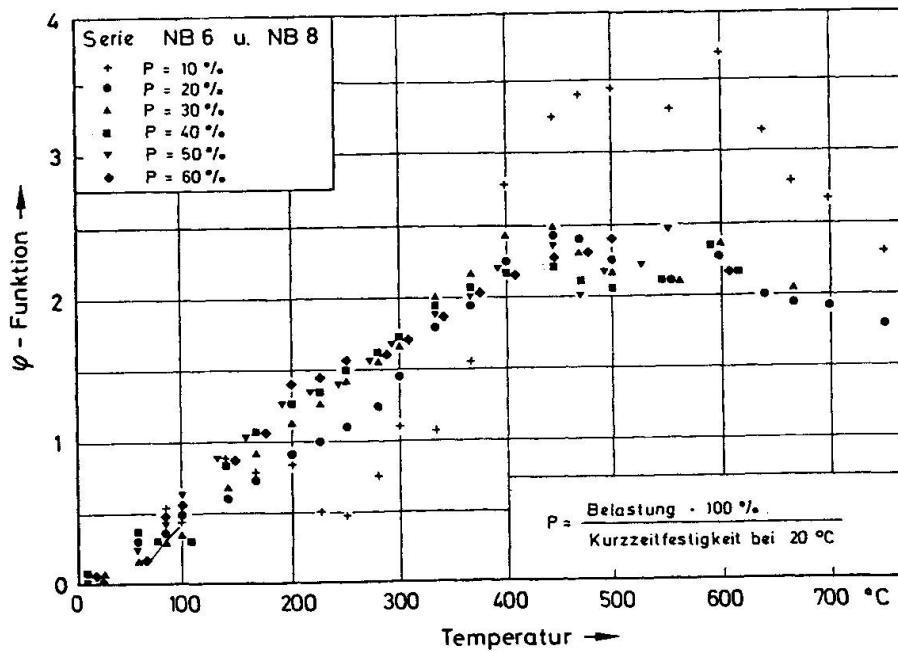


Fig. 1  $P = \frac{\text{load} \cdot 100\%}{\text{short time strength at } 20^\circ\text{C}}$